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Solution of the Radiation Problem by the B-Spline Higher Order Kelvin Panel Method for an Oscillating Cylinder Advancing in the Free Surface

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Abstract

Numerical solution of the forward-speed radiation problem for a half-immersed cylinder advancing in regular waves is presented by making use of the improved Green integral equation in the frequency domain. The B-spline higher order panel method is employed since the potential and its derivative are unknown at the same time. The present numerical solution of the improved Green integral equation by the B-spline higher order Kelvin panel method is shown to be free of irregular frequencies which are present in the Green integral equation using the forward-speed Kelvin-type Green function.

Keywords: improved Green integral equation, time-harmonic forward speed Green function, forward-speed radiation-diffraction problem, B-spline higher order Kelvin panel method

1 Introduction

The numerical solution of the three-dimensional forward-speed radiation-diffraction problem in the frequency domain for a surface ship advancing in the free surface using the Kelvin-type Green function(Brard 1948) has been reported by many researchers(Chang 1977, Guevel and Bougis 1982, Inglis and Price 1982, Chan 1990). It seems that all these methods are not conclusive as pointed out by Sclavounos and Nakos(1990). It has been shown that a rigorous derivation of the line integral leads to the exact Green integral equation with the Kelvin-type Green function in the frequency domain(Hong 2000).

The solution of this exact Green integral equation is still susceptible to be non-unique since there may be irregular frequencies. In order to avoid the tremendous computational effort to find the irregular frequencies in the three-dimensional problem for a floating body, the two-dimensional forward-speed radiation problem in the frequency domain for a surface-piercing cylindrical body analogous to the three-dimensional problem, is treated in this paper by making use of the two-dimensional Kelvin-type Green function(Haskind 1954).

In a steady translational problem in two dimensions, the linearized free surface boundary condition breaks down since the perturbation velocity is no longer a small order of the incoming flow velocity at the stagnation points on the free surface, i.e., the juncture points piercing the free surface. The present two dimensional problem has similar difficulties. However, the results of present investigation in a two dimensional mathematical problem can be extended in a straightforward manner and applied to the three dimensional problem of physical interest.

The B-spline higher order panel method is employed since the potential and its derivative are unknown at the same time in the integral equation containing the free surface line integrals. Simultaneous resolution of this kind of integral equation is hard to be carried out by the low order panel method. The present numerical results show irregular frequencies in the two-dimensional forward-speed Green integral equation associated with the Kelvin-type Green function in the frequency domain while the solution of the improved Green integral equation is free of the irregular frequencies.

2 Formulation of the problem

The fluid is bounded by the mean wetted surface S of a surface-piercing body and by the mean free surface F defined by $y = 0$,of deep water under gravity. The body performs simple harmonic oscillations of small amplitude with circular frequency ω about its mean position which is moving with a steady horizontal velocity $\overrightarrow{u}$. Cartesian coordinates(x,y) attached to the mean position of the body, are employed with the origin O in the waterplane W of the body at its mean position and the y axis vertically upwards.

With the usual assumptions of an incompressible fluid and irrotational flow without capillarity, the fluid velocity is given by the gradient of a velocity potential. The boundary condition is linearized by assuming that both the magnitude of unsteady flow and the magnitude of steady flow due respectively to the oscillation and steady translation of the cylinder are small enough to neglect their products. Since the problem is linear, the governing equations for the unsteady potential

$$\Phi(x,y,t) = Re\{\Psi(x,y)e^{-i\omega t}\}$$

can be given as follows:

$$\nabla^2\Psi = 0 \quad \text{in fluid region} \tag{1}$$

$$\frac{\partial\Psi}{\partial n} = [-i\omega a_1\overrightarrow{e}_1 + (-i\omega a_2 - ua_3)\overrightarrow{e}_2 - i\omega a_3\overrightarrow{e}_3 \times \overrightarrow{OM}]\cdot\overrightarrow{n} \quad \text{on} \quad S \tag{2}$$

$$[(-i\omega - u\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial y}]\Psi = 0 \quad \text{on} \quad y = 0 \tag{3}$$

where $a_k(k = 1,2,3)$ denote complex amplitude of surge, heave and pitch motions respectively, O the center of rotation of the body, n the normal vector directed into the fluid region from S and g the gravitational acceleration. The real motions are

$$a_k = Re\{a_k e^{-i\omega t}\}.$$

The potential must also satisfy the radiation condition at infinity.
Introducing elementary potentials $\psi_k(k = 1,2,3)$, Ψ can be expressed as follows

$$\Psi = -i\omega\sum_{k=1}^{3} a_k\psi_k - ua_3\psi_2 \tag{4}$$

Taking into account of (2) and (4), the following body boundary conditions for can be found:

$$\frac{\partial \psi_k}{\partial n} = n_k, \qquad k = 1, 2 \tag{5a}$$

$$\frac{\partial \psi_3}{\partial n} = (\vec{e}_3 \times \overrightarrow{OM}) \cdot \vec{n}, \qquad M \quad \text{on} \quad S \tag{5b}$$

Other boundary conditions for ψ_k are identical to those for Ψ.

3 Improved green integral equation

When a cylindrical body is present in the free surface, the fluid region D_e is bounded by the mean wetted surface of the body S, the outer free surface $F_e = F - W$ and some arbitrary surface S_∞ at infinity. Applying Green's theorem to the potential ψ and the Green function G over the fluid region D_e, the following Green integral equation(GIE) can be obtained in two dimensions. The similar derivation of a three dimensional problem is given in Appendix:

$$\begin{aligned} &\frac{\psi}{2} + \int_S \psi \frac{\partial G}{\partial n} dl - 2i\gamma[(\psi G)_C - (\psi G)_D] + \frac{u^2}{g}[(\psi \frac{\partial G}{\partial x})_C \\ &- (\psi \frac{\partial G}{\partial x})_D - (\frac{\partial \psi}{\partial x} G)_C + (\frac{\partial \psi}{\partial x} G)_D] = \int_S \frac{\partial \psi}{\partial n} G dl \quad \text{on} \quad S \end{aligned} \tag{6}$$

where $\gamma = \omega \cdot u/g$ denotes the Brard number and the suffixes C and D the two intersecting points of S and $y = 0$ as shown in the Figure 1, where two-dimensional line integrals are to be calculated.

The two dimensional Kelvin type Green function G was first presented by Haskind(1954) under integral form and its integrated form has been given by Hong(1978) as follows:

$$G(z_P, z_M) = \frac{1}{2\pi} Re\{log(\frac{z_P - z_M}{z_P - \bar{z}_M}) + I_1 + I_2\} + iIm\{I_1 - I_2\} \tag{7}$$

with

$$I_1 = \frac{1}{\sqrt{(1+4\gamma)}}\{e^{\zeta_2}[Em_1(\zeta_2) + 2i\pi] - e^{\zeta_1}[Em_1(\zeta_1) + 2i\pi]\} \tag{8}$$

$$I_2 = \begin{cases} \frac{1}{\sqrt{(1-4\gamma)}}\{e^{\zeta_4}[Em_1(\zeta_4) + 2i\pi] - e^{\zeta_3} Em_1(\zeta_3)\} & \text{for} \quad \gamma < \frac{1}{2}, \\ \frac{1}{\sqrt{(1-4\gamma)}}[e^{\zeta_4} E_1(\zeta_4) - e^{\zeta_3} E_1(\zeta_3)] & \text{for} \quad \gamma \geq \frac{1}{2}. \end{cases} \tag{9}$$

$$\zeta_j = -ik_j(z_P - \bar{z}_M), \qquad j = 1, 2, 3, 4 \tag{10}$$

$$k_j = \frac{g}{2u^2}[1 + 2\gamma + (-1)^j \sqrt{(1+4\gamma)}], \qquad j = 1, 2 \tag{11a}$$

$$k_j = \frac{g}{2u^2}[1 - 2\gamma + (-1)^j \sqrt{(1-4\gamma)}], \qquad j = 3, 4 \tag{11b}$$

where $z = x + iy$ is the complex plane and z_P, z_M denote source and field points respectively. $E_1(\zeta)$ is the complex exponential integral and $Em_1(\zeta)$ the modified complex exponential integral defined as follows:

$$Em_1(\zeta) = \begin{cases} E_1(\zeta) & \text{for } Im(\zeta) > 0, \\ E_1(\zeta) - 2i\pi & \text{for } Im(\zeta) < 0. \end{cases} \tag{12}$$

Numerical results of (6) using the Green function (7) have shown to be satisfactory for an oscillating circular cylinder advancing under the free surface where the line integrals are absent(Hong 1988). But for a surface-piercing body, it seems that some boundary conditions are missing in (6). According to the theory of integral equation, an integral equation must contain all the boundary conditions of the boundary value problem in question. Let the surface in contact with the fluid be the positive side of the boundary surface and the other side of the same surface outside D_e the negative side of the surface. The wetted surface will be denoted by S^+ hereafter. According to the potential theory, the potential jump across S which has been incorporated in (6) implies that the condition $\psi = 0$ is imposed on S^-, the negative side of S. In fact, it was necessary to impose $\psi(P) = 0$ when P lies on the negative side of the closed surface $S \bigcup F_e \bigcup S_\infty$ as it is done in the Green integral equation with the Rankine-type Green Function. Thus, in order to ensure the uniqueness of the solution, it is necessary to impose the following condition:

$$\psi = 0 \qquad \text{on} \qquad F_e^- \tag{13}$$

where F_e^- denotes the negative side of F_e.
The condition on S_∞ can be omitted since ψ on S_∞ vanishes in the limit.
But, since the integral over the boundary surface F_e was already replaced by line integrals, it is not desirable to reintroduce F_e into the present Green integral equation. Instead, let us impose the following supplementary condition for ψ which can compensate for the condition (13):

$$\psi(P) = 0 \qquad \text{for} \qquad P \in W \tag{14}$$

This condition is equivalent to

$$\begin{aligned}\int_S \psi \frac{\partial G}{\partial n} dl - 2i\gamma[(\psi G)_C - (\psi G)_D] + \frac{u^2}{g}[(\psi \frac{\partial G}{\partial x})_C - (\psi \frac{\partial G}{\partial x})_D \\ - (\frac{\partial \psi}{\partial x} G)_C + (\frac{\partial \psi}{\partial x} G)_D] = \int_S \frac{\partial \psi}{\partial n} G dl \quad \text{on} \quad W\end{aligned} \tag{15}$$

Combining the (6) with (15), we have the following integral equation, say, the improved Green integral equation(IGIE) of overdetermined type(Hong and Lee 1999):

$$\begin{aligned}\alpha(P)\psi(M) + \int_S \psi(M) \frac{\partial G(P,M)}{\partial n_M} dl_M - 2i\gamma[(\psi(M)G(P,M))_C - (\psi(M)G(P,M))_D] \\ + \frac{u^2}{g}\{[\psi(M)\frac{\partial G(P,M)}{\partial x_M}]_C - [\psi(M)\frac{\partial G(P,M)}{\partial x_M}]_D - [\frac{\partial \psi(M)}{\partial x_M} G(P,M)]_C \\ + [\frac{\partial \psi(M)}{\partial x_M} G(P,M)]_D\} = \int_S \frac{\partial \psi(M)}{\partial n_M} G(P,M) dl_M\end{aligned} \tag{16}$$

where

$$\alpha(P) = \begin{cases} \frac{1}{2} & \text{for } P \in S, \\ 0 & \text{for } P \in W. \end{cases} \tag{16a}$$

4 B-spline higher order panel method

We will represent the potential and its derivative as a weighted sum of B-spline basis functions as follows;

$$\psi = \sum_{j=0}^{N^v-1} \psi_j^v N_j(u) \quad \text{on} \quad S\bigcup W \tag{17}$$

$$\frac{\partial\psi}{\partial x} = \sum_{j=0}^{N^v-1} \psi_j^v \frac{\partial N_j(u)}{\partial x} \quad \text{at} \quad C \quad \text{and} \quad D \tag{18}$$

where $N_j(u)$ are the p-th degree B-spline basis functions, ψ_j^v the potential control vertices.

The unknowns of the problem are now the values of the potential vertices, ψ_j^v, which are not the potential in the physical sense. It should also be noted that the representation (18) permits us to solve the present problem in a rigorous manner, where the potential and its derivative are unknowns at the same time.

Discretization of the body surface in (16) into a set of curvilinear segments s, $(s = 0, 1, 2, , , N^S - 1)$, will then yield(Lee and Kerwin 1999)

$$\begin{aligned}
&\alpha(P)\sum_j \psi_j^v N_j + \sum_{s=0}^{N^S-1}\sum_{j=0}^{N^v-1} \psi_j^v \int_S N_j \frac{\partial G}{\partial n} dl - 2i\gamma[(\psi^v G)_C - (\psi^v G)_D] \\
&+ \frac{u^2}{g}[(\psi^v \frac{\partial G}{\partial x})_C - (\psi^v \frac{\partial G}{\partial x})_D] - \frac{u^2}{g}\{G_C[\sum_{j=0}^{N^v-1} \psi_j^v \frac{\partial N_j}{\partial x}]_C \\
&- G_D[\sum_{j=0}^{N^v-1} \psi_j^v \frac{\partial N_j}{\partial x}]_D\} = \sum_{s=0}^{N^S-1} \int_S \frac{\partial\psi}{\partial n} G dl, \quad P \in S\bigcup W
\end{aligned} \tag{19}$$

The number of unknown potential vertices N^v is greater than the number of the curvilinear segments or panels since $N^v = N^S + p$ according to the properties of the B-spline basis functions. Since the (16) is overdetermined, we can place any number of control points N^P on $S \bigcup W$ which is greater than or equal to N^v. This linear system will be solved by the usual Gauss elimination when $N^P = N^v$ and by a least square approach when $N^P > N^v$(Hong and Lee 1999).

5 Numerical results and discussion

The hydrodynamic pressure forces due to the unsteady potential can be obtained as

$$\vec{F} = -\int_S p\vec{n}\, dl \tag{20}$$

The pressure can be obtained by making use of the Bernoulli equation:

$$p = -\rho(\frac{\partial\Phi}{\partial t} - u\frac{\partial\Phi}{\partial x}) \tag{21}$$

Substituting (21) into (20) and introducing non-dimensional added-mass and wave-damping coefficients M_{ik} and D_{ik}, we have the following expression for F_i:

$$F_i = -\rho A \sum_{k=1}^{3} [M_{ik}\ddot{a}_k + \omega D_{ik}\dot{a}_k], \qquad i = 1, 2 \tag{22}$$

where denotes the sectional area of the cylinder.

The hydrodynamic coefficients of a half immersed circular cylinder are computed for various Froude numbers, $F_n = \frac{u}{\sqrt{gD}}$, based on the diameter of the circle $D = 1$. They are all plotted as functions of $K = \omega\sqrt{D/g}$.

After extensive numerical tests for the convergence, 30 higher order panels with 60 control points in S are employed as well as 38 control points in W to show the present numerical values for four different Froude numbers: $F_n = 0.0001$. $F_n = 0.05$, $F_n = 0.075$ and $F_n = 0.099$. The last Froude number is chosen in order to see the behaviour of the numerical solution in the vicinity of the critical Brard number $\gamma_c = 1/4(K \cdot F_n = 1/4)$ and to compare the present results with those in the time-domain with Rankine panel method reported by Prins and Hermans(1994). The first Froude number is chosen to compare the present results with numerical results at zero forward speed(for example, (Hong and Lee 1999) and it has been found to be practically equal.

We have found numerically that the irregular frequency exists in the Green integral equation (6) while the improved Green integral equation (16) is free of the irregular frequency as shown in Figures 2 through 13. The hydrodynamic coefficients calculated from the solution of (6) are denoted by GIE and those from (16) by IGIE in the figures. We see also continuous fluctuation in the curves denoted by GIE for $F_n \geq 0.075$.

The changes of hydrodynamic coefficients calculated from the solution of the improved Green integral equation, with respect to the Froude number have been presented in Figures 14 through 17. We see that the curves for $F_n \leq 0.075$ are smooth but the curves for $F_n = 0.099$ fluctuate as the Brard number approaches $\gamma_c(K \simeq 2.5 \quad \text{for} \quad F_n = 0.099)$. The peak values at γ_C are not surprising since the Green function fails there as shown in (9). But for $\gamma > \gamma_C$, the curves denoted by $F_n = 0.099$ are not reliable. The present body intersects at right angle with the mean free surface and the horizontal derivative of the potential is well known divergent at C and D. Therefore even though the numerical solution of (16) is bounded, its derivative with respect to x may not be determined. Thus the derivative intervening in the calculation of the pressure in (21), would result in the unreliable values of hydrodynamic coefficients for relatively high Froude numbers where the role of line integrals gets more important. In Figures 18 through 21 the real and imaginary values of potential as a function of x-coordinates for $F_n = 0.0001$ and $F_n = 0.099$, are presented to verify the diverging phenomena of the horizontal derivative of the potential at C and D.

The present numerical results are considered similar to those of Prins and Hermans for $F_n = 0.099$, but clearly differ in values for $\gamma < \gamma_C$. Prins and Hermans have not reported their numerical results for $\gamma > \gamma_C$. Direct comparison between Prins and Hermans and ours is not advisable, since they linearized the free surface boundary condition with respect to the steady double-body potential. This is the major source of differences between the two numerical results.

Some numerical results using the low order panel method has also been obtained but they are not comparable to the results computed by the B-spline higher panel method since they do not converge no matter how great the number of panels may be. It seems that it is hard to obtain useful results by making use of the low order panel method when the potential and its derivative are unknown at the same time in the integral equation.

6 Conclusions

1. The exact formulation of the Green integral equation associated with the Kelvin-type Green function in the frequency domain for an oscillating surface-piercing body advancing in the free surface, has been presented with correct free surface boundary conditions.

2. It has been shown that there exist irregular frequencies in the solution of the two-dimensional Green integral equation for a surface-piercing cylinder, which occur far more frequently than the irregular frequencies in the zero-speed Green integral equation.

3. The solution of the improved Green integral equation for an oscillating surface-piercing cylinder in steady horizontal translation, is shown to be free of irregular frequencies.

4. Finally, it has been shown that the present numerical method, say, the B-spline higher order Kelvin panel method provides accurate numerical solutions of the integral equations where the potentials and their derivatives are unknown at the same time.

The present method is equally applicable to the three-dimensional problem, and the result will be less critical, even for higher Froude numbers, to the numerical procedure since the line integrals are less singular than in two-dimensional problem.

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Appendix

Exact Formulation of the Green Integral Equation for the Three-Dimensional Forward-Speed Radiation-Diffraction Problem

A.1 Linearized boundary-value problem in the frequency domain

A ship is moving with mean forward speed U in the free surface of deep water under gravity and in the presence of plane progressive sinusoidal incident wave of small amplitude a_0. Let $oxyz$ be a Cartesian co-ordinate system attached to the mean position of the ship, with z vertically upward, x in the direction of forward motion and o in the mean waterplane W. The ship performs simple harmonic oscillations of small amplitude about its mean position with circular frequency ω which is equal to the encounter frequency of incident wave. It is assumed that the disturbance of the free surface due to the forward motion is also small.

With the usual assumptions of the incompressible, inviscid fluid and irrotational flow without capillarity, the fluid velocity can be given by the gradient of a velocity potential Φ which satisfies the Laplace equation,

$$\nabla^2\Phi = 0 \tag{A1}$$

in the fluid region.

Under the assumptions given above, Φ at P in the fluid region can be decomposed as follows:

$$\Phi(P,t) = \Phi_S(P) + Re\{\Psi(P)e^{-i\omega t}\} \tag{A2}$$

where Φ_S denotes a steady potential known as the Neumann-Kelvin potential, Ψ a complex-valued unsteady potential and ω the encounter frequency of the incident wave. The velocity potential of incident wave is as follows:

$$\Phi_0 = Re\{\Psi_0 e^{-i\omega t}\} \tag{A3}$$

where

$$\Psi_0 = -\frac{a_0 g}{\omega_0} e^{k_0[z+i(x\cos\beta+y\sin\beta)]} \quad \text{for} \quad \omega = (\omega_0 - Uk_0\cos\beta) > 0 \tag{A4}$$

and

$$\Psi_0 = -\frac{a_0 g}{\omega_0} e^{k_0[z-i(x\cos\beta+y\sin\beta)]} \quad \text{for} \quad \omega = (Uk_0\cos\beta - \omega_0) > 0 \tag{A5}$$

where g is the gravitational acceleration, β the angle between the phase velocity of the incident wave and the forward velocity of the ship, ω_0 the circular frequency of incident wave and $k_0 = \frac{\omega_0^2}{g}$ the wavenumber expressed in a space-fixed co-ordinate system $\overline{oxyz}$ given as follows:

$$\overline{x} = x + Ut, \quad \overline{y} = y, \quad \overline{z} = z \tag{A6}$$

The equation of the mean free surface is

$$z = 0 \tag{A7}$$

and the linearized free surface boundary condition for Φ on $z = 0$ is as follows:

$$\left[(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}\right]\Phi = 0 \quad \text{on} \quad z = 0 \tag{A8}$$

Substitution of (A2) to (A8) yields the following free surface boundary conditions for Φ_S and Ψ respectively:

$$\left[U^2\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial z}\right]\Phi_S = 0 \quad \text{on} \quad z = 0 \tag{A9}$$

$$\left[(-i\omega - U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}\right]\Psi = 0 \quad \text{on} \quad z = 0 \tag{A10}$$

The forward speed U is of $O(1)$. Under the assumption of small amplitude oscillation, the displacement vector $\overrightarrow{A}(M)$ of a point M on the wetted surface S of the ship in its mean position is of $O(\varepsilon)$ where ε, being as small as the wave slope, is the measure of smallness in the present study. The expression of $\overrightarrow{A}(M)$ is as follows:

$$\overrightarrow{A}(M) = Re\{\overrightarrow{\alpha}(M)e^{-i\omega t}\}, \quad M \in S \tag{A11}$$

$$\overrightarrow{\alpha}(M) = \sum_{k=1}^{3} a_k \overrightarrow{e}_k + \overrightarrow{\theta} \times \overrightarrow{OM}, \quad M \in S \tag{A11a}$$

$$\overrightarrow{\theta} = \sum_{k=4}^{6} a_k \overrightarrow{e}_{k-3} \tag{A11b}$$

where $a_k(k = 1, 2, , , 6)$ denotes complex valued amplitude of surge, sway, heave, roll, pitch, yaw respectively and O the center of rotation of the ship.
It should be noted that the time-harmonic quantities correspond to the real part of terms involving $e^{-i\omega t}$ which will not be shown hereafter unless its presence is necessary.
Applying impermeability condition on S, the following body boundary condition can be found:

$$(\vec{n} + \vec{\theta} \times \vec{n}) \cdot \nabla(\Phi_S + \Psi) = (\vec{n} + \vec{\theta} \times \vec{n}) \cdot (U\vec{e_1} - i\omega\vec{\alpha}) \tag{A12}$$

where $\vec{n}$ denotes a unit normal to S directed into the fluid region, at its mean position and $(\vec{n} + \vec{\theta} \times \vec{n})$ the Taylor expansion of the normal at its instantaneous position to the first order. The above condition can also be found from its alternative expression given in the literature(Timman and Newman 1962).
Assuming Φ_S is of $O(\varepsilon)$ and neglecting second-order quantities, the following linearized body boundary condition for Φ_S and Ψ can be found respectively:

$$\frac{\partial \Phi_S}{\partial n} = Un_1 \quad \text{on} \quad S \tag{A13}$$

$$\frac{\partial \Psi}{\partial n} = -i\omega\vec{\alpha} \cdot \vec{n} + U(a_5 n_3 - a_6 n_2) \quad \text{on} \quad S \tag{A14}$$

With these linearized boundary conditions on S and on $z = 0$, the unsteady potential and the steady potential problems can be solved independently and the latter will be dropped from the present study.
The unsteady potential Ψ can further be decomposed as follows:

$$\Psi = \Psi_0 + \Psi_7 + \Psi_R \tag{A15}$$

where the sum of Ψ_0 and Ψ_7 is known as the diffraction potential and Ψ_R the radiation potential which can be decomposed as follows:

$$\Psi_R = -i\omega \sum_{k=1}^{6} a_k \Psi_k - U(a_6\Psi_2 - a_5\Psi_3) \tag{A16}$$

Then the body boundary conditions for $\Psi_k(k = 1, 2, , , 7)$ are

$$\frac{\partial \Psi_k}{\partial n} = n_k \quad \text{on} \quad S, \quad k = 1, 2, 3 \tag{A17a}$$

$$\frac{\partial \Psi_k}{\partial n_0} = (\vec{e}_{k-3} \times \overrightarrow{OM}) \cdot \vec{n} \quad \text{on} \quad S, \quad k = 4, 5, 6 \tag{A17b}$$

$$\frac{\partial \Psi_7}{\partial n} = -\frac{\partial \Psi_0}{\partial n} \quad \text{on} \quad S \tag{A18}$$

The potentials $\Psi_k(k = 1, 2, , , 7)$ also satisfy the free surface boundary condition given by the equation (A10):

$$\left[(-i\omega - U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}\right] \Psi_k = 0 \quad \text{on} \quad F, \quad k = 1, 2, , , 7 \tag{A19}$$

It is also assumed that they vanish at infinity as $\frac{1}{r^\infty}$ where r^∞ denotes the distance from the ship. They must also satisfy the radiation condition presented by Brard(1948).

Here, the potential boundary-value problem for $\Psi_k(k = 1, 2, , , 7)$ will be solved by making use of the Green integral equation.

A.2 Brard's green function

The Green function derived by Brard(1948) characterizes the potential induced at P by a pulsating source of unit strength at M advancing under the free surface with uniform velocity $U\vec{e_1}$. The point M is the so-called source point and P the field point. It has been obtained as follows:

$$G(P, M, t) = Re\{G(P, M)e^{-i\omega t}\} \tag{A20}$$

where

$$G(P, M) = G_0(P, M) - G_1(P, M) + G_f(P, M) \tag{A21}$$

$$G_j(P, M) = -\frac{1}{4\pi}\frac{1}{r_j}, \quad j = 0, 1 \tag{A22}$$

$$r_j = \{(x_P - x_M)^2 + (y_P - y_M)^2 + (z_P - (-1)^j z_M)^2\}^{\frac{1}{2}}, \quad j = 0, 1 \tag{A23}$$

$$G_j(P, M) = \frac{1}{4\pi^2}(H_1 + H_2) \tag{A24}$$

$$H_l = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta \int_0^{\infty} \frac{1}{D_j} e^{\bar{\zeta}} gkdk, \quad l = 1, 2 \tag{A25}$$

$$D_l = [\omega - (-1)^{(l+1)}Uk\cos\theta]^2 - gk + i\nu[\omega - (-1)^{(l+1)}Uk\cos\theta], \quad l = 1, 2 \tag{A26}$$

$$\zeta = k\{z_P + z_M + i[(x_P - x_M)\cos\theta + (y_P - y_M)\sin\theta]\} \tag{A27}$$

where ν is an artificial damping parameter infinitely small, positive, which will determine the path of integration in the complex plane K associated with the variable k shown in the expressions of H_1 and H_2.
The function G_0 is the Rankine-type Green function which is singular when $P = M$ and regular otherwise. The function G_i and G_f are regular for $z_P \leq 0$.

Brard's Green function satisfies the following equations:

$$\nabla_P^2 G(P, M) = 0 \quad \text{for} \quad P \neq M \tag{A28}$$

$$\left[(-i\omega - U\frac{\partial}{\partial x_P})^2 + g\frac{\partial}{\partial z_P}\right] G(P, M) = 0 \quad \text{for} \quad z_M < 0 \quad \text{and} \quad z_P \leq 0 \tag{A29}$$

In fact, Brard derived first a Green function $F(\overline{x}_M, \overline{y}_M, \overline{z}_M, \overline{x}_P\overline{y}_P\overline{z}_P, t)$ in a space-fixed coordinate system $\overline{oxyz}$, associated with a source whose intensity and horizontal speed are arbitrary functions of time. The so-called damped free surface condition given below was used to construct $F(P, M, t)$:

$$\left(\frac{\partial^2}{\partial t^2} + \nu\frac{\partial}{\partial t} + g\frac{\partial}{\partial \overline{z}_P}\right) F(P, M, t) = 0 \quad \text{on} \quad z_P = 0 \tag{A30}$$

Next, $G(P, M, t)$ was derived from $F(P, M, t)$. Finally, he presented a radiation condition for $G(P, M, t)$ through the analysis of its far-field behavior by making use of the Cauchy-Lord Kelvin's principle of stationary phase. Thus, it can be said that the radiation condition for $G(P, M)$ is satisfied when the artificial damping parameter is present in the denominators D_1 and D_2. Since the Green function $G(P, M)$ is of $O(\frac{1}{r})$, it tends to zero as $r \to \infty$.

The Green function $G(P,M)$ also satisfies the so-called adjoint free surface condition

$$\left[(-i\omega + U\frac{\partial}{\partial x_M})^2 + g\frac{\partial}{\partial z_M}\right] G(P,M) = 0 \quad \text{for} \quad z_P < 0 \quad \text{and} \quad z_M \leq 0 \qquad \text{(A31)}$$

which can be derived from the free surface condition (A29) according to the reciprocal property of the forward-speed Green function(Timman and Newman 1962, Brard 1972).

The integrations with respect to k in H_1 and H_2 can be done analytically by making use of the complex exponential integral $E_1(\zeta)$ Hong(1978). This method of integration was generalized by Guevel(1979) and was applied to the three-dimensional radiation-diffraction problem with forward speed by Bougis(1980).

A.3 Green integral equation

When a ship is present in the free surface, the fluid region D_e is bounded by the mean wetted surface of the ship S, the outer free surface $F_e = F - W$ and some arbitrary surface S_∞ at infinity. Let C and C_∞ denote the closed intersection contours of F with S and S_∞ respectively. Applying Green's theorem to the potential Ψ and the Green function G over the fluid region D_e, the following integral identities can be obtained:

$$\alpha\Psi(P) = -\int\int_{BS} [\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)]ds, \quad \text{for} \quad z_P \leq 0 \qquad \text{(A32)}$$

where $\vec{n}$ denotes a unit normal to the boundary surface directed into the fluid region D_e and $BS = S \bigcup F^e \bigcup S^\infty$.

The number α in the left-hand side of (A32) takes the value of 1, $\frac{1}{2}$ or 0 according as the field point P lies inside, on and outside the closed surface BS. The Ψ and $\frac{\partial\Psi}{\partial n}$ on the boundary surface denote the densities of sources and normal doublets known as the fundamental hydrodynamic singularities, distributed over there.

Since Ψ and G tend to zero as $\frac{1}{r^\infty}$, the integral over S_∞ vanishes in the limit and, in D_e, we have

$$\Psi(P) = -\int\int_{S} [\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)]ds - I_F, \quad \text{for} \quad z_P \leq 0 \qquad \text{(A33)}$$

where

$$I_F = \int\int_{F_e} [\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)]ds, \quad \text{for} \quad z_P \leq 0 \qquad \text{(A34)}$$

Substitution of the free surface condition (A19) and the adjoint free surface condition (A31) into the normal derivatives of Ψ and G in (A34) respectively, yields

$$I_F = I_\gamma + I_U \qquad \text{(A35)}$$

where

$$\begin{aligned} I_\gamma &= -2i\gamma \int\int_{F_e} [\Psi(M)\frac{\partial G(P,M)}{\partial x_M} + \frac{\partial \Psi(M)}{\partial x_M} G(P,M)]ds \\ &= -2i\gamma \int\int_{F_e} \frac{\partial}{\partial x_M}[\Psi(M)G(P,M)]ds, \quad \text{for} \quad z_P \leq 0 \end{aligned} \qquad \text{(A36)}$$

and

$$I_U = \frac{U^2}{g} \int\int_{F_e} \frac{\partial}{\partial x_M} [\Psi(M) \frac{\partial G(P, M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P, M)] ds, \quad z_P \leq 0 \qquad \text{(A37)}$$

The γ in (36) is a non-dimensional parameter known as the Brard number.

$$\gamma = \frac{U\omega}{g} \qquad \text{(A38)}$$

Application of Stokes's theorem to (A36) yields

$$I_\gamma = -2i\gamma \int_{C\infty} \Psi(M)G(P, M)dy_M + 2i\gamma \int_C \Psi(M)G(P, M)dy_M, \quad z_P \leq 0 \qquad \text{(A39)}$$

where the positive directions around both C and C_∞ are defined counterclockwise when one would see them from above the free surface.
The line integral of the product Ψ and G along C_∞ vanishes in the limit since both Ψ and G tend to zero as $\frac{1}{r\infty}$ and we have

$$I_\gamma = 2i\gamma \int_C \Psi(M)G(P, M)dy_M, \quad z_P \leq 0 \qquad \text{(A40)}$$

Similarly, application of Stokes's theorem to (A37) yields

$$I_U = -\frac{U^2}{g} \int_C [\Psi(M) \frac{\partial G(P, M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P, M)] dy_M, \quad z_P \leq 0 \qquad \text{(A41)}$$

Substitution of (A40) and (A41) into (A35) yields

$$\begin{aligned} I_F = 2i\gamma \int_C \Psi(M)G(P, M)dy_M - \frac{U^2}{g} \int_C [\Psi(M) \frac{\partial G(P, M)}{\partial x_M} \\ - \frac{\partial \Psi(M)}{\partial x_M} G(P, M)] dy_M, \quad z_P \leq 0 \end{aligned} \qquad \text{(A42)}$$

Substituting the final expression of I_F into the integral relation (A33) and taking account of the potential jump across S, we can obtain the following Green integral equation for Ψ:

$$\begin{aligned} \frac{1}{2}\Psi(P) + \int\int_S \Psi(M) \frac{\partial G(P, M)}{\partial n_M} ds + 2i\gamma \int_C \Psi(M)G(P, M)dy_M \\ - \frac{U^2}{g} \int_C [\Psi(M) \frac{\partial G(P, M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P, M)] dy_M \\ = \int\int_S \frac{\partial \Psi(M)}{\partial n_M} G(P, M) ds, \quad P \in S \end{aligned} \qquad \text{(A43)}$$

The derivative of Ψ with respect to x_M can be decomposed as follows:

$$\frac{\partial \Psi(M)}{\partial x_M} = \vec{e}_1 \cdot [\frac{\partial \Psi(M)}{\partial n_M} \vec{n}_M + \frac{\partial \Psi(M)}{\partial l_M} \vec{l}_M + \frac{\partial \Psi(M)}{\partial \tau_M} \vec{\tau}_M], \quad M \in S \qquad \text{(A44)}$$

where $\overrightarrow{l}$ is a unit vector tangent to C whose direction along which one, traveling in D_e, would proceed in keeping W to his left, is defined positive and τ a unit vector tangent to S forming a right-hand vector triad $\overrightarrow{\tau} = \overrightarrow{l} \times \overrightarrow{n}$.
Substitution of (A44) into (A43) yields

$$\begin{aligned} &\frac{1}{2}\Psi(P) + \int\int_S \Psi(M)\frac{\partial G(P,M)}{\partial n_M}ds + 2i\gamma\int_C \Psi(M)G(P,M)dy_M \\ &-\frac{U^2}{g}\int_C \left(\Psi(M)\frac{\partial G(P,M)}{\partial x_M} - \overrightarrow{e}_1\cdot[\frac{\partial\Psi(M)}{\partial l_M}\overrightarrow{l}_M + \frac{\partial\Psi(M)}{\partial \tau_M}\overrightarrow{\tau}_M]G(P,M)\right)dy_M \\ &= \int\int_S \frac{\partial\Psi(M)}{\partial n_M}G(P,M)ds - \frac{U^2}{g}\int_C \frac{\partial\Psi(M)}{\partial n_M}G(P,M)\overrightarrow{e}_1\cdot\overrightarrow{n}_M dy_M, \quad P\in S \end{aligned} \tag{A45}$$

It should be noted that the expression of the Green function in the line integral can be reduced as follows

$$G(P,M) = G_f(P,M) \quad \text{on} \quad C \tag{A45a}$$

since the first and second terms in the right-hand side of (A21) cancel out when $z_M = 0$.

Bougis(1980) presented a Green integral equation for Ψ. But, in his Green integral equation, the sign of the line integral involving the Brard number is minus since he used the following condition:

$$[(-i\omega - U\frac{\partial}{\partial x_M})^2 + g\frac{\partial}{\partial z_M}]G(P,M) = 0 \quad \text{for} \quad z_P < 0 \quad \text{and} \quad z_M \le 0 \tag{A46}$$

It is evident that Brard's Green function does not satisfy the condition (A46) which is different from the adjoint free surface condition defined by (A31). Besides, he had to assume that

$$\int\int_{F_e} \frac{\partial\Psi(M)}{\partial x_M}G(P,M)ds = 0 \tag{A47}$$

in order to obtain the line integral associated with the Brard number. There is no reason that (A47) holds. Thus the Green integral equation by Bougis is not complete. More recently, the same mistake has been made by Hong in his study on the two-dimensional radiation problem of a cylinder advancing in the free surface(Hong 1995).

The equation (A45) is the Green integral equation which contains the correct free surface boundary conditions(Hong 2000).

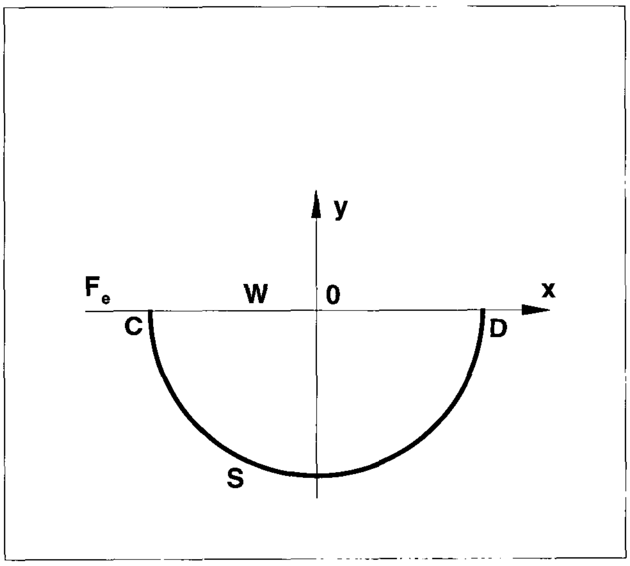

Figure 1: Coordinate system

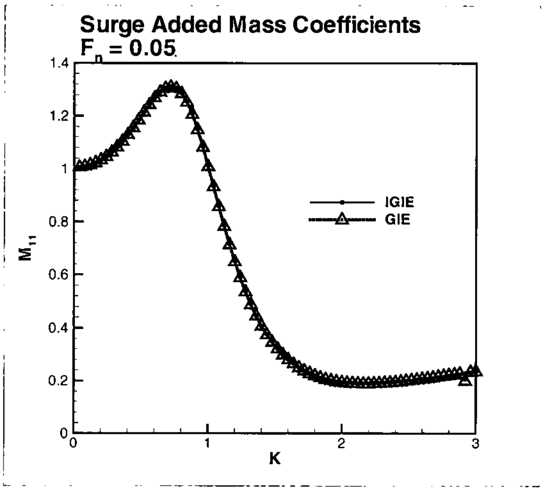

Figure 2: Surge added mass coefficients computed by IGIE and GIE for $Fn = 0.05$

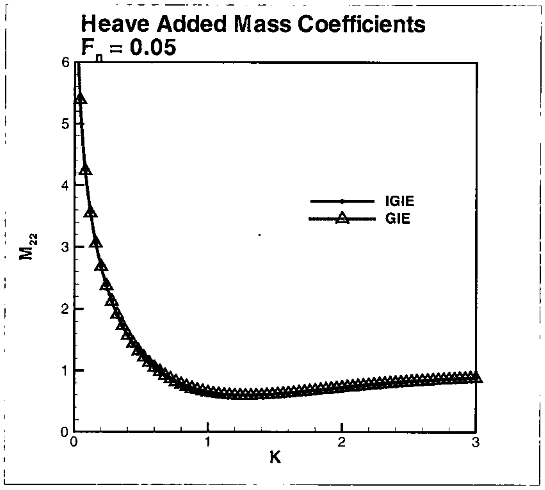

Figure 3: Heave added mass coefficients computed by IGIE and GIE for $Fn = 0.05$

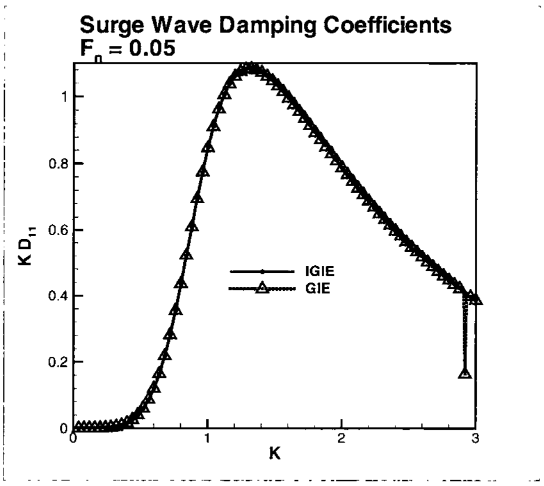

Figure 4: Surge wave damping coefficients computed by IGIE and GIE for $Fn = 0.05$

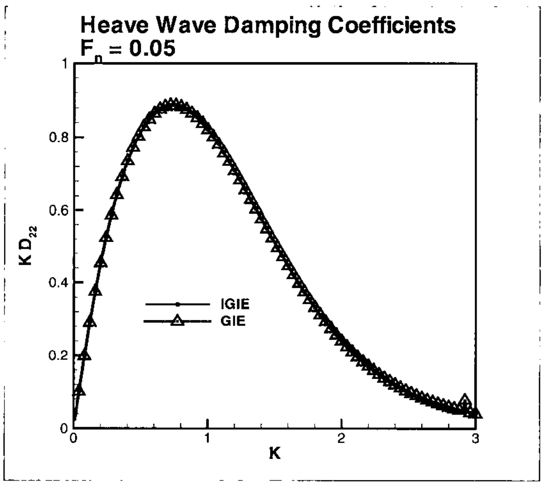

Figure 5: Heave wave damping coefficients computed by IGIE and GIE for $Fn = 0.05$

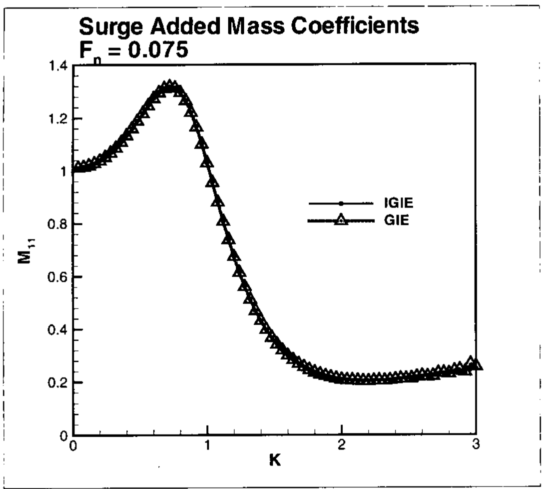

Figure 6: Surge added mass coefficients computed by IGIE and GIE for $Fn = 0.075$

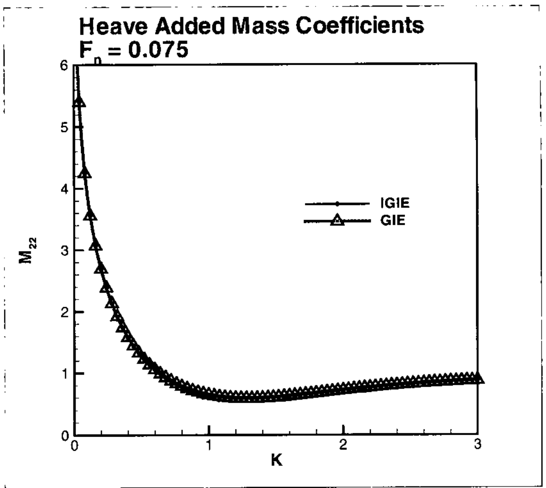

Figure 7: Heave added mass coefficients computed by IGIE and GIE for $Fn = 0.075$

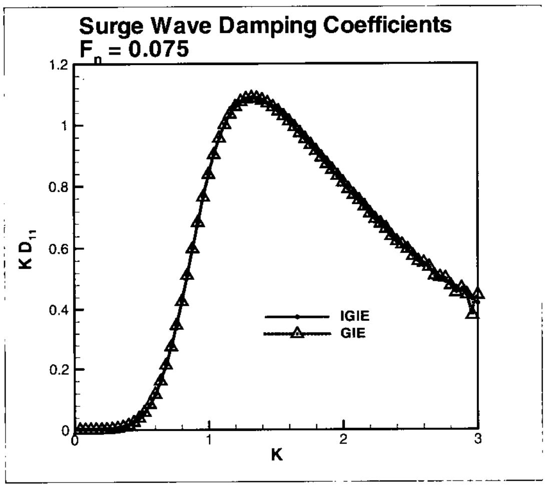

Figure 8: Surge wave damping coefficients computed by IGIE and GIE for $Fn = 0.075$

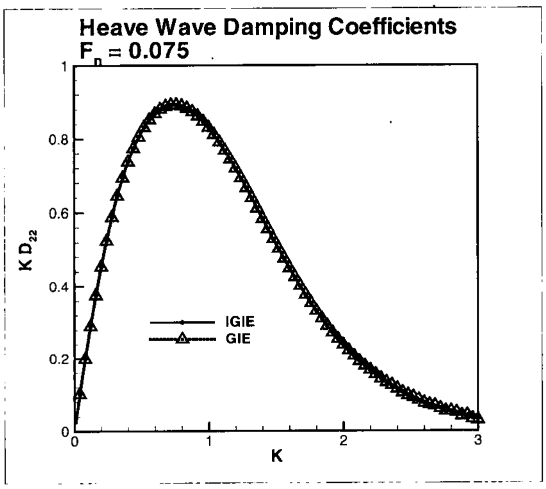

Figure 9: Heave wave damping coefficients computed by IGIE and GIE for $Fn = 0.075$

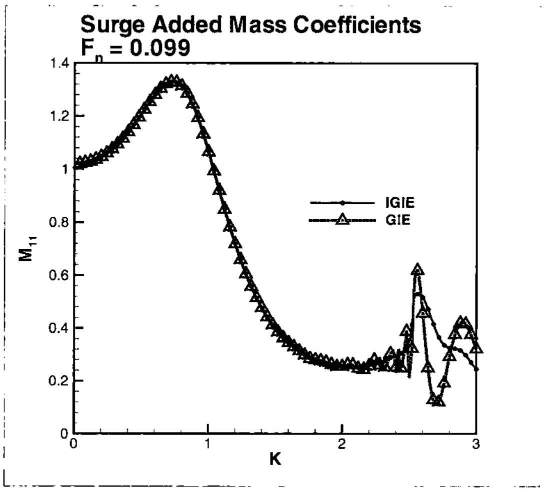

Figure 10: Surge added mass coefficients computed by IGIE and GIE for $Fn = 0.099$

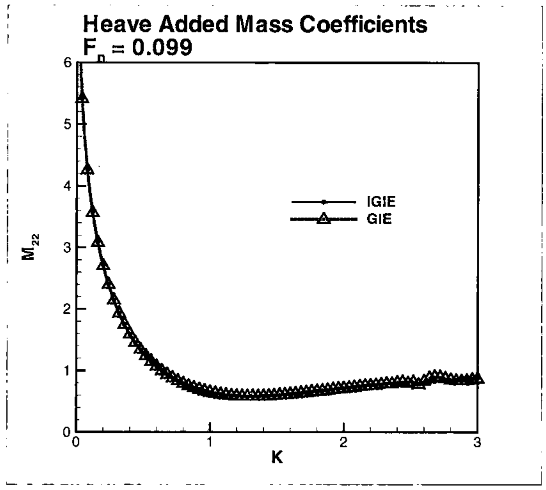

Figure 11: Heave added mass coefficients computed by IGIE and GIE for $Fn = 0.099$

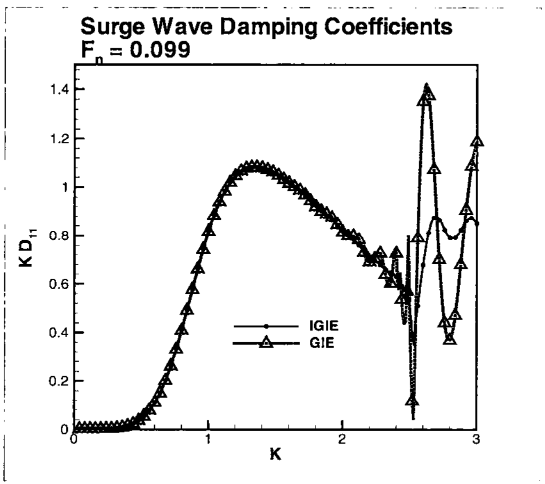

Figure 12: Surge wave damping coefficients computed by IGIE and GIE for $Fn = 0.099$

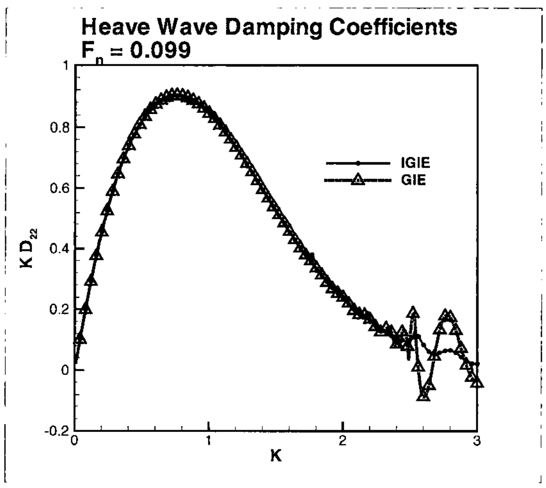

Figure 13: Heave wave damping coefficients computed by IGIE and GIE for $Fn = 0.099$

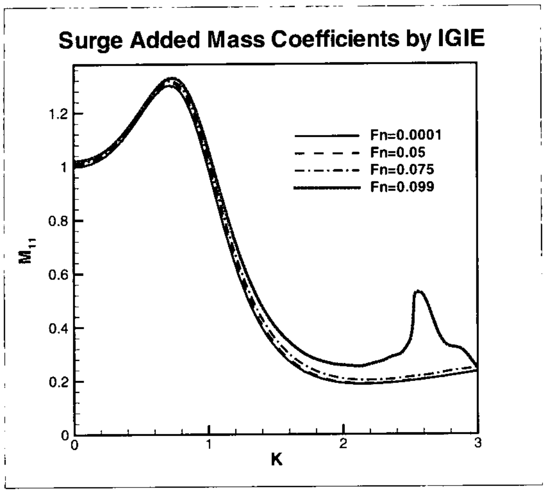

Figure 14: Surge added mass coefficients computed by IGIE

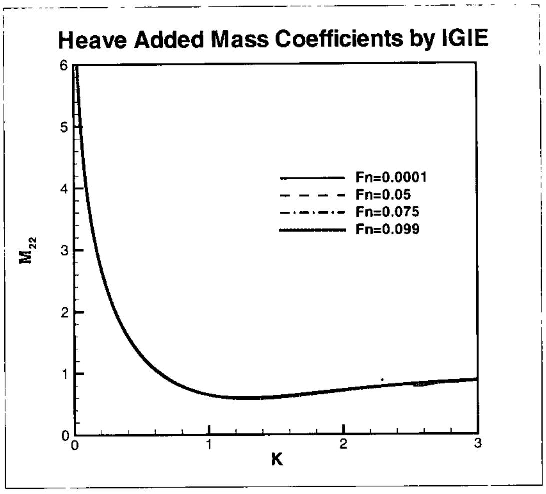

Figure 15: Heave added mass coefficients computed by IGIE

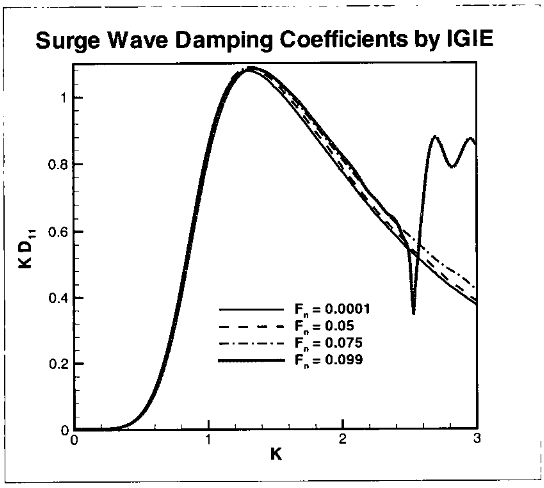

Figure 16: Surge wave damping coefficients computed by IGIE

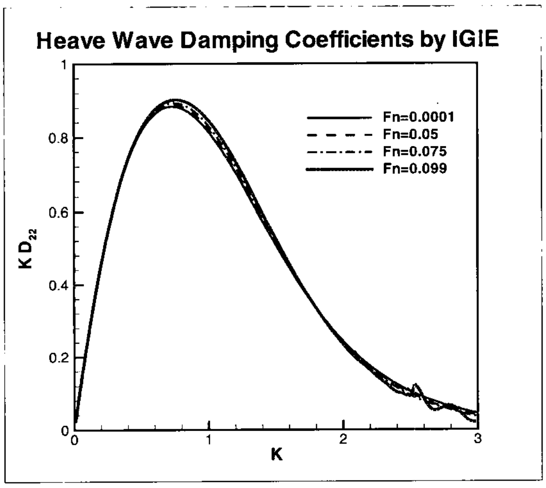

Figure 17: Heave wave damping coefficients computed by IGIE

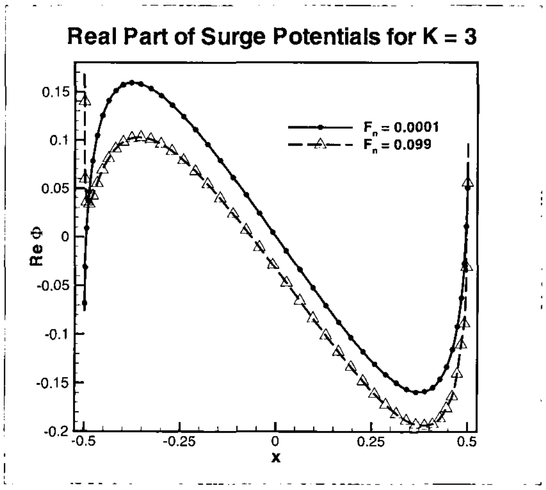

Figure 18: Real part of surge potentials on the wetted surface for $K = 3$

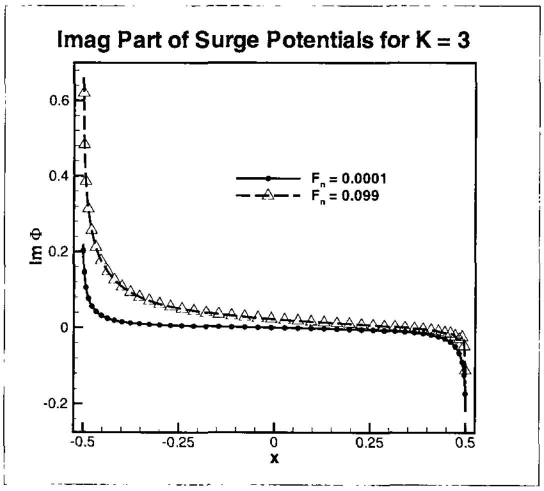

Figure 19: Imaginary part of surge potentials on the wetted surface for $K = 3$

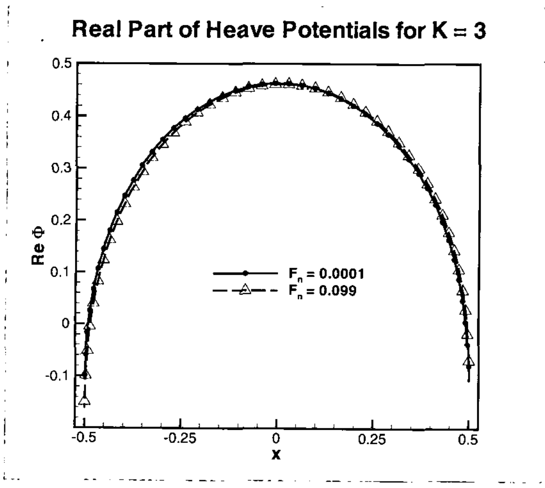

Figure 20: Real part of heave potentials on the wetted surface for $K = 3$

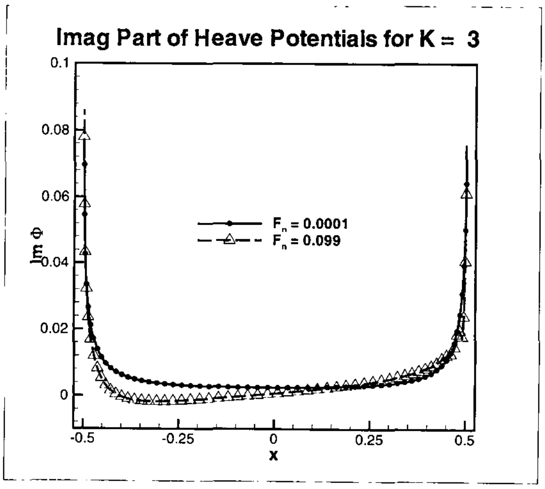

Figure 21: Imaginary part of heave potentials on the wetted surface for $K = 3$