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Envelope Correlation of Wideband Signals in Nakagami-Rice Fading Channel

나카가미-라이스 페이딩 채널에서 광대역 신호의 진폭 상관

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Abstract

In this paper, we analyzed the envelope correlation of wideband signals modulated on two separate subcarriers in a Nakagami-Rice fading channel with exponential power delay profile. The results show that the envelope correlation of resolvable paths is implicated by the ratio of the specular wave power to the mean multipath power as well as the signal bandwidth and the rms delay spread of the multipath waves.

요 약

본 논문에서는 지수함수적인 전력지연 특성을 갖는 나카가미-라이스 페이딩 채널에서 두 개의 반송파로 변조된 광대역 신호의 진폭특성을 분석한다. 그 결과, 분해 가능한 경로 신호들 간의 진폭상관은 신호의 대역폭과 다중경로 신호의 rms지연확산 뿐만 아니라 직접파의 전력과 다중경로 신호의 평균전력의 비에 영향을 받는다.

Key words: envelope correlation, wideband signals, Nakagami-Rice fading

I. Introduction

The diversity reception provides redundancy to be exploited in reducing the impact of fading on system performance. The fading signals are statistically independent when the subchannels are sufficiently separated from each other, rather it is advantageous to use smaller separation of subchannels than that allowed for independent fading, because the closer separation can exploit the benefit of higher order diversity for a given resource. Therefore, the knowledge of received signal envelope correlation is needed to determine potential diversity gain in the system. In multicarrier systems[2], for example, envelope correlation coefficients of overlapping signals among subcarriers are necessary to evaluate the performance of such systems employing RAKE receivers with frequency diversity.

A few works in the literature consider the analysis of envelope correlation characteristics in some conditions[3]-[5]. In [3] and [4], the envelope correlation statistics of narrowband signals were analyzed in Nakagami-Rice fading and Rayleigh fading, respectively, and that of wideband signals in Rayleigh

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fading was analyzed in [5]. DS-CDMA is the most promising multiple access scheme for the next generation mobile communications, IMT-2000 and has been attracted much interests in the application such as high-capacity wireless systems in micro-cellular or indoor communication environments. Therefore, this paper presents an analysis of envelope correlation of wideband signals in Nakagami-Rice fading channel. To do that, we obtain the correlation coefficient of two resolvable path signals belonging to two separate carriers f_1 and f_2 each, and investigate the effect of rms delay spread, signal bandwidth and frequency separation of subchannels on the envelope correlation.

II. Channel and Signal Envelope Model

The impulse response $g(t,\tau)$ of Nakagami-Rice fading channel can be expressed as the sum of a specular wave component given by $m(\tau)$ and a Gaussian random process $h(t,\tau)$ that is sum of scattered wave components of the channel. $g(t,\tau)$ is described by

$$g(t,\tau)=m(\tau)+h(t,\tau) \tag{1}$$

A specular wave component, $m(\tau)$ can be expressed as

$$m(\tau)=\rho\delta(\tau) \tag{2}$$

where ρ is the amplitude of specular wave component and $\delta(\tau)$ dirac delta function. A scattered channel, $h(t,\tau)$ is assumed to be a wide-sense stationary uncorrelated scattering(WSSUS) mobile radio channel in which the propagation takes places by scattering. The sum of all the individual scattered components produces the low pass equivalent impulse response of the scattered channel described by

$$h(t,\tau)=\sum_n \beta_n e^{-(2\pi\upsilon_n t+2\pi f_c\tau_n+\partial\tau_n)}\delta(\tau-\tau_n) \tag{3}$$

where f_c denotes the carrier frequency and β_n the path amplitude which is Rayleigh distributed, $\partial\tau_n$ the perturbation term and $2\pi f_c\tau_n$ phase term which are both uniformly distributed over the interval $[-\pi,\pi)$, υ_n the Doppler shift and all these random variables are mutually uncorrelated[6].

The exponential power delay profile and uniform distribution in the arrival angle of incident power are assumed. Therefore, the power delay profile can be written as

$$p(\alpha,\tau)=p_D\delta(\tau)+p_R(\alpha,\tau) \tag{4}$$

$$p_R(\alpha,\tau)=\frac{P_R}{2\pi\sigma_\tau}e^{-\frac{\tau}{\sigma_\tau}} \tag{5}$$

where σ_τ, τ and α are rms delay spread of the channel, arrival times and angle of arrival of the signal respectively. p_D represents the power of direct component and P_R is the average received power of the multipath components over the range of channel delay and angle of arrivals, and can be written as

$$P_R=\int_0^{2\pi}\int_0^{\infty}p_R(\alpha,\tau)d\tau d\alpha \tag{6}$$

Let the two replicas of the same wideband signal $s(t)$ modulated on two subcarriers f_1 and f_2 be denoted as $s_1(t)$ and $s_2(t)$, and the corresponding received signals as $r_1(t)$ and $r_2(t)$ respectively. Wideband signal is referred to the direct sequence spread spectrum(DS-SS) signal with bandwidth equal to or larger than channel coherence bandwidth. Because the frequency components of a wideband signal transmitted under the frequency selective fading channel experience different attenuation and phase shift, no single envelope or phase for $r_1(t)$ and $r_2(t)$ can be defined.

Therefore, the correlation between signal envelope of one resolvable path belonging to $r_1(t)$ with that of the corresponding resolvable path belonging to $r_2(t)$ is derived as a function of frequency separation.

Assuming L paths are resolvable and specular signal components are in each resolvable path, Eqs. (2) and (3) can be expressed as

$$m(\tau)=\sum_{i=1}^{L} m_i(\tau)=\sum_{i=1}^{L}\rho_i\delta(\tau-\tau_i) \quad (7)$$

$$h(t,\tau)=\sum_{i=1}^{L} h_i(t,\tau) \quad (8)$$

where the maximum number of resolvable paths between the transmitter and receiver, L is given $L=\lfloor \frac{\sigma_\tau}{T_c}\rfloor+1$. Here, σ_τ and T_c is the rms delay spread of the channel and the chip duration of DS-SS signal respectively, and $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. Through Eqs. (3), (7) and (8), the impulse response of ith resolvable path can be written as

$$m_i(\tau)=\rho_i\delta(\tau-\tau_i),\quad \tau_i=(i-1)T_c+T_c/2 \quad (9)$$

$$h_i(t,\tau)=\sum_{n=l_i}^{l_i'}\beta_n e^{-j(2\pi\upsilon_n t+2\pi f_c\tau_n+\partial\tau_n)}\delta(\tau-\tau_n) \quad (10)$$
$$(i-1)T_c\le\tau_n\le iT_c$$

In Eqs. (9) and (10), $m_i(\tau)$ means the specular signal path arrived at $\tau_i=(i-1)T_c+T_c/2$ and $h_i(t,\tau)$ means the sum of scattered signal paths in multipath channel which arrives in the interval $(i-1)T_c\le\tau_n\le iT_c$. l_i and l_i' are the first and last number of scattered signal path within that interval, respectively. With some manipulation, Eq. (9) and Eq. (10) can be written as

$$m_i(\tau)=\delta[\tau-(i-1)T_c]*m'(\tau) \quad (11)$$

$$h_i(t,\tau)=e^{-j2\pi f_c(i-1)T_c}\delta[\tau-(i-1)T_c]*h_i'(t,\tau) \quad (12)$$

where * means convolutional operation, and

$$m_i'(\tau)=\rho_i\delta[\tau-T_c/2] \quad (13)$$

$$h_i'(t,\tau)=\sum_{n=l_i}^{l_i'}\beta_n e^{-j(2\pi\upsilon_n t+2\pi f_c\tau_n'+\partial\tau_n')}\delta(\tau-\tau_n') \quad (14)$$
$$0\le\tau_n'\le T_c$$

Therefore, the channel impulse response of ith resolvable path is

$$\begin{aligned} g_i(t,\tau)&=m_i(\tau)+h_i(t,\tau)\\ &=\delta[\tau-(i-1)T_c]*m_i'\\ &\quad+\delta[\tau-(i-1)T_c]e^{-j2\pi f_c(i-1)T_c}*h_i'(t,\tau)\end{aligned} \quad (15)$$

In Eq. (15), each $h_i'(t,\tau)$ acts as an independent channel with maximum delay spread of T_c and maximum delay profile of

$$p_i'(\alpha,\tau)=P_{D,i}\delta(\tau-T_c/2)+A_i e^{-\frac{\tau}{\delta_\tau}} \quad (16)$$
$$0\le\tau\le T_c$$

$$A_i=\frac{P_R}{2\pi\sigma_\tau}e^{-\frac{(i-1)}{\sigma_\tau}} \quad (17)$$

which is the power delay profile of each resolvable paths and extracted from Eqs. (4) and (5).

Let us define $r_1(\tau)$ and $r_2(\tau)$ as the received signals when DS-SS signal $s_1(\tau)$ and $s_2(\tau)$ are transmitted via $g_i(t,\tau)$. Since the effect of phase shift and delay on the envelope correlation in Eq. (15) is irrelevant, only the response of $[m_i'(\tau)+h_i'(\tau)]$ to $s_1(\tau)$ and $s_2(\tau)$, denoted r_{i1}' as and r_{i2}' respectively, is considered. Therefore, r_{i1}' and r_{i2}' can be obtained as

$$\begin{aligned} r_{i1}'(t)&=\rho_{i1}+\sum_{n=l_i}^{l_i'}\beta_n e^{-j(2\pi\upsilon_n t+2\pi f_1\tau_n'+\partial\tau_n')}\\ &=V_{i1}(t)e^{-j\vartheta_{i1}(t)}\end{aligned} \quad (18a)$$

$$\begin{aligned} r_{i2}'(t)&=\rho_{i2}+\sum_{n=l_i}^{l_i'}\beta_n e^{-j(2\pi\upsilon_n t+2\pi f_2\tau_n'+\partial\tau_n')}\\ &=V_{i2}(t)e^{-j\vartheta_{i2}(t)}\end{aligned} \quad (18b)$$

where $V_{i1}(t)$ and $V_{i2}(t)$ are envelopes of the received signals via $g_i(t,\tau)$.

III. Joint Densities and Envelope Correlation

The received signals in Eq. (18) can be expressed as

$$r_{i1}'=x_{c1}(t)-jx_{s1}(t) \quad (19a)$$

$$r'_{i2} = x_{c2}(t) - jx_{s2}(t) \tag{19b}$$

where $x_{c1}(t) = V_{i1}(t)\cos\vartheta_{i1}(t)$, $x_{s1}(t) = V_{i1}(t)\sin\vartheta_{i1}(t)$, $x_{c2}(t) = V_{i2}(t)\cos\vartheta_{i2}(t)$ and $x_{s2}(t) = V_{i2}(t)\sin\vartheta_{i2}(t)$. Next let us determine the joint probability densities of $V_1 = V_{i1}(t)$, $V_2 = V_{i2}(t - t_\triangle)$, $\vartheta_1 = \vartheta_{i1}(t)$ and $\vartheta_2 = \vartheta_{i2}(t - t_\triangle)$ from the joint probability density of the non-zero mean Gaussian random variables $x_{c1} = x_{c1}(t)$, $x_{s1} = x_{s1}(t)$, $x_{c2} = x_{c2}(t - t_\triangle)$, and $x_{s2} = x_{s2}(t - t_\triangle)$. It is necessary to find the covariance of these random variables to obtain joint probability density function. The variance of x_{c1} is defined as $\sigma^2_{x_{c1}} = E[V_1^2\cos^2\vartheta_1] - E^2[V_1\cos\vartheta_1]$, and obtained by applying Eq. (18) as

$$\begin{aligned}\sigma^2_{x_{c1}} &= \frac{1}{2}\sum_{n=l_i}^{l_i} E[\beta_n^2] \\ &= \frac{1}{2}\sum_p\sum_q p'_{R,i}(\alpha_p, \tau'_q)(\triangle\alpha)(\triangle\tau')\end{aligned} \tag{20}$$

because the random variables are assumed to be uncorrelated and $2\pi f_1\tau'_n$ and $\partial\tau_n'$ are uniformly distributed[6]. If the number of scattered signal path is large enough, Eq. (20) is

$$\begin{aligned}\sigma^2_{x_{c1}} &= \frac{1}{2}\int_0^{2\pi}\int_0^{T_c} p'_{R,i}(\alpha_p, \tau')d\tau' d\alpha \\ &= \pi A_i\sigma_\tau[1 - e^{-\frac{T_c}{\sigma_\tau}}]\end{aligned} \tag{21}$$

Similarly, it can be obtained that

$$\sigma^2_{x_{c1}} = \sigma^2_{x_{c2}} = \sigma^2_{x_{s1}} = \sigma^2_{x_{s2}} = \sigma^2_x \tag{22}$$

Using the assumption of uncorrelated distribution for ν_n, τ'_n and $\partial\tau_n'$, and uniform distribution for $\partial\tau_n'$, the covariance of x_{c1} and x_{c2}, $R_c(\tau_\triangle)$ is given by

$$\begin{aligned}R_c(\tau_\triangle) &= E[x_{c1}x_{c2}] - E[x_{c1}]E[x_{c2}] \\ &= E\Big[\sum_{n=l_i}^{l_i}\beta_n^2\cos[\tau_\triangle\omega_m\cos(\alpha_n) - 2\pi(\triangle f)\tau'_n]\Big]\end{aligned} \tag{23}$$

where $\triangle f = f_1 - f_2$ is frequency separation between two subcarriers and $\omega_m = 2\pi f_m$ is maximum Doppler shift. By some manipulations and assuming a large number of paths,

$$\begin{aligned}R_c(\tau_\triangle) &= \frac{1}{2}\int_0^{T_c}\int_0^{2\pi} p'_{R,i}(\alpha, \tau') \\ &\quad \cos[\tau_\triangle\omega_m\cos(\alpha) - 2\pi(\triangle f)\tau']d\alpha d\tau' \\ &= J_0(\omega_m\tau_\triangle)\frac{\pi\sigma_\tau A_i}{1 + 4\pi^2\sigma_\tau^2(\triangle f)^2} \\ &\quad \cdot\Big\{1 - e^{-\frac{T_c}{\sigma_\tau}}[2\pi(\triangle f)\sigma_\tau\sin(2\pi(\triangle f)T_c) \\ &\quad - \cos(2\pi(\triangle f)T_c)]\Big\}\end{aligned} \tag{24}$$

in which $J_0(\cdot)$ is a zero order Bessel function. Similarly, the covariance x_{c1} and x_{c2} is

$$\begin{aligned}R_c(\tau_\triangle) &= E[x_{c1}x_{s2}] - E[x_{c1}]E[x_{s2}] \\ &= J_0(\omega_m\tau_\triangle)\frac{\pi\sigma_\tau A_i}{1 + 4\pi^2\sigma_\tau^2(\triangle f)^2} \\ &\quad \cdot\Big\{2\pi(\triangle f)\sigma_\tau - e^{-\frac{T_c}{\sigma_\tau}}[2\pi(\triangle f)\sigma_\tau \\ &\quad \sin(2\pi(\triangle f)T_c) + \sin(2\pi(\triangle f)T_c)]\Big\}.\end{aligned} \tag{25}$$

It can also be shown that $R_s(\tau_\triangle) = E[x_{s1}x_{s1}] = R_c(\tau_\triangle)$ and $R_{sc}(\tau_\triangle) = E[x_{s1}x_{c2}] = -R_{cs}(\tau_\triangle)$.

Let Λ denotes the covariance matrix of the random variables x_{c1}, x_{s1}, x_{c2} and x_{s2}. Thus, the joint probability density function V_1, V_2, ϑ_1 and ϑ_2 is given by [7]

$$\begin{aligned}&p(V_1, \vartheta_1, V_2, \vartheta_2) \\ &= \frac{V_1V_2}{4\pi^2|\Lambda|^2}\exp\Big\{\frac{1}{2}|\Lambda|^2 \\ &[\sigma_x^2\{V_1^2 + V_2^2 - 2\rho_1V_1\cos\vartheta_1 \\ &- 2\rho_2V_2\cos\vartheta_2 + \rho_1^2 + \rho_2^2\} \\ &- 2R_c(\tau_\triangle)\{V_1V_2\cos(\vartheta_2 - \vartheta_1) \\ &- \rho_2V_1\cos\vartheta_{1-\rho_1}V_2\cos\vartheta_2 + \rho_1\rho_2\} \\ &- 2R_{cs}(\tau_\triangle)\{V_1V_2\sin(\vartheta_2 - \vartheta_1) \\ &- \rho_1V_2\sin\vartheta_2 + \rho_2V_1\sin\vartheta_1\}]\end{aligned} \tag{26}$$

where $|\Lambda|$ is the determinant of the covariance matrix given by $|\Lambda| = [\sigma_x^4 - R_c^2(\tau_\triangle) - R_{cs}^2(\tau_\triangle)]^2$.

The envelope correlation function of two resolvable path signals may now be calculated. Let us define

$$\begin{aligned}R_c(\triangle f, \tau_\triangle) &= E[V_1V_2] \\ &= \int_0^\infty\int_0^\infty V_1V_2p(V_1V_2)dV_1dV_2\end{aligned} \tag{27}$$

where $p(V_1, V_2)$ can be obtained by integrating Eq.

(26). The expected value $E[V_1]$, $E[V_2]$ and variance $\sigma^2_{V_1}$, $\sigma^2_{V_2}$ of Nakagami-Rice distributed signal envelope V_1 and V_2 [7] is obtained by

$$E[V_1]=E[V_2]=\sqrt{\frac{\pi\sigma_x^2}{2}}\exp\left(\frac{-\rho^2}{4\sigma_x^2}\right)\left\{\left(1+\frac{\rho^2}{2\sigma_x^2}\right)I_0\left(\frac{\rho^2}{4\sigma_x^2}\right)+\frac{\rho^2}{2\sigma_x^2}I_1\left(\frac{\rho^2}{4\sigma_x^2}\right)\right\} \quad (28)$$

and

$$\sigma^2_{V_1}=\sigma_x^2\left(2+\frac{\rho^2}{\sigma_x^2}\right)-E^2[V_1]$$
$$\sigma^2_{V_2}=\sigma_x^2\left(2+\frac{\rho^2}{\sigma_x^2}\right)-E^2[V_2] \quad (29)$$

in which $I_0(\cdot)$ and $I_1(\cdot)$ are zero and first order modified Bessel function, respectively and $\rho=\rho_1=\rho_2$ is assumed. Therefore, using Eqs. (27), (28) and (29), the envelope correlation can be obtained as a function of the time difference and frequency separation[8] as

$$\rho_e(\triangle f,\tau_\triangle)=\frac{R_e(\triangle f,\tau_\triangle)-E[V_1]E[V_2]}{\sigma_{V_1}\sigma_{V_2}} \quad (30)$$

IV. Numerical Results

At $\tau_\triangle=0$, Eq. (30) expresses the spaced frequency envelope correlation between wideband signals of two resolvable paths belonging to two disjoint subcarriers in Nakagami-Rice fading channel with exponential power delay profile. Fig. 1 is a plot of the envelope correlation coefficient calculated using Eq. (30) together with the result from [3] as a function of $\triangle f$ for different signal bandwidths of (a)30MHz, (b)10MHz and (c)1MHz and for different values k, the ratio of specular wave power to mean multipath power.

It is found that the spaced frequency envelope correlation of wideband signals decreases as k increases, while that of narrowband signals is independent of k. The percentage decreases, however, is small for larger values of k than 1. In addition, it is shown that the envelope correlation of wideband signals depends on rms delay spread as well as signal bandwidth.

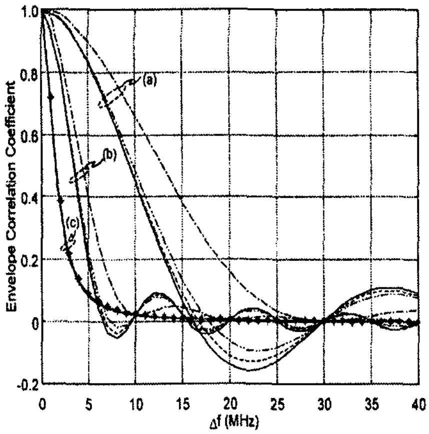

Fig 1. Envelope correlation as function of frequency separation for exponential power delay profile and $\sigma_\tau=0.1\mu$sec. (a) is for $T_c=\sigma_\tau/3$, (b) is for $T_c=\sigma_\tau$ and (c) is for $T_c=10\sigma_\tau$.

- — — — — — k = 0

- — — — — k = 1

– – – – – – – – k = 2

———————— k = 1

*********** from [3]

V. Conlusions

We have analyzed the spaced frequency envelope correlation of wideband signals modulated on two separate carriers in frequency selective Nakagami-Rice fading channels with exponential power delay profile. The results show that the correlation of wideband signals decrease as the ratio of specular wave power to mean multipath power increase. This is comparable with the fact that envelope correlation of narrowband signals is not influenced by specular wave power in [3]. In addition, the envelope correlation of wideband signals depends on rms delay spread as well as signal bandwidth.

These results can be used to select subcarrier spacing to improve the performance of DS-CDMA systems employing RAKE receivers with frequency diversity in indoor or micro-cellular channel environments.

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