Thermal Analysis on Triple-Passage Heat Exchangers for a Continuous Hot-Steel Tube Cooling System

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Key words: Triple-passage heat exchanger, Nusselt number, Friction factor, Radius ratio, Velocity ratio

Abstract

The objective of present study is to analyze a concentric triple-passage heat exchanger for an optimal design of a continuous hot steel-tube cooling system, where a hot-steel tube line is passing through an antioxidant gas with a constant speed. Velocities and temperatures of the inert gas flowing between inner and outer tubes are calculated theoretically for laminar and numerically for turbulent flow regimes. From their profiles Nusselt numbers and friction factors are calculated for various ratios of inner/outer tube radii and relative velocities. With these Nusselt numbers triple-passage heat exchangers are investigated for their thermal characteristics. It is shown that heat transfer coefficients based on ratios of average heat fluxes from inner and outer tubes might result in great errors for the temperature distributions of the flows, since local heat transfer coefficients for flows through an annulus are dependent on local wall heat flux ratios.

Nomenclature

C^* : ratio of heat capacity
C_p : specific heat
D_e : hydraulic diameter
f : friction factor
k : thermal conductivity
Nu : Nusselt number
p : pressure
q'' : heat flux
r : radial coordinate
r^* : ratio of radii, r_i/r_o
Re : Reynolds number
t : temperature
u : axial velocity
u_i : speed of inner tube
u_r : ratio of velocities, u_i/V
U : overall heat transfer coefficient
V : bulk velocity of the antioxidant gas
x : axial coordinate

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Greek symbols

α : thermal diffusivity
μ : viscosity
θ : influence coefficient
Θ : dimensionless temperature
ξ : dimensionless axial coordinate

Subscripts

i : inner tube
ii : constant heat flux boundary condition for inner tube and insulation condition for outer tube
in : inlet
m : bulk
o : outer tube
oo : constant heat flux boundary condition for outer tube and insulation condition for inner tube
out : outlet

1. Introduction

The objective of this research is to analyze heat transfer mechanisms to optimize a continuous hot steel-tube cooling system as a concentric triple-passage heat exchanger, whose inner tube is moving with a constant speed.

A hot steel-tube cooling process is very important to anneal the tube after reducing and heating in the production process of single-walled steel tubes for automobile brake tubing,[1] as briefly shown in Fig. 1.

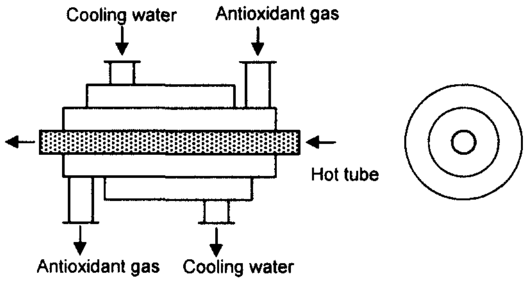


Fig. 2 Schematic diagram of three fluid heat exchangers for the tube cooling process.

The schematic structure of the cooling system is shown in Fig. 2. The cooling system is composed of three passage flows: (1) hot steel tube to be cooled with a constant passing speed, (2) antioxidant gas to control the cooling rate of the tube and to prevent the tube from being oxidized, (3) cooling water flowing through the outer water jacket. This system is different from conventional triple passage heat exchangers[2-6] in that the inner tube is moving with a constant velocity.

For the optimal design of the cooling system it is necessary to understand the heat transfer mechanism from the inner hot steel tube to the outer cooling water. There are, however, no previous studies[8-10] on the heat transfer coefficients when the inner tube has a velocity. In this study, therefore, heat transfer characteristics such as Nusselt numbers for the moving system will be obtained theoretically for laminar and numerically for turbulent flow regimes. With these heat transfer coefficients the temperature distributions of the system will be

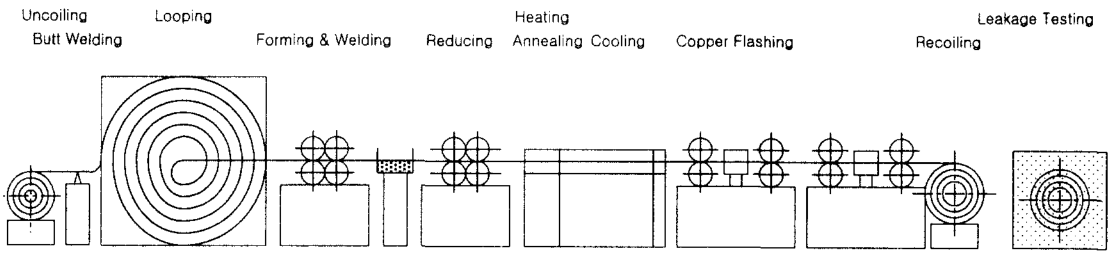


Fig. 1 Production process of hot steel tubes.

calculated to estimate the cooling process.

2. Friction factors and heat transfer coefficients

The triple passage cooling system in Fig. 2 can be simplified as shown in Fig. 3 for the theoretical analysis. It is assumed that the inner tube has a constant velocity, the fluid and thermal fields between the annulus are fully developed, and the boundary conditions for heat transfer are constant heat flux or insulated ones. In addition, the radiative heat transfer is neglected here, which is very important for a practical cooling system design.

2.1 Laminar flow

2.1.1 Friction coefficient

Flow through the annulus in Fig. 3 can be described by the governing equation and relevant boundary conditions as,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)=\frac{dp}{dx} \tag{1}$$

$$u=0 \quad \text{at} \quad r=r_o$$
$$u=u_i \quad \text{at} \quad r=r_i$$

where u_i is the velocity of the inner tube (hot steel tube). Then, the solution of Eq. (1) can be obtained analytically as,

$$u=V\left[\frac{2}{M}(1+u_rD)\left\{1-\left(\frac{r}{r_o}\right)^2+B\ln\frac{r}{r_o}\right\}+u_r\frac{\ln(r/r_o)}{\ln r^*}\right] \tag{2}$$

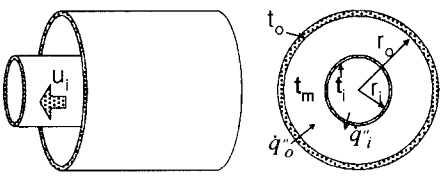


Fig. 3 Flow in a circular-tube annulus.

where V is the bulk velocity of the antioxidant gas in the annulus, u_r is the ratio of inner tube and average gas velocities $(u_r=u_i/V)$, r^* is the ratio of radii $(r^*=r_i/r_o)$, and other parameters are $B=\frac{r^{*2}-1}{\ln r^*}$, $M=1+r^{*2}-B$, $D=\frac{1}{2\ln r^*}+\frac{r^{*2}}{1-r^{*2}}$.

Then, Moody's friction factor is formulated as,

$$f=\frac{64}{\mathrm{Re}}(1-r^{*2})\frac{1}{M}(1+u_rD) \tag{3}$$

2.1.2 Heat transfer coefficient

The governing energy equation for the flow in the annulus with constant heat flux boundary conditions is simplified as,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)=\frac{u}{\alpha}\frac{\partial t_m}{\partial x} \tag{4}$$

$$\text{i) } r=r_i,\ t=t_i \qquad r=r_o,\ \frac{dt}{dr}=0$$
$$\text{ii) } r=r_i,\ \frac{dt}{dr}=0 \qquad r=r_o,\ t=t_o$$

where case (i) implies that the inner wall temperature is t_i and the outer wall is insulated, and case (ii) does that the outer wall temperature is t_o and the inner wall is insulated. It, however, is not meant that the wall temperature distributions along the flow direction are not varying. With Eq. (2) for the velocity of the flow the temperature distribution is obtained from Eq. (4) analytically as, for boundary condition (i):

$$t=\frac{V}{\alpha}\frac{dt_m}{dx}\left\{\frac{(A-A\cdot B-C)}{4}\left(r^2-r_i^2+r_0^2\ln\frac{r_i}{r}\right)-\frac{A}{16r_o^2}(r^4-r_i^4)+\frac{A\cdot B}{4}r^2\ln\frac{r}{r_0}-\frac{A\cdot B}{4}r_i^2\ln\frac{r_i}{r_0}+\frac{Cr^2}{4}\ln\frac{r}{r_0}-\frac{Cr_i^2}{4}\ln\frac{r_i}{r_0}\right\}+t_i \tag{5}$$

for boundary condition (ii):

$$t=\frac{V}{a}\frac{dt_m}{dx}\left\{\frac{(A-A\cdot B-C)}{4}(r^2-r_0^2)-\frac{A}{16r_0^2}(r^4-r_0^4)+\frac{(A\cdot B+C)}{4}r^2\ln\frac{r}{r_0}+\frac{r_i^2}{4r_o^2}\ln\frac{r}{r_0}\left(Ar_i^2-(2A-A\cdot B-C)r_0^2-2(A\cdot B+C)r_0^2\ln\frac{r_i}{r_0}\right)\right\}+t_0 \quad (6)$$

where

$$A=\frac{2}{M}(1+u_rD),\quad B=\frac{r^{*2}-1}{\ln r^*},$$

$$C=\frac{u_r}{\ln r^*},\quad D=\frac{1}{2\ln r^*}+\frac{r^{*2}}{1-r^{*2}}.$$

Nusselt numbers for cases (i) and (ii) are,

$$\mathrm{Nu}_{ii}=\frac{V}{a}\frac{dt_m}{dx}\frac{(r_o^2-r_i^2)(r_o-r_i)}{(t_i-t_m)r_i} \quad (7)$$

$$\mathrm{Nu}_{\infty}=\frac{V}{a}\frac{dt_m}{dx}\frac{(r_o^2-r_i^2)(r_o-r_i)}{(t_o-t_m)r_o} \quad (8)$$

For general cases where both the heat flux boundary conditions exist, Nusselt numbers[9,10] for the inner and outer walls can be obtained as functions of influence coefficients (θ_i, θ_o),

$$\mathrm{Nu}_i=\frac{\mathrm{Nu}_{ii}}{1-(\dot{q}_o''/\dot{q}_i'')}\theta_i \quad (9)$$

$$\mathrm{Nu}_o=\frac{\mathrm{Nu}_{\infty}}{1-(\dot{q}_i''/\dot{q}_o'')}\theta_o \quad (10)$$

2.2 Turbulent flow

Due to the difficulty in the analysis on the turbulent flow inside the annulus, STAR-CD 3.0 code[11] has been utilized and $k-\varepsilon$ model is applied to obtain turbulent flow and thermal fields. The same procedure as that for laminar flow is taken for turbulent friction factors and Nusselt numbers.

3. Thermal design of triple-passage heat exchangers

Assumptions for the thermal analysis of triple-passage heat exchangers are:

(1) heat conduction in the axial direction and thermal radiation are neglected,

(2) all the properties are constant,

(3) the cross section of the annulus is uniform along the axial direction,

(4) overall heat transfer coefficient is described by the gas flow only.

The governing equations for the analysis are nondimensionalized as,

$$\frac{d\Theta_1}{d\xi}=NTU_1\,C^*_{2,1}(\Theta_2-\Theta_1) \quad (11)$$

$$i_2\frac{d\Theta_2}{d\xi}=-NTU_1(\Theta_2-\Theta_1)-NTU_2(\Theta_2-\Theta_3) \quad (12)$$

$$\frac{d\Theta_3}{d\xi}=NTU_2\,C^*_{2,3}(\Theta_2-\Theta_3) \quad (13)$$

where

$$\Theta_j=\frac{T_j-T_{1,in}}{T_{2,in}-T_{1,in}}\ \text{for}\ j=1,2,3\ \text{and}\ \xi=\frac{x}{L}$$

$$C^*_{2,1}=\frac{(\dot{m}c_p)_2}{(\dot{m}c_p)_1};\quad C^*_{2,3}=\frac{(\dot{m}c_p)_2}{(\dot{m}c_p)_3}$$

$$NTU_1=\frac{(UA)_{1,2}}{(\dot{m}c_p)_2};\quad NTU_2=\frac{(UA)_{3,2}}{(\dot{m}c_p)_2}$$

The directional index in Eq. (12), i_2, is $+1$ for parallel flow and -1 for counterflow conditions. According to assumption (4) overall heat transfer coefficients based on fluid 2 are

$$U_{1,2}=\frac{\mathrm{Nu}_{\infty}}{1-(\dot{q}_i''/\dot{q}_o'')\theta_o}\frac{k}{D_e} \quad (14)$$

$$U_{3,2}=\frac{\mathrm{Nu}_{ii}}{1-(\dot{q}_o''/\dot{q}_i'')\theta_i}\frac{k}{D_e} \quad (15)$$

where the heat flux ratio, $\dot{q}_o''/\dot{q}_i''$, is the ratio

Table 1 Dimensionless boundary conditions

j	Parallel-flow		Counter-flow	
	ξ	Θ	ξ	Θ
1	0	0	0	0
2	0	1	1	1
3	0	0	0	0

of heat fluxes for outer and inner walls. Both fluxes can vary arbitrarily along the axial direction. Boundary conditions for Eqs. (11)~(13) in this study are given in Table 1. Solutions of this system are obtained numerically, using the 4th order Runge-Kutta algorithm with a shooting method.

4. Results and discussion

Friction factors for laminar flow inside an annulus are shown in Fig. 4 with respect to velocity and radius ratios. As expected, the factors decrease significantly when the inner tube velocity approaches the bulk gaseous flow velocity. The variation of the factors becomes more prominent for larger radius ratios, since the channel depth gets smaller with the increase of the radius ratio. Friction factors for stationary inner tube case are available in literatures[10] and depicted by 'X' symbols. This symbol will be given in subsequent figures, wherever previous studies are available.

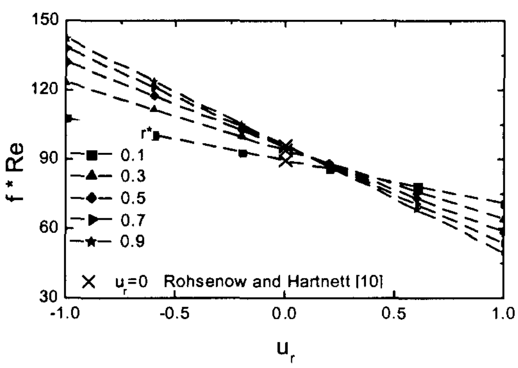


Fig. 4 Friction factors with respect to velocity ratios for laminar flow.

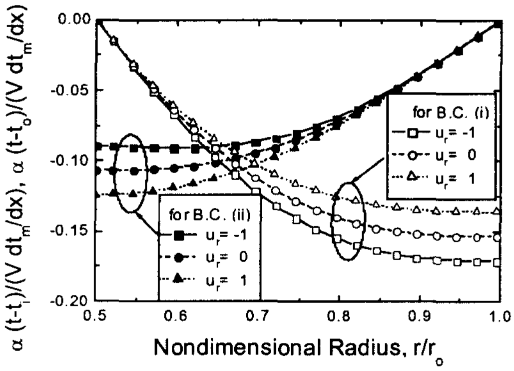


Fig. 5 Temperature profiles in the annulus for velocity ratios and boundary conditions.

Fig. 5 shows the temperature distributions in the radial direction for $r^*=0.5$. For case (i) in Eq. (4) temperature gradients at the inner surface become smaller, as the velocity ratio increases, while temperature gradients at the outer surface become larger for case (ii). These surface temperature gradients have an effect on the bulk gas temperatures and thus Nusselt numbers through Eqs. (7) and (8). Fig. 6 compares Nusselt numbers for laminar convective heat transfer at inner and outer surfaces. Nu_{ii} increases slightly with velocity ratios, while Nu_{oo} decreases, which are implied in the discussion for Fig. 5 and Eq. (7) and (8). In addition, Nu_{ii} is strongly dependent on the radius ratios, while Nu_{oo} is slightly influenced. Similar

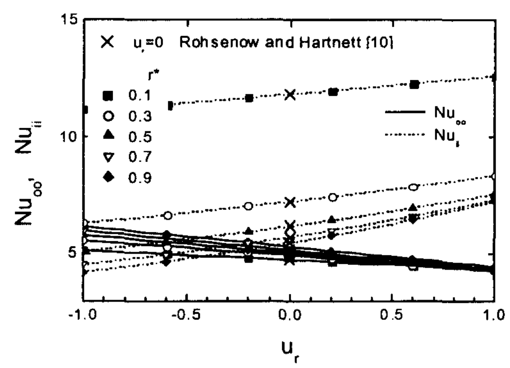


Fig. 6 Nusselt numbers with respect to velocity ratios for laminar flow.

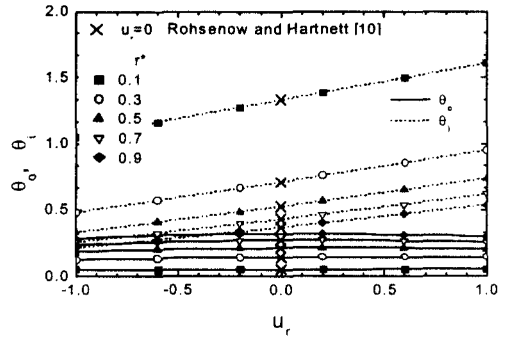


Fig. 7 Influence coefficients with respect to velocity ratios for laminar flow.

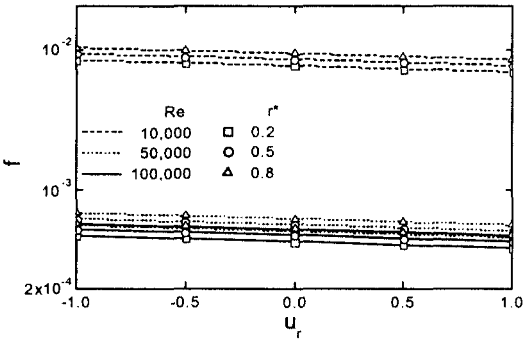


Fig. 8 Friction factors with respect to velocity ratios for turbulent flow.

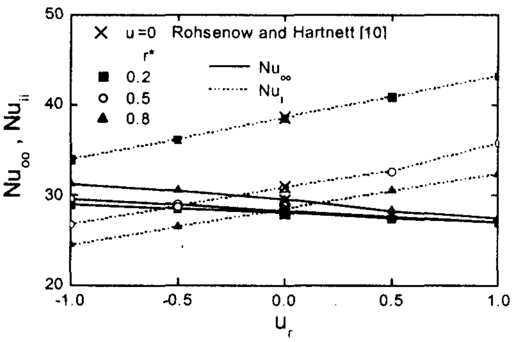


Fig. 9 Nusselt numbers with respect to velocity ratios at Re=10000.

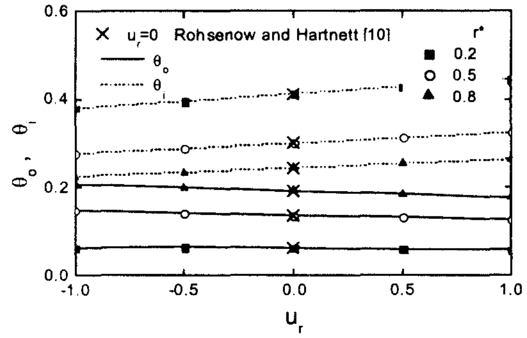


Fig. 10 Influence coefficients with respect to velocity ratios at Re=10000.

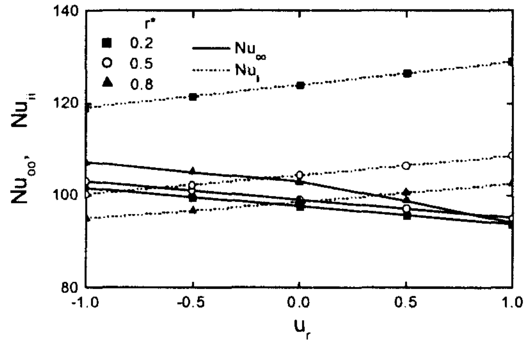


Fig. 11 Nusselt numbers with respect to velocity ratios at Re=50000.

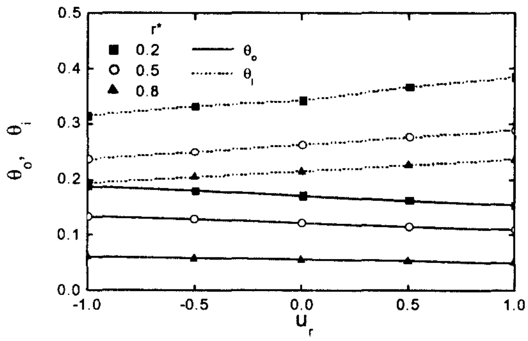


Fig. 12 Influence coefficients with respect to velocity ratios at Re=50000.

behavior has been observed for the influence coefficients (θ), as shown in Fig. 7. These coefficients will be used to calculate Nusselt numbers, Eqs. (9) and (10), under general conditions and thus to estimate the overall heat transfer coefficients using Eqs. (14) and (15).

Under turbulent flows condition thermal and flow fields are obtained using STAR-CD 3.0 code. Fig. 8 implies that the factors are not significantly influenced by the velocity ratios.

Figs. 9 and 10 describe Nusselt numbers and their influence coefficients for Reynolds number

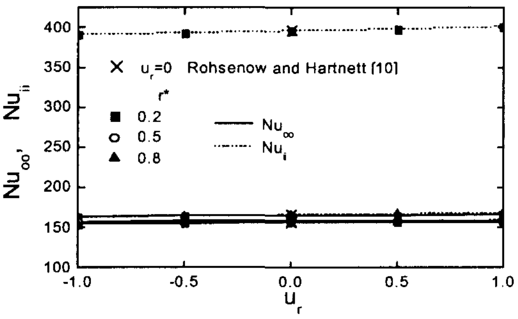


Fig. 13 Nusselt numbers with respect to velocity ratios at Re=100000.

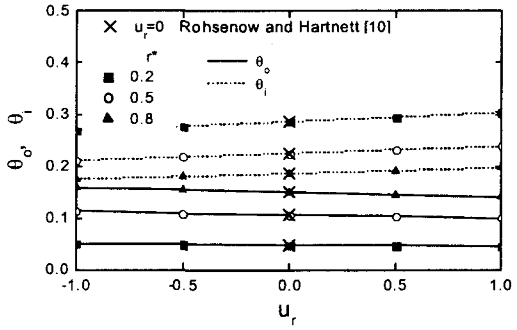
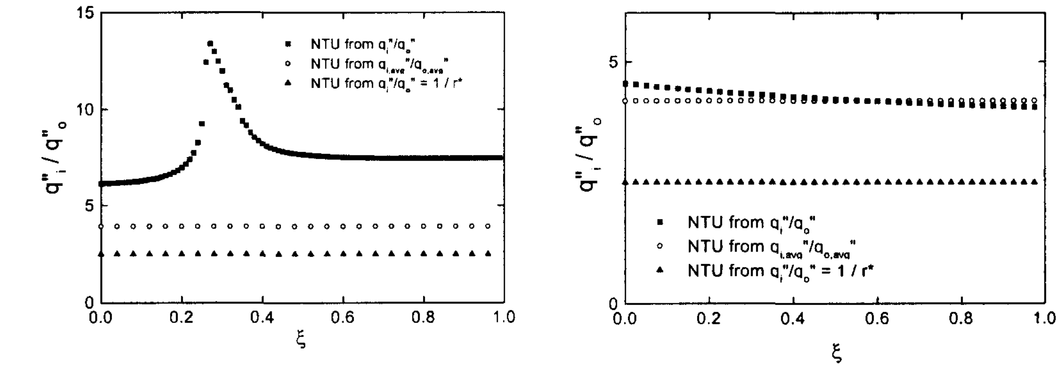


Fig. 14 Influence coefficients with respect to velocity ratios at Re=100000.

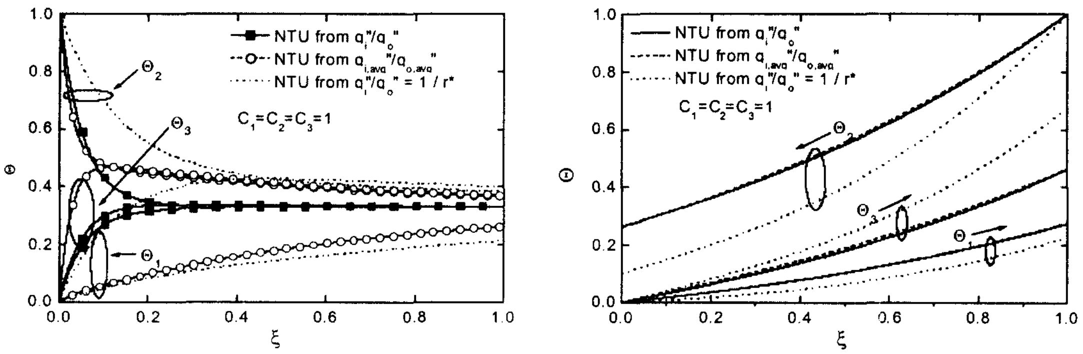
10000. Nusselt numbers are varying considerably with velocity ratios, while influence coefficients are influenced very slightly. Similar trend has been observed for Reynolds number 50000, as shown in Figs. 11 and 12, For Reynolds number 100000 the dependency on velocity ratios is not clear, as shown in Figs. 13 and 14.

With Nusselt numbers obtained in this study and Eqs. (14) and (15), the governing equations for a triple-passage heat exchanger of Eqs. (11)~(13) are solved numerically. There exists, however, an uncertainty in applying Eqs. (14) and (15), since heat fluxes at both inner and outer surfaces, $\dot{q}_i''$ and $\dot{q}_o''$ are assumed uniform along the flow direction, when these equations are formulated. Since there is no better available formulation on this triple-passage problem, these formulation will be used to estimate the temperature distribution along the tube axis with relevant assumptions on $\dot{q}_i''$ and $\dot{q}_o''$. The assumptions are as follows: (1) $\dot{q}_i''/\dot{q}_o''$ is a geometric average value, given as $\dot{q}_i''/\dot{q}_o'' = 1/r^*$, (2) $\dot{q}_i''/\dot{q}_o''$ is an actual average value along the tube axis, (3) $\dot{q}_i''$ and $\dot{q}_o''$ are nonuniform and actually obtained by iteration. For these three cases, wall heat fluxes are described in Figs. 15(a) and (b) for parallel and counter flows, respectively. In general, wall heat fluxes are uniform along the tube axis, except for case (3) of the parallel flow. The temperature distributions are calculated and shown in Figs. 16(a) and (b). Although the temperature



(a) Parallel flow (b) Counter flow

Fig. 15 Heat flux ratio distributions based on averaged and local wall heat flux ratios.



(a) Parallel flow (b) Counter flow

Fig. 16 Temperature distributions based on averaged and local wall heat flux ratios.

distributions for parallel flows are different from each other, the distributions for counter flows are very similar to each other, especially for cases (2) and (3).

5. Conclusions

A continuous cooling system for hot-steel tubes is investigated in the thermal aspect, using a triple-passage heat exchanger model. However, the inner tube wall has a velocity to simulate the hot tube passing through the cooling system. Since there are no thermal data available on this case, a theoretical study is carried out to obtain friction factors as well as Nusselt numbers for laminar and turbulent flows.

Laminar friction factors vary with velocity and radius ratios considerably, while turbulent ones are relatively uninfluenced. Nusselt numbers increase with the increase of Reynolds numbers and have similar behaviors for wide range of velocity and radius ratios. Nusselt numbers vary significantly for inner wall heating case, while they change slightly for outer wall heating case.

With these Nusselt numbers a theoretical model is established for the triple-passage heat exchangers. To calculate the overall heat transfer necessitates the estimation of the ratio of heat fluxes at the inner and outer walls. Three cases for the ratio: geometrical average, actual average, and actual local distribution obtained by iteration are applied. The geometric average heat flux ratio gives a significantly different temperature distribution from other cases for both parallel and counter flows. Actual average ratio gives a temperature distribution very close to that obtained from actual local heat flux ratios for counter flows.

Acknowledgements

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