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Correction and Addendum : On Asymptotic Property of Matheron's Spatial Variogram Estimators

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First, we add in the footnote in the first page the acknowledgement "The research was supported by KOSEF through SRCCS at Seoul National University". The mailing address of the first author should read "Statistical Research Center for Complex Systems(SRCCS), Seoul 151-747, Korea".

Finally, we give some corrections for Theorem 3.2 and its proof.

Theorem 3.2. *For an 1-dimensional intrinsic stationary process* $\{Z(s) : s \in \mathbb{R}\}$ *having the variogram model,* $2\gamma_0(|h|)$, *assume that there exists a sequence* $\{ C_{2k} \}$ *of constants satisfying the following conditions.*

i) $\limsup_{k\to\infty} \left|\frac{C_{2k}}{(2k)!}\right|^{1/(2k)} = 0.$

ii) There exists a positive constant M, *such that* $\gamma_0(\cdot)$ *is analytic on* $[M,\infty)$ *and*

$$\int_{t>M} \left(\frac{\partial^{2k}}{\partial t^{2k}}\gamma_0(t)\right)^2 dt < C_{2k}^2, \quad \forall \ k \geq 1 \ .$$

Then, the normalized variogram estimator $\sqrt{N_n(h)}\cdot 2\hat{\gamma}_n(h)$ *has finite asymptotic variance,*

$$\sum_{u\in\mathbb{Z}} 2 \cdot \{2\gamma_0(|u|) - \gamma_0(|u+h|) - \gamma_0(|u-h|)\}^2.$$

Proof of Theorem 3.2

Under condition ii) $\gamma_0(t)$ is analytic for sufficiently large values of t, and here,

$$\gamma_0(t+h) = \sum_{k=0}^{\infty} \frac{{\gamma_o}^{(k)}(t)}{k!}h^k, \quad \forall\, t > M, \quad t+h > M \qquad \text{(A.4)}$$

when ${\gamma_o}^{(k)}(\cdot)$ means k-th derivative of $\gamma_o(\cdot)$.

From (A.4),

$$\left(2\gamma_0(t) - \gamma_0(t-h) - \gamma_0(t+h)\right)^2 = 4\left[\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} \frac{{\gamma_o}^{(2k)}(t){\gamma_o}^{(2l)}(t)}{(2k)!\,(2l)!} h^{2k+2l}\right], \quad \text{(A.5)}$$

for t, $t+h$, $t-h \in [M,\infty)$. The RHS term in (A.5) is obtained by simple arithmatics. The RHS has nonzero term only when both of indices k' and l' in the exponent of h are even. We used reparametrized the indices with $k' = 2k$ and $l' = 2l$.

In asymptotic variance, we need to sum (A.5) over the region $\{s : |s| > M\}$. This summation is approximated by 1-dimmensional integeration because of the symetric property of $\gamma_0(\cdot)$. Thus the asymptotic variance is bounded by

$$d\,M \;+\; d\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\int_{t>M} \left|\gamma_0^{(2k)}(t)\gamma_0^{(2l)}(t)\right| dt \frac{h^{2k+2l}}{(2k)!\,(2l)!}, \quad \text{(A.6)}$$

for a suitable constant $d > 0$. The integral term in (A.6) has *Cauchy-Schwarz* bound;

$$\int_{t>M}\left|\gamma_0^{(2k)}(t)\gamma_0^{(2l)}(t)\right| dt \le \left(\int_{t>M}\left(\gamma_0^{(2k)}(t)\right)^2 dt\right)^{\frac{1}{2}} \left(\int_{t>M}\left(\gamma_0^{(2l)}(t)\right)^2 dt\right)^{\frac{1}{2}}. \quad \text{(A.7)}$$

Now, the summation with respect to k and l in (A.6) are separated and we have terms :

$$\sum_{k=1}^{\infty}\left(\int_{t>M}\left(\gamma_0^{(2k)}(t)\right)^2 dt\right)^{\frac{1}{2}} \frac{h^{2k}}{(2k)!} \quad \text{and} \quad \sum_{l=1}^{\infty}\left(\int_{t>M}\left(\gamma_0^{(2l)}(t)\right)^2 dt\right)^{\frac{1}{2}} \frac{h^{2l}}{(2l)!} \quad \text{(A.8)}$$

To guarantee that the series in (A.8) have finite values for all h , the series in (A.8) have to have infinite radii of convergence with respect to h which is ensured by i).