헬리콥터 회전날개깃의 안정성 해석과 제어

김종선*, Aditi Chattopadhyay**

Aeromechanical stability analysis and control of helicopter rotor blades

J. S. Kim*, Aditi Chattopadhyay**

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요 약

복합재로 된 회전날개깃을 상자보로 모델링하고 수동/능동 감쇠를 주기 위해 ACL(Active Constrained Damping Layer)을 상하양면에 부착하고 복합변위이론에 기초한 유한요소방법을 이용하여 구조해석을 수행하였다. 이 이론은 ACL내의 복합재와 점탄성층 그리고 압전층의 전단변형효과를 정확하게 모델링하는데 효과적이다. Hankel 의 특이값을 이용해 축차모델을 유도하였으며 축차모델과 측정된 출력에 기초한 LQG 제어기를 설계하였다. 그러나 LQG 제어기는 공칭 운전속도에서는 좋은 성능을 보여주었으나 운전속도가 변하는 상황에 대해서는 강인안정성을 보여주지 못했다. 이 LQG제어기의 강인안정성을 개선하기 위하여 루프전달회복을 통한 강인한 제어기를 설계하였다. 수치 예를 통해 제시된 제어기가 회전날개깃의 공기역학적인 안정성을 개선하는데 효과적이며 동체모드와 연계된 리드-래그 모드감쇠를 증가시켜 회전날개깃의 진동을 효과적으로 억제하는 것을 보였다.

* 한국항공대학교 항공우주 및 기계공학부 교수
** Department of Mechanical and Aerospace Engineering
Arizona State University, USA

Abstract

The rotor blade is modeled using a composite box beam with arbitrary wall. The active constrained damping layers are bonded to the upper and lower surfaces of the box beam to provide active and passive damping. A finite element model, based on a hybrid displacement theory, is used in the structural analysis. The theory is capable of accurately capturing the transverse shear effects in the composite primary structure, the viscoelastic and the piezoelectric layers within the ACLs. A reduced order model is derived based on the Hankel singular value. A linear quadratic Gaussian (LQG) controller is designed based on the reduced order model and the available measurement output. However, the LQG control system fails to stabilize the perturbed system although it shows good control performance at the nominal operating condition. To improve the robust stability of LQG controller, the loop transfer recovery (LTR) method is applied. Numerical results show that the proposed controller significantly improves rotor aeromechanical stability and suppresses rotor response over large variations in rotating speed by increasing lead-lag modal damping in the coupled rotor-body system.

I. INTRODUCTION

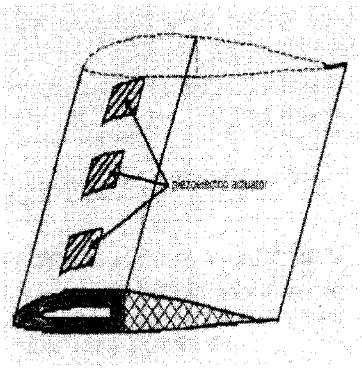
Increase of lead-lag damping in rotor blades has been investigated for many years to improve helicopter aeroelastical and aeromechanical stability. Recent research has shown that improvements in helicopter vibration reduction, aeroelastic stability and aeromechanical stability can be achieved by using smart materials and active control techniques [1]. The use of segmented active constrained layer (SCL) damping treatment for passive augmentation of ground and air resonance stability was investigated by Chattopadhyay et al.[2]. The study indicates that significant improvement in lead lag damping can be achieved through the use of this type of damping treatment.

The use of active constrained layer (ACL) damping treatment has been investigated by many researchers in the context of vibration control. The piezoelectric layers have sensing and control capabilities that actively tune the shear of the viscoelastic layer based on the structural response. Considerable amount of research has been performed in modeling ACL, as summarized by Ro and Baz [3]. It is also well known that segmentation of the constrained layers provides an effective means of increasing passive damping in low frequency vibration modes by increasing the number of high shear regions[4]. A more comprehensive approach to model sparse segmented ACL damping treatment on composite plates of arbitrary thickness was developed by Chattopadhyay et al. [5]. The rotary wing applications can be found in be [6]. In Ref. 2, the segmented ACL configuration was used to investigate improvement in passive inplane damping in rotor blades; no active control technique was employed. An active control method based on the linear quadratic Gaussian (LQG) technique was developed in Ref. 7. To deal with the time-variant characteristics of the dynamic model due to rotor rotation, a transformation matrix was introduced in that work. However, although the proposed controller was very effective in improving aeromechanical stability, the model reduction and the associated robustness issues were not addressed.

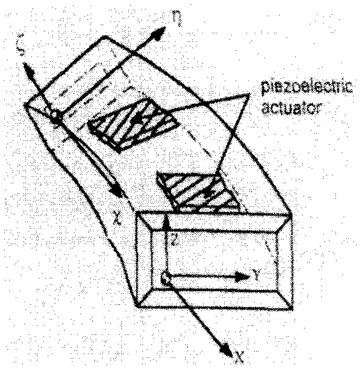
The objective of this paper is to investigate the robustness of the reduced order controller by taking into account some of the associated uncertainties, based on Ref. [7], for a smart rotor blade built around a composite box beam with segmented active constrained layers. An air resonance model is used to investigate the coupled rotor-body stability. Both model reduction and robust stability, which are important factors for real-time implementation, are addressed. A balanced model reduction method using Hankel singular values is used for the model reduction. A LQG controller is then designed based on the reduced order model. As shown numerically, the controller is capable of stabilizing the unstable open loop system at nominal operating condition. However, the LQG controller based on the reduced order model does not exhibit robust stability under model uncertainties caused by the variation in the rotor rotating speed. To improve the robust stability of the LQG design, the loop transfer recovery (LTR) technique is introduced. Through numerical study, it is shown that the proposed controller effectively stabilizes the perturbed systems as well as the nominal system.

II. MODELING OF SMART COMPOSITE ROTOR BLADES

The principal rotor load-carrying member is represented by a composite box beam of arbitrary wall thickness as shown in Figure 1. A hybrid displacement theory developed by Chattopadhyay et al. [5] is used to model surface bonded ACL on a composite plate.



a) Rotary wing with ACLs



b) Composite box beam

Figure 1 Configuration of composite box beam with ACLs

The air resonance model is considered. Only rigid body pitch and roll rotation degrees of freedom are taken into account in this model. A fundamental flap modal displacement and a fundamental lead-lag modal displacement are considered. The blade pitch degree of freedom is not included in the analysis. ACLs are bonded on the top and bottom surfaces of the composite box beam, which represent the load-carrying member of the rotor blade. It is assumed that the blade mass is distributed uniformly along the blade span and the plat form is rectangular. It is also assumed that there is no geometric twist. It is assumed that there is no structural coupling between flap and lead-lag motions. The individual blade flap and lead-lag motions are combined together and are transferred to the nonrotating coordinate system through multiblade transformation. The aerodynamic forces are calculated based on quasi-steady lifting line theory, combined with a dynamic inflow model.

The finite element model consists of 528 degrees of freedom, which results in 1056-th order open loop model. In order to reduce the full model for real time control problem, a 14-th order reduced order model is derived. By combining equations of structural dynamics and air resonance model, the equation of motion becomes the linear time-variant system in the state space form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + F(t)v(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t)$, $v(t)$ and $y(t)$ are the state, input and output vectors, respectively, and A and C are the system and output matrices, respectively. The control matrix $F(t)$ is periodic, that is $F(t) = F(t+\tau)$ with period $\tau = 2\pi/\Omega$, due to the rotating nature of the blade with rotational speed Ω. It is assumed that only two lead-lag angles are measured.

Ⅲ. ROBUST CONTROLLER DESIGN

Using the transformation matrix defined as

$$T(t) = \begin{bmatrix} \cos\Omega t & -\sin\Omega t \\ \sin\Omega t & \cos\Omega \end{bmatrix} \tag{2}$$

the time varying system can be transformed to the time-invariant design problem by using $v(t) = T(t)u(t)$. The detailed trnaformation procedure is shown in reference 7.

A balanced model reduction method based on Hankel singular values is used in order to obtain the reduced order model. The Hankel singular value is a measure of the significance of the associated mode in the dynamic model. In order to use the balanced model reduction, a projection method is applied. The method decomposes the minimal realization model G(s) to stable and antistable part as follows [10]

$$G(s) = [G(s)]_- + [G(s)]_+ \tag{3}$$

where $[G(s)]_-$ and $[G(s)]_+$ are the unstable and stable subset of the full model, respectively. The unstable subset should be included in the reduced order model. A balanced model reduction procedure is applied to the stable part $[G(s)]_+$. After obtaining the reduced order model $[G(s)]_{m+}$ of $[G(s)]_+$, a reduced order model $[G(s)]_m$ of $[G(s)]$ can be obtained as follows

$$G(s)_m = [G(s)]_- + [G(s)]_{m+} \tag{4}$$

The state space equation of reduced order model is derived as follows

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u(t) \\ y(t) &= C_m x_m(t) \end{aligned} \tag{5}$$

and the performance index can be stated as

$$J_m = E\{\lim_{T\to\infty}\frac{1}{T}\int_0^T [x'_m(t)Q_m x_m(t) + \rho u'(t)u(t)]dt\} \quad (6)$$

where x_m is the reduced state and A_m, B_m and are C_m the state, input and output matrices of the reduced order model, respectively. The control problem is to find the output feedback control input u(t) in terms of output y(t) so as to minimize the performance index, J_m where Q_m is symmetric and positive semi-definite and ρ is a positive scalar. In this research, a transformation matrix is used to transform the time-variant system to the time-invariant system, the equation of LQG controller based on the reduced order model has the following form

$$\begin{aligned} \dot{x}_m(t) &= A_m\hat{x}_m(t) + B_m u(t) + H_m[y(t) - C_m\hat{x}_m(t)] \\ u(t) &= -K_m\hat{x}_m(t) \end{aligned} \quad (7)$$

where $\hat{x}_m$ is the estimated value of the reduced state x_m. The control gain matrix K_m and filter gain matrix H_m are determined from the linear quadratic control theory and the Kalman filter theory, respectively. The matrices K_m and H_m are obtained from two algebraic Riccati equations; control algebraic Riccati equation and filter algebraic Riccati equation [8, 9]

Numerical investigation shows that the LQG controller stabilizes the unstable lead-lag modes at nominal operating condition. However, it fails to guarantee robust stability under model uncertainties. To ensure robust stability, the loop transfer recovery technique is applied by using the method of Doyle and Stein [8]. The loop transfer function of the linear quadratic controller

$$G_L(s) = K_m(sI - A_m)^{-1}B_m \quad (8)$$

is recovered through the loop transfer recovery procedure. The loop transfer recovery property means that for stabilizable , observable minimum phase plants satisfying

$$G_L(s) = C_m(sI - A_m)^{-1}B_m \text{ is nonsingular in } Re\ s \geqq 0 \quad (9)$$

the loop gain transfer matrix in a full-state feedback design, equation (8), is recovered in a full-order state estimate feedback design under a certain limiting operation.

Ⅳ. RESULTS AND DISCUSSIONS

The dimensions of the box beam are such that the length of 6m, width of 0.17m, and height of 0.043m. All walls are assumed to have the same stacking sequence. In the ACL configuration, the piezoelectric layer thickness is 1.96mm and the viscoelastic layer thickness is 0.98 mm. The aeromechanical behavior of a rotor blade built around the composite box beam, with one pair of top and bottom surface bonded ACLs, is studied. The coupled rotor-body system poles for the seven system modes are as follows: lead-lag regressive mode (LR), lead-lag advancing mode (LA), flap regressive mode (FR), flap advancing mode (FA), gyroscopic mode (GS), dynamic inflow mode (DI) and zero root mode.
To perform model reduction, the Hankel singular values are calculated as shown in Figure 2. The first four singular values contributed by the lead-lag states are dominant compared with the other singular values.

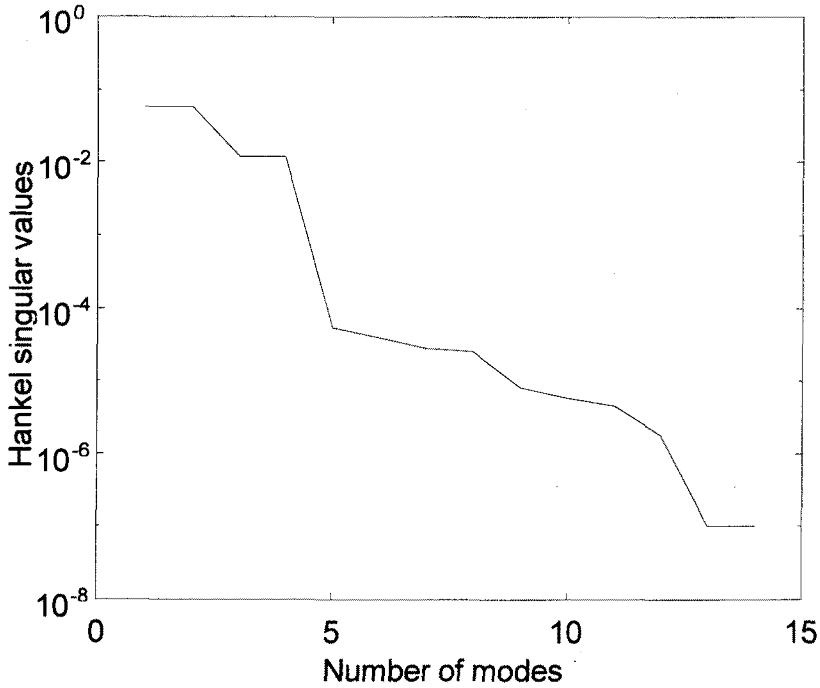


Figure 2 Hankel singular values

Actually, the lead-lag modes are more important since these modes often make the system unstable. Based on the obtained singular values, a 4th order reduced order model is derived as follows.

$$A_m = \begin{bmatrix} -0.610 & -0.140 & 0.402 & 0.902 \\ 0.960 & 0.495 & -0.996 & -0.279 \\ -0.604 & 0.692 & 0.575 & 0.009 \\ -0.129 & 1.160 & 0.753 & -0.446 \end{bmatrix}$$

$$B_m = \begin{bmatrix} -7.130e-6 & -1.764e-4 \\ 1.831e-4 & 4.941e-5 \\ -4.765e-5 & 1.120e-4 \\ -1.340e-4 & 5.097e-5 \end{bmatrix}$$

$$C_m = \begin{bmatrix} -0.106 & 0.479 & -0.724 & 0.777 \\ 0.593 & 0.502 & 0.723 & 0.382 \end{bmatrix}$$

A LQG controller is designed based on the reduced order model. The reduced order controller stabilizes the full model as well as the reduced order model. However, the LQG controller fails to stabilize the perturbed system when the rotating speed changes. In order to improve the robust stability, a loop transfer recovery procedure is applied to the LQG design and it is represented in Figure 3. When the parameter q, weighting for fictitious noise in Doyle and Stein's loop transfer recovery procedure, becomes large, the designed LQG/LTR controller almost recovers the loop transfer characteristics of the state feedback control system. As is well known, the state feedback controller based on the LQ design guarantees the robust stability under model uncertainties.

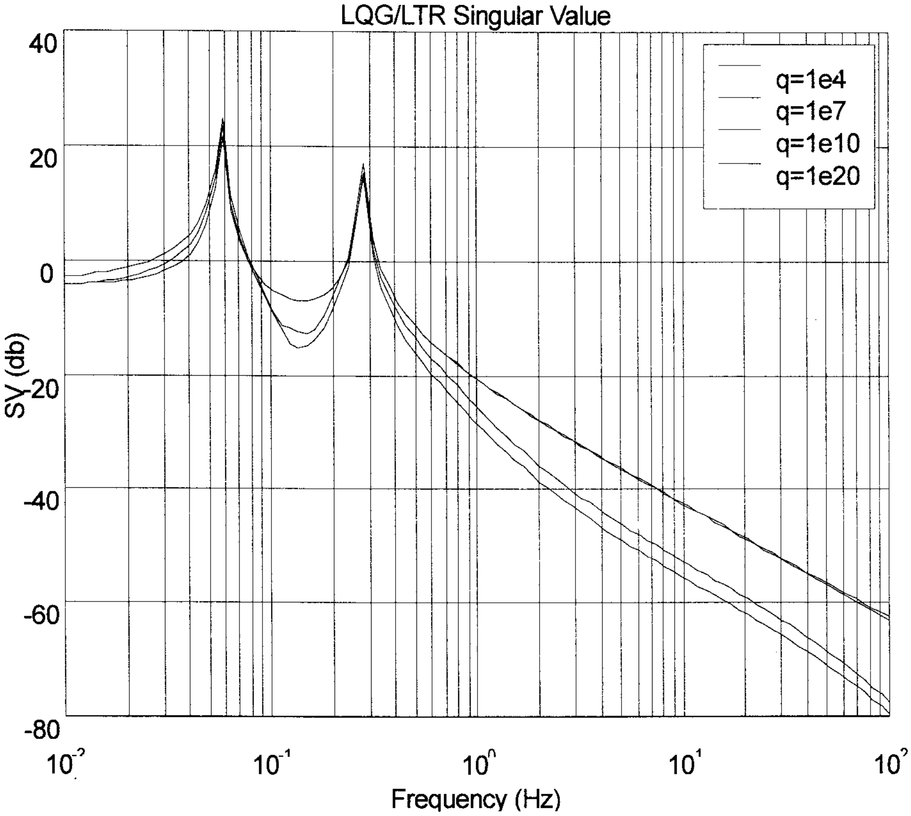


Figure 3 LQG/LTR singular values with respect to weight q

The system poles of open loop, passive control, LQG and LQG/LTR systems at nominal rotating speed are shown in Figure 4. As shown in Figure 4, the open loop system is unstable without sufficient rotor mechanical lead-lag damping. The unstable modes are the LR and LA modes. It is well known that lead-lag motion is associated with lower modal damping due to less aerodynamic loads. With the application of active control methods to the coupled system, the closed loop system is stabilized. The lead-lag damping ratios of the closed loop system are about three times larger than the corresponding passive system at nominal operating condition. To examine the robustness of the proposed control scheme, the poles of the open loop and closed loop systems at other operating speeds are calculated. The LQG and LQG/LTR control systems show almost similar performance at nominal operating condition. As the rotor rotating speed decreases, the LQG controller does not guarantee stability any longer. However, the LQG/LTR controller stabilizes the perturbed system by locating all closed loop poles in the left half plane as shown in Figure 4. While the relative stability of the lead-lag regressive mode becomes worse as the rotating speed increases, it is shown that the closed loop system remains stable under (25 % variations in speed.

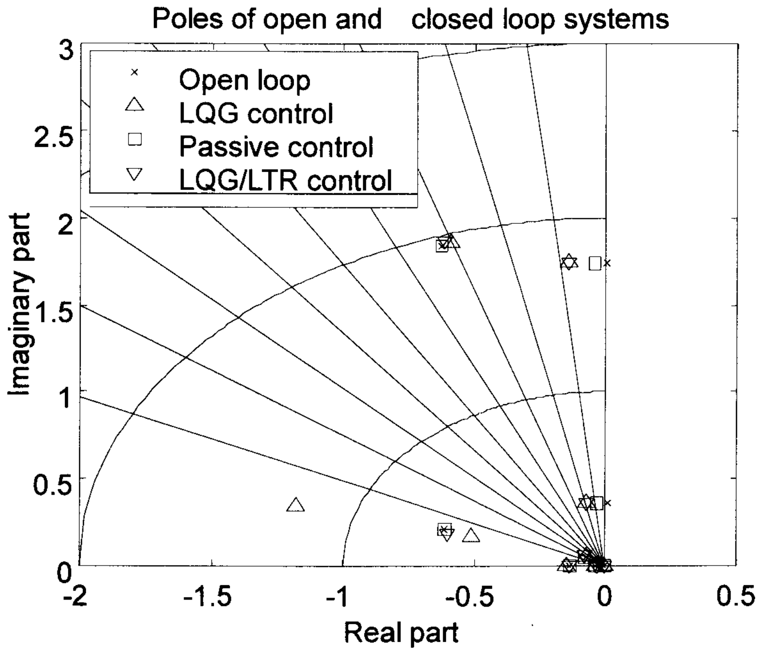
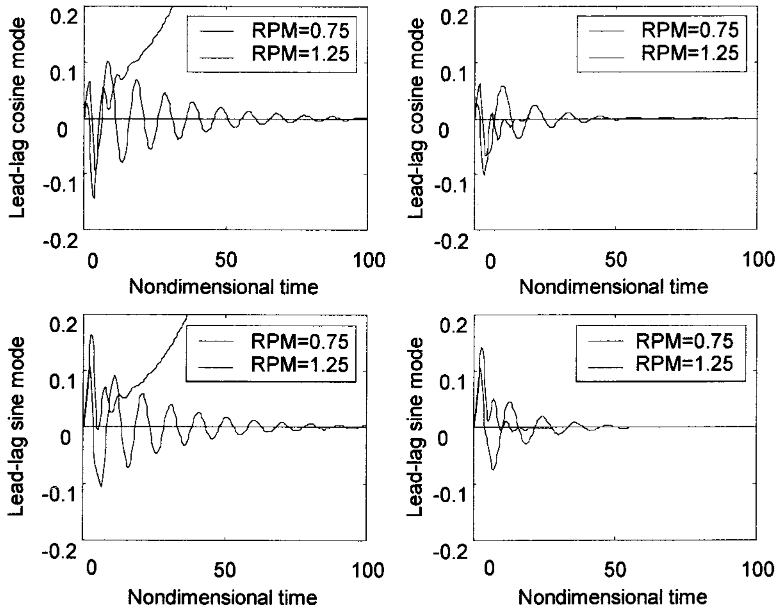
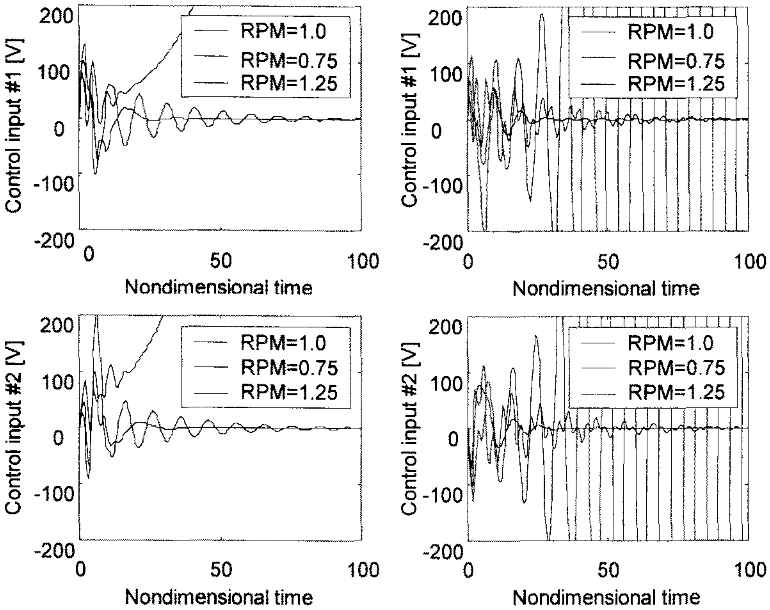


Figure 4 Poles of open and closed loop systems



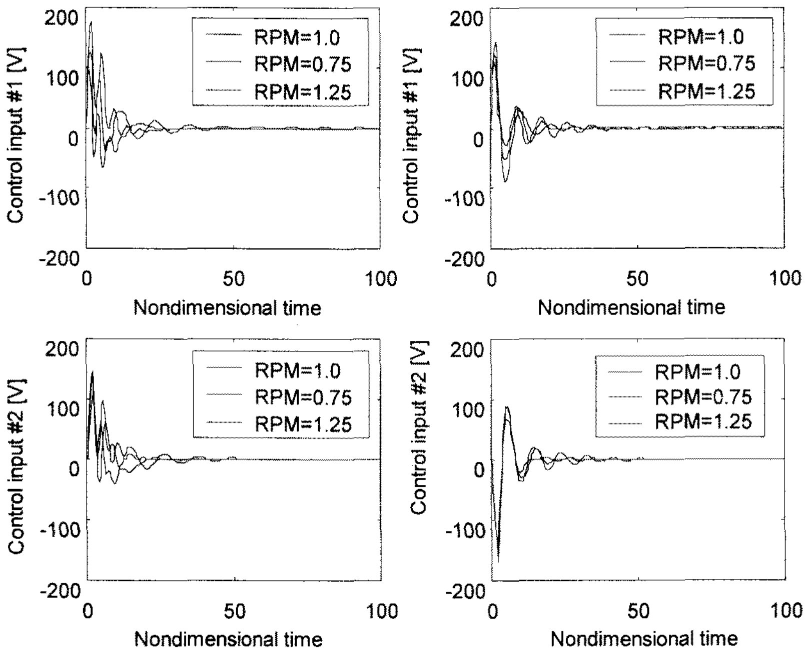
a) LQG controller, b) LQG/LTR controller

Figure 5 Response of the control systems (Ω=0.75, 1.25)



a) Calculated control input [u(t)], b) Actual control input [v(t)]

Figure 6 Control input of LQG controller



(a) Calculated control input [u(t)], (b) Actual control input [v(t)]

Figure 7 Control input of LQG/LTR controller

Figures 5 shows the impulse response of the closed loop system at the perturbed operating points. The LQG/LTR control system is sufficient to suppress the unstable lead-lag vibrations over the wide operating range, while the LQG controller fails to stabilize the body coupled instability at lower rotating speed. In order to examine the required control energy, the electric voltages for four pairs of actuators are also shown in Figures 6 and 7. The 3rd and 4th control inputs are the negative values of the 1st and 2nd control inputs, respectively, because of the geometric symmetry of the rotor and the actuator. The control inputs in Figure 6, calculated based on the LQG control scheme, show increase in control energy due to the instability of the control system as the rotating speed decreases. However, the control inputs in Figure 7, calculated based on the LQG/LTR control scheme, converge to zero asymptotically and the magnitudes are smaller than the LQG control systems.

V. CONCLUSIONS

Based on the Hankel singular value analysis, a reduced order model is derived and a LQG controller is designed based on that model. To improve the robust stability of the LQG design under model uncertainties, caused by the variation in rotor rotating speed, the loop transfer recovery (LTR) technique is introduced. Numerical results indicate that the surface bonded ACL actuators with LQG/LTR control significantly increase rotor lead-lag regressive and advancing modal damping in the coupled rotor-body system over a wide range of rotating speed. The following important observations are made from the present study.

(1) The segmented constrained layers, bonded on the top and bottom surfaces of the rotor blade, significantly improve the damping of the lead-lag modes in the air resonance model.
(2) The combined use of the Hankel singular value analysis and robust control method, shows that the 14th order control system can be reduced to the 4th order while preserving the control performance of the 14th order control system.
(3) The LQG/LTR design based on the reduced order model is efficient in improving the robust stability of aeromecanical rotor response over a wide range of rotating speed.

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