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σ-COHERENT FRAMES

SEUNG ON LEE

ABSTRACT. We introduce a new class of σ-coherent frames and show that $\mathcal{H}A$ is a σ-coherent frame if A is a σ-frame. Based on this, it is shown that a frame is σ-coherent iff it is isomorphic to the frame of σ-ideals of a σ-frame. Finally we show that **σCohFrm** and **σFrm** are equivalent.

0. Introduction.

It is well known that for any topological space X, its topology $\Omega(X)$ is a frame. Many efforts have been made to generalize continuous lattices, and to extend corresponding properties to them as appeared in studies of Banaschewski [1, 2], Hoffmann [3, 4], and Madden and Vermeer [7].

Using the way below relation, it is shown that the category **Co-hFrm** of coherent frames and coherent homomorphisms is coreflective in the category **Frm** of frames and frame homomorphisms, Finally **CohFrm** and **DLatt**, where **DLatt** is the category of distributive lattices and lattice homomorphisms, are equivalent. The purpose of this paper is to introduce a new class of σ-coherent frames as a generalization of coherent frames, and examine its properties. To do so, we introduce σ-frames and countably approximating frames. Throughout this paper, a lattice means a bounded lattice, i.e., a lattice with the top element e and the bottom element 0. For terminology not introduced here, we refer to [5].

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1. σ-frames.

Recall that a subset D of a poset A is called *directed (countably directed, resp.)* if every finite (countable, resp.) subset of D has an upper bound in D. D is called a *down set* if $D = \downarrow D$ where $\downarrow D = \{y \in A | y \leq x \text{ for some } x \in D\}$, and D an *ideal (σ-ideal, resp.) of* A if it is a directed (countably directed, resp.) down set. Let $P(X)$ denote the power set lattice of a set X endowed with the inclusion relation $\subseteq$. Then $Fin(X)$ denotes the ideal of finite subsets of X in $P(X)$ and $Count(X)$ the σ-ideal of countable subsets of X in $P(X)$. For a distributive lattice A, the set of all ideals (σ-ideals, resp.) is denoted by $\mathcal{J}A$ ($\mathcal{H}A$, resp.), and $\mathcal{J}A$ ($\mathcal{H}A$, resp.) is closed under directed (countably directed, resp.) unions and arbitrary intersections; hence $\mathcal{J}A$ and $\mathcal{H}A$ are complete lattices. If A is a distributive lattice (σ-frame, resp.) then $\mathcal{J}A$ and $\mathcal{H}A$ are frames([6]).

DEFINITION 1.1. 1) For x, y in a complete lattice A, x is said to be *way below (countably way below, resp.)* y, in symbols $x \ll y$ ($x \ll_c y$, *resp.)*, if for any directed (countably directed, resp.) subset D of A with $y \leq \bigvee D$, there is $d \in D$ with $x \leq d$.

If $x \ll x$ ($x \ll_c x$, resp.), then x is said to be a *compact (Lindelöf, resp.) element of* A.

2) A complete lattice A is said to be a *compact (Lindelöf, resp.) lattice* if the top element e of A is a compact (Lindelöf, resp.) element of A.

The set of all compact elements of A will be denoted by $K(A)$, and the set of all Lindelöf elements by $L(A)$.

3) A complete lattice A is said to be a *frame* if for any $x \in A$ and $S \subseteq A$,

$$x \wedge (\bigvee S) = \bigvee \{x \wedge s | s \in S\}.$$

4) For frames X and Y, a map $f : X \to Y$ is said to be a *frame*

homomorphism if f preserves arbitrary joins and finite meets.

The class of all frames and frame homomorphisms between them form a category which will be denoted by **Frm**.

Remark 1.2 ([6]). Let A be a complete lattice and $x, y \in A$. Then the following are equivalent:

1) $x \ll_c y$.

2) If $y \leq \bigvee X (X \subseteq A)$, then there is $K \in Count(X)$ with $x \leq \bigvee K$.

3) If $y \leq \bigvee I (I \in \mathcal{H}A)$, then $x \in I$.

Remark 1.3 ([6]). Let A be a frame, then the following are equivalent:

1) $x \ll_c y$.

2) If $y = \bigvee X (X \subseteq A)$, then there is $K \in Count(X)$ with $x \leq \bigvee K$.

3) If $y = \bigvee I (I \in \mathcal{H}A)$, then $x \in I$.

PROPOSITION 1.4 ([6]). *In a complete Lattice A, one has the following:*

1) If $x \ll_c y$, then $x \leq y$. $(x, y \in A)$

2) If $u \leq x \ll_c y \leq v$, then $u \ll_c v$. $(x, y, u, v \in A)$

3) For any sequence (x_n) in A such that $x_n \ll_c y$ $(n \in N)$, $\bigvee\{x_n | n \in N\} \ll_c y$.

4) $0 \ll_c x$. $(x \in A)$

5) If $x \ll y$, then $x \ll_c y$. $(x, y \in A)$

COROLLARY 1.5 ([6]). *Let A be a complete lattice, then one has:*

1) 0 is a Lindelöf element.

2) If (x_n) is a sequence of Lindelöf elements, then $\bigvee x_n$ is again a Lindelöf element.

Remark 1.6. 1) For a complete lattice A, $\downarrow_c x = \{y \in A | y \ll_c x\}$ is a σ-ideal of A, which is contained in $\downarrow x$.

2) For a lattice A, $\downarrow a$ is a Lindelöf element of $\mathcal{H}A$ for any $a \in A$, because for any countably directed subset $\mathcal{E}$ of $\mathcal{H}A$, $\downarrow a \subseteq \bigvee \mathcal{E}$ iff

$a \in \bigcup \mathcal{E}$ iff $a \in S$ for some $S \in \mathcal{E}$ iff $\downarrow a \subseteq S$ for some $S \in \mathcal{E}$; hence $\downarrow a$ is a Lindelöf element of $\mathcal{H}A$. Thus $\{\downarrow a | a \in A\} \subseteq L(\mathcal{H}A)$.

DEFINITION 1.7. 1) A lattice A is said to be *distributive* if

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

for any $x, y, z \in A$.

2) A lattice with countable joins A is said to be a *σ-frame* if

$$x \wedge (\bigvee K) = \bigvee \{x \wedge k | k \in K\}$$

for any $x \in A$ and $K \in Count(A)$.

EXAMPLE 1.8. 1) Every σ-frame is a distributive lattice, but a distributive lattice need not be a σ-frame. Let $\Gamma(R)$ be the set of all closed subsets of R, where R is the real line endowed with the usual topology. Then $\Gamma(R)$ is a distributive lattice, but not a σ-frame. For $x = \{0\}$ and $K = \{x_n = [\frac{1}{n}, 3 - \frac{1}{n}] | n \in N\}, x \wedge (\bigvee K) = \{0\}$ but $\bigvee\{x \wedge x_n | x_n \in K\} = \phi$.

2) A frame is a σ-frame, but the converse need not be true in general. Let $\Gamma(R_c)$ be the set of all closed subsets of R, where R_c is the real line endowed with the cocountable topology. Then $\Gamma(R_c)$ is a σ-frame but not a frame. For $S = \{\{s\} | s \in R - \{1\}\}$ and $x = \{1\}$, $x \wedge (\bigvee S) = x$ but $\bigvee\{x \wedge t | t \in S\} = \phi$.

DEFINITION 1.9. For σ-frames X and Y, a map $f : X \to Y$ is said to be a *σ-frame homomorphism* if f preserves countable joins and finite meets.

The class of all σ-frames and σ-frame homomorphisms between them forms a category which will be denoted by σ**Frm**.

PROPOSITION 1.10. *Let A be a σ-frame, then Lindelöf elements of $\mathcal{H}A$ are precisely principal ideals.*

Proof. Take any Lindelöf element I of $\mathcal{H}A$, then $I = \bigvee\{\downarrow x | x \in I\}$. Since $I \ll_c I$, there is $K \in Count(I)$ such that $I \leq \bigvee\{\downarrow x | x \in K\}$ in $\mathcal{H}A$. Let $a = \bigvee K$, then $\bigvee\{\downarrow x | x \in K\} = \downarrow a$; hence $I = \downarrow a$. Conversely, $\downarrow a$ is a Lindelöf element of $\mathcal{H}A$ by 2) of Remark 1.6.

2. δ-frames.

In this section, we introduce a concept of δ-frames and study the relations between Lindelöf regular δ-frames and countably approximating frames.

DEFINITION 2.1. A lattice with countable meets is said to be a δ-*frame* if

$$x \vee (\bigwedge K) = \bigwedge\{x \vee k | k \in K\}$$

for any $x \in A$ and $K \in Count(A)$.

EXAMPLE 2.2. Every δ-frame is a distributive lattice, but a distributive lattice need not be a δ-frame. Let $\Omega(R)$ be the open set frame of the real line endowed with the usual topology, then $\Omega(R)$ is a distributive lattice but not a δ-frame. So, a frame need not be a δ-frame. Clearly a δ-frame need not be a frame. The σ-frame $\Gamma(R_c)$ in 2) of Example 1.8 is a δ-frame but not a frame.

DEFINITION 2.3. 1) A complete lattice A is said to be a *countably approximating lattice* if

$$x = \bigvee\{u \in A | u \ll_c x\}$$

for all $x \in A$, equivalently $x = \bigvee \downarrow_c x$.

2) A frame A is said to be *regular* if $a = \bigvee\{t \in A | t \prec a\}$ for all $a \in A$, where $t \prec a$ iff $t \wedge x = 0$ and $a \vee x = e$ for some $x \in A$,

or equivalently, $a \vee t^* = e$ for the pseudocomplement $t^* = \bigvee\{s \in A | t \wedge s = 0\}$ of $t \in A$.

In a frame A and $x_n \in A$ $(n \in N)$, $x_n \prec a$ does not imply $\bigvee x_n \prec a$. In fact, in the open set frame $\Omega(R)$ of the real line endowed with the usual topology, $x_n = (\frac{1}{n}, 3 - \frac{1}{n}) \prec (0,3)$ for any $n \in N$ but $\bigvee x_n = (0,3) \not\prec (0,3)$. In a compact regular frame, $x \ll y$ iff $x \prec y$. But in a Lindelöf regular frame, $x \ll_c y$ does not imply $x \prec y$ in general. In the Lindelöf regular frame $\Omega(R)$, $(0,3) \ll_c (0,3)$ but $(0,3) \not\prec (0,3)$. If A is a δ-frame, $x_n \prec a$ implies $\bigvee x_n \prec a$; hence $\{t \in A | t \prec a\}$ is a σ-ideal of A. So we have the following:

Remark 2.4. Let A be a Lindelöf frame, then we have:

1) If $x \prec y$, then $x \ll_c y$ $(x, y \in A)$.

2) If A is a regular δ-frame, then $x \prec y$ iff $x \ll_c y$ $(x, y \in A)$.

Proof. 1) Suppose $x \prec y$ and $y \leq \bigvee S$ for any $S \subseteq A$. Then $x^* \vee y = e$ implies $x^* \vee (\bigvee S) = e$. Since A is a Lindelöf frame, there is $K \in Count(S)$ with $x^* \vee (\bigvee K) = e$. Hence $x \leq \bigvee K$ for some $K \in Count(S)$. So $x \ll_c y$.

2) Let $x \ll_c y$ and $y = \bigvee\{t \in A | t \prec y\}$. Then $x \in \{t \in A | t \prec y\}$, because $\{t \in A | t \prec y\}$ is a σ-ideal of A. So $x \prec y$. □

Remark 2.5. Every Lindelöf regular frame is countably approximating, because for any $a \in A$, $a = \bigvee\{x \in A | x \prec a\} \leq \bigvee \downarrow_c a \leq \bigvee \downarrow a \leq a$ implies $a = \bigvee \downarrow_c a$. But the converse need not be true. The open set frame $\Omega(R_c)$ of the real line with the cocountable topology is countably approximating, but not regular.

3. σ-coherent frames.

In this section, we introduce a concept of σ-coherent frames and study the relations between σ-coherent frames and σ-frames.

DEFINITION 3.1. 1) A frame A is said to be *coherent* if $K(A) = \{a \in A | a$ is a compact element$\}$ is a sublattice of A and $K(A)$ generates A.

2) A frame A is said to be *σ-coherent* if $L(A) = \{a \in A | a$ is a Lindelöf element$\}$ is a subσ-frame of A and $L(A)$ generates A.

In a frame A, $L(A)$ is closed under countable joins. Thus we have:

$L(A)$ is a subσ-frame of A iff $e \in L(A)$ and for $x, y \in L(A)$, $x \wedge y \in L(A)$.

Remark 3.2. 1) The open set frame $\Omega(R_c)$ in Remark 2.5 is a σ-coherent frame. In case, $L(A) = A$.

2) A σ-coherent frame need not be a coherent frame. For example, the complete chain $[0, 1]$ with the usual order $\leq$ is a σ-coherent frame but not a coherent frame.

3) Every σ-coherent frame A is a countably approximating frame, because for any $a \in A$, $a = \bigvee(\downarrow a \cap L(A)) \leq \bigvee \downarrow_c a \leq a$ implies $a = \bigvee \downarrow_c a$.

4)Let $T = [0, \Omega) \cup \{z_1, z_2\}$, where Ω is the first uncountable ordinal, $x \leq z_1, z_2$ for all $x \in [0, \Omega)$ and $[0, \Omega)$ is a chain with the ordinal order $\leq$. Then $\mathcal{D}T = \{U \subseteq T | U = \downarrow U\}$ is a countably approximating frame and $L(\mathcal{D}T) = \{T, \downarrow z_1, \downarrow z_2\} \cup \{\downarrow x | x \in [0, \Omega)\}$. Consider $\downarrow z_1$ and $\downarrow z_2$ are Lindelöf elements of $\mathcal{D}T$, but $\downarrow z_1 \cap \downarrow z_2 = [0, \Omega)$ is not a Lindelöf element of $\mathcal{D}T$, because $[0, \Omega) = \bigcup\{\downarrow x | x \in [0, \Omega)\}$ has no countable subcover. So $\mathcal{D}T$ is not a σ-coherent frame, because $L(\mathcal{D}T)$ is not a subσ-frame of $\mathcal{D}T$. But $L(\mathcal{D}T)$ generates $\mathcal{D}T$.

Consider $\downarrow z_1$ and $\downarrow z_2$ are Lindelöf elements of $\mathcal{D}T$, but $\downarrow z_1 \cap \downarrow z_2$ is not a Lindelöf elements of $\mathcal{D}T$. So in a frame A, the relation $\ll_c$ is not closed under finite meets. That is, $x \ll_c a$ and $x \ll_c b$ need not imply $x \ll_c a \wedge b$ for some $x, a, b \in A$, where A is a frame.

Let A be a frame, $\bigvee : \mathcal{H}A \to A$ is the map defined by $\bigvee(I) = \bigvee I$, and $\downarrow$: $A \to \mathcal{H}A$ the map defined by $d(x) = \downarrow x$. Then the frame homomorphism $\bigvee : \mathcal{H}A \to A$ is a left adjoint of the map $\downarrow : A \to \mathcal{H}A$. In case, $\bigvee$ preserves arbitrary joins and $\downarrow$ preserves arbitrary meets.

PROPOSITION 3.3. *Let A be a complete lattice, then the followings are equivalent:*

1) A is a countably approximating lattice.

2) For each $x \in A$, the set $\downarrow_c x$ is the smallest σ-ideal I with $x \leq \bigvee I$.

3) For each $x \in A$, there is a smallest σ-ideal I with $x \leq \bigvee I$.

4) The map $\bigvee : \mathcal{H}A \to A$ has a left adjoint.

5) The map $\bigvee : \mathcal{H}A \to A$ preserves arbitrary meets and joins.

A is a countably approximating lattice iff for each $x \in A$, the set $\downarrow_c x$ is the smallest σ-ideal I with $x \leq \bigvee I$. So $\downarrow_c$: $A \to \mathcal{H}A$ is a left adjoint of the map $\bigvee : \mathcal{H}A \to A$ where A is a countably approximating lattice. So the map $\downarrow_c$: $A \to \mathcal{H}A$ preserves arbitrary joins if A is a countably approximating lattice.

PROPOSITION 3.4. *In* **Frm**, *let A be a regular δ-frame. Then A is a Lindelöf frame iff the frame homomorphism $\bigvee : \mathcal{H}A \to A$ has a right inverse.*

Proof. ($\Leftarrow$) Let $h : A \to \mathcal{H}A$ be a right inverse of $\bigvee : \mathcal{H}A \to A$. Then $\bigvee \circ h = 1_A$ and h is an 1-1 frame homomorphism. So A is isomorphic to $h(A)$ and $h(A)$ is a subframe of $\mathcal{H}A$. Since $\mathcal{H}A$ is a Lindelöf frame, A is also a Lindelöf frame.

($\Rightarrow$) Define $h : A \to \mathcal{H}A$ as $h(a) = \{t \in A | t \prec a\}$ for all $a \in A$, then $h(a)$ is a σ-ideal in A, because A is a δ-frame; hence h is a map. By Remark 2.5, $h(a) = \downarrow_c a$ and $\bigvee \circ h = 1_A$. Since the relation $\prec$ is closed under finite meets, the map h is closed under finite meets. Furthermore, $\downarrow_c$: $A \to \mathcal{H}A$ is a left adjoint of the map $\bigvee : \mathcal{H}A \to A$, so the map h preserves arbitrary joins. In all, h is a frame homomorphism. □

DEFINITION 3.5. For σ-coherent frames X and Y, a frame homomorphism $h : X \to Y$ is said to be *σ-coherent* if $h(L(X)) \subseteq L(Y)$.

The class of all σ-coherent frames and σ-coherent homomorphisms between them forms a category which will be denoted by σ**CohFrm**.

PROPOSITION 3.6. *Let A be a σ-frame, then we have:*
1) $\mathcal{H}A$ is a σ-coherent frame.
2) $L(\mathcal{H}A)$ is a σ-frame.
3) The down map $\downarrow: A \to L(\mathcal{H}A)$ is an isomorphism.

Proof. 1) $L(\mathcal{H}A) = \{\downarrow a | a \in A\}$ by Proposition 1.10. Consider $\downarrow a \wedge \downarrow b = \downarrow (a \wedge b) \in L(\mathcal{H}A)$ and $\mathcal{H}A$ is a Lindelöf frame. For any $I \in \mathcal{H}A$, $I = \bigvee\{\downarrow x | x \in I\}$. Thus $\mathcal{H}A$ is σ-coherent.

2) It follows from 1) together with the fact that $L(\mathcal{H}A)$ is a subset of a frame $\mathcal{H}A$

3) By Proposition 1.10, the down map $\downarrow$ is an $1-1$ onto map which is closed under arbitrary meets. Moreover $\downarrow (\bigvee K) = \bigvee\{\downarrow k | k \in K\}$ for any $K \in Count(A)$, so $\downarrow$ is a σ-frame homomorphism. Hence $\downarrow$ is an isomorphism. □

Remark 3.7. Let A be a σ-coherent frame, then we have:
1) $\downarrow (\downarrow a \cap L(A)) = \downarrow_c a$ for all $a \in A$.
2) $\downarrow (\downarrow a \cap L(A)) = \downarrow a$ for all $a \in L(A)$.

Proof. 1) Take any $x \in \downarrow (\downarrow a \cap L(A))$, then there is $y \in \downarrow a \cap L(A)$ with $x \leq y$; hence $x \leq y \ll_c y \leq a$. Thus $x \ll_c a$; hence $x \in \downarrow_c a$. Conversely, take any $x \in \downarrow_c a$, then $x \ll_c a$, and $a = \bigvee(\downarrow a \cap L(A))$ since A is σ-coherent. Thus $a = \bigvee(\downarrow (\downarrow a \cap L(A)))$ and $\downarrow (\downarrow a \cap L(A))$ is a σ-ideal of A; hence $x \in \downarrow (\downarrow a \cap L(A))$.

2) It follows from 1) together with the fact that for $a \in L(A)$, $\downarrow_c a = \downarrow a$. □

PROPOSITION 3.8. *A frame is σ-coherent if and only if it is isomorphic to the frame of σ-ideals of a σ-frame.*

Proof. ($\Rightarrow$) Let A be a σ-coherent frame, then $L(A)$ is a σ-frame and $\mathcal{H}L(A)$ is a σ-coherent frame. Since the inclusion map $i : L(A) \to A$ is a σ-frame homomorphisms, there is a unique frame homomorphism $f : \mathcal{H}L(A) \to A$ with $f \circ \downarrow = i$ for the down map $\downarrow : A \to \mathcal{H}L(A)$. In case, $f(I) = \bigvee I$. Define $g : A \to \mathcal{H}L(A)$ by $g(a) = \downarrow a \cap L(A)$. Then $0 \in g(a)$ and $g(a)$ is a down set. Furthermore, for $x_n \in g(a)$ $(n \in N)$, $\bigvee\{x_n | n \in N\} \in g(a)$; hence g is well-defined. Take any $I \in \mathcal{H}L(A)$, then $g(f(I)) = I$ and $f(g(a)) = a$ for all $a \in A$. So f is an isomorphism.

($\Leftarrow$) It is immediate from 1) of Proposition 3.6. $\square$

THEOREM 3.9. *σ**Frm** and σ**CohFrm** are equivalent.*

Proof. For a σ-frame homomorphism $h : A \to B$, there is a unique frame homomorphism $\mathcal{H}h : \mathcal{H}A \to \mathcal{H}B$ $(\mathcal{H}h(I) = \downarrow h(I))$. $\mathcal{H}h$ is σ-coherent, because for any $\downarrow a$ $(a \in A)$, $\mathcal{H}h(\downarrow a) = \downarrow h(a) \in L(\mathcal{H}B)$. Thus $\mathcal{H} : \sigma\mathbf{Frm} \to \sigma\mathbf{CohFrm}$ $(A \to \mathcal{H}A)$ is a functor. For any σ-coherent homomorphism $f : C \to D$, $L(f) : L(C) \to L(D)(L(f)(x) = f(x))$ is a σ-frame homomorphism. Thus $\mathcal{L} : \sigma\mathbf{CohFrm} \to \sigma\mathbf{Frm}$ $(C \to L(C))$ is a functor. For each σ-frame A, the correspondence $A \to \mathcal{H}A$ is functorial and we have a functor $\mathcal{H} : \sigma\mathbf{Frm} \to \sigma\mathbf{CohFrm}$. For each σ-coherent frame M, the correspondence $M \to L(M)$ is also functorial and we have a functor $\mathcal{L} : \sigma\mathbf{CohFrm} \to \sigma\mathbf{Frm}$. The functor $\mathcal{H}$ takes each σ-frame A to $\mathcal{H}A$ with a map $\eta_A : A \to L(\mathcal{H}A)$ $(\eta_A(a) = \downarrow a)$. Moreover, η_A is an isomorphism for all A. For any σ-frame homomorphism $h : A \to B$, $L(\mathcal{H}h) : L(\mathcal{H}A) \to L(\mathcal{H}B)$ $(L(\mathcal{H}h)(\downarrow a) = \downarrow h(a))$ is a σ-coherent homomorphism and $\eta_B \circ h = L(\mathcal{H}h) \circ \eta_A$. Thus $(\eta_A)_A : 1 \to \mathcal{L} \circ \mathcal{H}$ is a natural isomorphism for all $A \in \sigma\mathbf{Frm}$. The functor $\mathcal{L}$ takes each σ-coherent frame M to

$L(M)$ with a map $\epsilon_M : \mathcal{H}L(M) \to M$ ($\epsilon_M(I) = \bigvee I$). ϵ_M is an isomorphism for all M. For any σ-coherent homomorphism $g : M \to P$, $\mathcal{H}L(g) : \mathcal{H}L(M) \to \mathcal{H}L(P)$ ($\mathcal{H}L(g)(I) = \downarrow g(I)$) is a σ-frame homomorphism and $g \circ \epsilon_M = \epsilon_P \circ \mathcal{H}L(g)$. Thus $(\epsilon_M)_M : \mathcal{H} \circ \mathcal{L} \to 1$ is a natural isomorphism for all $M \in \sigma$**CohFrm**. In all $\mathcal{H}$ is an equivalence between σ**Frm** and σ**CohFrm**. □

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DEPARTMENT OF MATHEMATICS
CHUNGBUK NATIONAL UNIVERSITY
CHEONGJU, CHUNGBUK, 361-763, KOREA

E-mail: soleeQchungbuk.ac.kr