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Estimating Diameter and Height Growth for *Pinus densiflora* S. et Z. Using Non-linear Algebraic Difference Equations[1*]

Sang-Hyun Lee[2※]

非線形 代數差分 方程式을 利用한 소나무 直徑 및 樹高 生長 推定[1*]

李 尙 鉉[2※]

ABSTRACT

Pinus densiflora S. et Z. has widely been distributed, and is one of the important main foret resources in Korea. Diameter and height growth patterns were estimated using non-linear algebraic difference equation, which requires two-measurement times T_1 and T_2. To maximize data use, all possible measurement interval data were derived using Lag and Put statements in the SAS. In results, of the algebraic difference equations applied, the Schumacher and the Gompertz polymorphic equations for diameter and height, respectively showed the higher precision of the fitting. In order to allow more precise estimation of growth than those of the basic Schumacher and the Gompertz, further refinement that combine biological realism as input into the equation would be necessary.

Key words : Pinus densiflora S.et Z., *algebraic difference equation, anamorphic and polymorphic function, diamater and height growth*

要 約

우리나라에 전국적으로 분포하고 중요한 산림자원인 소나무 (*Pinus densiflora* S. et Z.)의 직경 및 수고 생장함수를 유도하였다. 모형 유도방법은 두 측정간격 T_1과 T_2를 필요로 하는 대수 차분 방정식을 이용하였고, 데이터 이용의 극대화를 위하여 SAS에서 Lag와 Put 문장을 사용한 프로그램을 이용하여 모든 가능한 생장 측정 기간을 포함하는 데이터를 사용하였다. 적용된 동형 및 다형 차분 방정식 중 Schumacher 다형 방정식이 직경 생장을 추정하는데 적합한 것으로 나타났고, 수고 생장 추정은 Gompertz 다형식이 적합한 것으로 나타났다. 보다 정밀한 추정을 위해서는 이들 식에 생물학적인 변수들을 동반한 연구가 필요할 것으로 판단된다.

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[2] Faculty of Forest Science (The Institute of Agricultural Science & Technology), College of Agriculture, Chonbuk National University, Chonju 561-756, Korea 전북대학교 산림과학부 (농업과학기술연구소).
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※ Corresponding author : leesh@moak.chonbuk.ac.kr

INTRODUCTION

Growth of the trees by age follows sigmoid-shaped curves, and deriving growth function to presume this growth pattern supplies efficient utilization of forest resources and basis for forest management. The clue to successful timber management is a proper understanding of growth processes, and for this various growth functions and models have been used.

And satisfactory of growth models may depend on the availability of high quality data. Growth models have mostly been developed on the basis of empirical data collected in trials of varying design. The data from permanent sample plots have been used commonly for the growth and yield modeling. Because the long-term observation of plots provide a very important database for developing growth models. Another data from temporary plots can produce a faster solution not only in the circumstances nothing has been known about forest development and also high costs of maintenance are required for the permanent sample plots. Especially temporary plots are used very consciously for developing growth models in situations where permanent sample plots data are not available.

Suitable method utilizing the data from two successive measurements is the algebraic difference form of a growth function that has been used by number of researchers (Clutter *et al.*, 1983; Borders *et al.*, 1984; Lee, 1998, 2000). It usually starts with the basic models which is the form of $Y_2 = f(Y_1, T_1, T_2)$. Where the response variable Y_2 measured at time T_2 is described as a function of the same variable measured at initial time T_1 and a measure of elapsed time as a function of T_1 and T_2. The Variable Y could be basal area, top height and stems per hectare or any stand variables.

However, when there are few sample plot remeasurements available for using the algebraic difference equation, how one solves this problem? These can be rearranged as all possible intervals for each unit.

The objectives of this study, therefore, are developing a method of sufficient projection form data through the basic data, which are from stem analysis, and to provide basic information for prediction of diameter and height growth in *Pinus densiflora* Sieb. et Zucc. after fitting sigmoid-shaped projection function to the projection data.

MATERIALS AND METHODS

Data for this study came from *Pinus densiflora* Sieb. et Zucc. temporary plots grown in Beonsan peninsular of Chonbuk province. All of 20 plots, which were 20m×20m size each plot, were used for analysis. From the each plot 1 sample tree was selected and cut, after cutting the sample trees diameter and height were measured using stem analysis.

Mean age, diameter and height were 35 years, 24.2 Cm and 16 m, respectively. The sample plots were with the gradients of 15-25 degrees, and soil type were moderately moist brown forest soil and mostly loam and clay loam. A summary of relevant plot statistics is given in Table 1.

Table 1. A summary of sample plots statistics.

Number of Plots	Mean ages (years)	Mean DBH (cm)	Mean height (m)	Slope (°)	Soil type
20	35	24.2	16	15 - 25	B_3

The basic data, which get from stem analysis, were transformed into projection format, and the data structure for all possible growth intervals was also created with a Statistical Analysis System (SAS) program for maximum use the data. If n times record of diameter and height were get, there are ${}_nC_2$ combinations of different intervals between time T_1 and T_2 that can be derived and used to build equation.

The methods used for this study were difference equation (Borders *et al.*, 1984) which has been used widely for growth and yield modeling studies.

The main standard statistical procedures used were non-linear least-squares regression based on PROC NLIN in Statistical Analysis System (SAS Inc, 1990). Among the algorithms of PROC NLIN procedures used to estimate parameters the derivative -free method (DUD), which was found to be best in convergence, was adopted for non-linear least-squares regression (Ralston and Jennrich, 1979).

The PROC UNIVARIATE procedure was also used to examine the residuals and provide several statistics that are valuable for making inferences about residuals patterns. The important values utilized in the analysis of this study were such as mean of residuals, skewness, kurtosis and extreme values. In addition, graphical charts and plots were used to check the distributions of residuals with regard to normality of errors. Residual errors were plotted against predicted values to determine goodness of fit. Because whether or not the residual patterns lay normally about the zero references line was the important criterion for judging the independent distribution.

The commonly adopted projection equations are log-reciprocal (Schumacher, 1939; Woollons and Wood, 1992), Chapman-Richard (Piennar and Turnbull, 1973; Goulding, 1979), Gompertz (Whyte and Woollons, 1990), Weibull (Yang *et al.*, 1978; Goulding and Shiley, 1979) and Hossfeld (Liu Xu, 1990). There are two types of projection functions used for tree growth models, namely anamorphic and polymorphic functions. Firstly, several frequently used and their accuracy of estimation proved anamorphic equations were assayed such as Schumacher, Chapman-Richard, Hossfeld and Gompertz functions. The functional forms of anamorphic projection equations used are presented in Table 2. Then, polymorphic forms of Schumacher, Chapman-Richard, Hossfeld and Gompertz equations were fitted to the data. The functional forms of polymorphic projection equations are presented in Table 3.

RESULTS AND DISCUSSION

1. Prediction of diameter growth projection function

Most anamorphic equations generally produced biased residuals patterns, though Schumacher anamorphic function proved little bit superior in

Table 2. General form of anamorphic projection equations applied to data.

Equation name	Equation Forms*
Schumacher anamorphic	$Y_2 = Y_1 \exp(-\beta(1/T_1^{\gamma} - 1/T_2^{\gamma}))$
Hossfeld anamorphic	$Y_2 = 1/((1/Y_1) + \beta(1/T_2^{\gamma} - 1/T_1^{\gamma}))$
Chapman-Richards Anamorphic	$Y_2 = Y_1((1 - \exp(-\beta T_2)) / (1 - \exp(-\beta T_1)))^{\gamma}$
Gompertz anamorphic	$Y_2 = Y_1 \exp(-\beta(\exp(\gamma T_2) - \exp(\gamma T_1)))$

* Y_1 = Diameter and height of trees at age T_1　　Exp = exponential function
Y_2 = Diameter and height of trees at age T_2　　ln = natural logarithm
α, β, γ are coefficients to be estimated

Table 3. General form of polymorphic projection equations applied to data.

Equation name	Equation Forms*
Schumacher	$Y_2 = \exp(\mathrm{In}(Y_1)(T_1/T_2)^{\beta} + \alpha(1 - (T_1/T_2)^{\beta}))$
Chapman-Richards	$Y_2 = (\alpha/\gamma)^{[1/(1-\beta)]}(1 - (1 - (\gamma/\alpha)Y_1^{(1-\beta)})\exp(-\gamma(1 - \beta)(T_2 - T_1))^{[1/(1-\beta)]}$
Gompertz	$Y_2 = \exp(\mathrm{In}(Y_1)\exp(-\beta(T_2 - T_1) + \gamma(T_2^2-T_1^2) + \alpha(1-\exp(-\beta(T_2-T_1) + \gamma(T_2^2-T_1^2))))$
Hossfeld	$Y_2 = 1/((1/Y_1)(T_1/T_2)^{\gamma} + (1/\alpha)(1 - (T_1/T_2)^{\gamma}))$

* Y_1 = Diameter and height of trees at age T_1　　Exp = exponential function
Y_2 = Diameter and height of trees at age T_2　　ln = natural logarithm
α, β, γ are coefficients to be estimated

Table 4. Statistics of residuals with the anamorphic equations fitted to data.

Equation name	MSE	Mean of residuals	Skewness	Kurtosis
Schumacher	5.3359	0.4643	0.3367	1.6460
Chapman-Richard	5.3871	0.4620	0.3561	1.6074
Hossfeld	26.6864	3.3304	0.3720	1.1947
Gompertz	15.9265	1.6261	0.0182	1.1408

statistics of residuals and residuals patterns to other anamorphic functions. The statistics of residuals of the anamorphic equations fitted are presented in Table 4 with corresponding mean square error values (MSE).

A plot of residuals against predicted values for Schumacher equation, which is proved fitting well among the anamorphic functions, shown in Figure 1 that fitted unsatisfactory with apparent bias. The mean of the average residuals was 0.46 Cm, which represents a slight underestimation and an absolute residual of 1.62 Cm which means that the equation would predict diameter with an average error of 1.62 Cm. Skeweness and kurtosis values were 0.34 and 1.65, respectively.

Then, polymorphic forms of Schumacher, Chapman-Richard, Hossfeld and Gompertz equations were fitted to the data. Most of the polymorphic equations generally fitted well without apparent bias, except Gompertz function that showed bias, in residuals pattern and showed better fit than anamorphic forms of equations. In the Chapman-Richard function, the confidence interval of the coefficients of β and γ were not significant at the α = 0.05. Comparing residuals pattern and mean square error values, the Schumacher polymorphic function, equation (1), with mean square error (MSE) 1.840 was found to represent better than the other equation. The fitted coefficients and mean square error are shown in Table 5.

$$D_2 = \exp(\ln(D_1)(T_1/T_2)^{\beta} + \alpha(1-(T_1/T_2)^{\beta})) \quad (1)$$

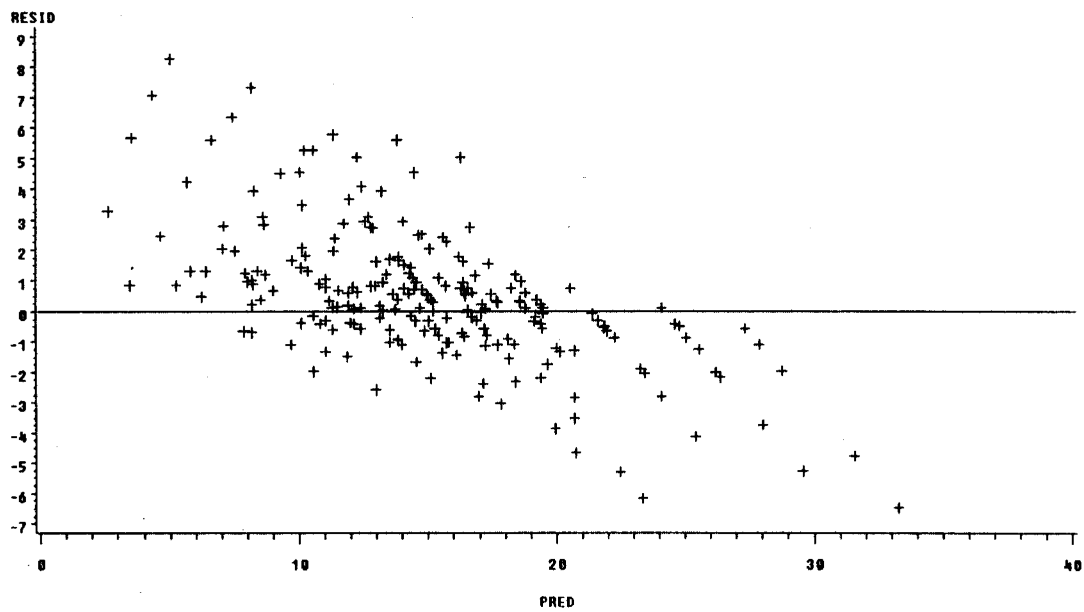

Figure 1. A plot of residuals against the predicted for diameter anamorphic projection equation.

Table 5. Coefficients for polymorphic equation fitted to data.

Model Name	Coefficients			MSE
	α	β	γ	
Schumacher	3.830	0.742	-	1.840
Chapman-Richards	5.812	-0.225	0.039	4.804
Gompertz	15.357	0.002	-0.001	17.096
Hossfeld	28.507	-	1.828	2.439

A plot of residual values against predicted values is given in Figure 2. A plotting of residuals against predicted values indicated that a random pattern around zero with little biased trend. An absolute residual of 0.98 Cm which means that the equation would predict diameter with an average error of 0.98 Cm. PROC UNIVARIATE in SAS showed that residual statistics were satisfactory as it contained -0.66 value for skewness and 1.41 value for kurtosis. The skeweness and kurtosis of a normal distribution is zero, but in practice values of these lesser or greater than zero result from least-square regression. A Shapiro-Wilk test for normality was totally accepted as 0.95 that is very closed to 1 of normal distribution.

2. Prediction of height growth projection function

The anamorphic and polymorphic functions were applied, such as the log-reciprocal equation, Chapman-Richards, Gompertz and Hossfeld. The fitted coefficients and mean square errors are shown in Tables 6 and 7.

Table 6. Coefficients for anamorphic equation fitted to data.

Model Name	Coefficients			MSE
	α	β	γ	
Schumacher	-	8.6936	0.3692	4.2120
Chapman-Richards	-	0.0447	-1.7735	4.2830
Hossfeld	-	-6.9687	1.6386	27.3819
Gompertz	-	0.0305	-0.8984	24.0461

Table 7. Coefficients for polymorphic equation fitted to data.

Model Name	Coefficients			MSE
	α	β	γ	
Schumacher	4.1287	0.5218	-	2.0369
Chapman-Richards	3.2040	0.6927	1.3113	4.4803
Gompertz	3.1794	0.0916	0.008	1.5958
Hossfeld polymorphic	22.5543	-	1.8221	1.8860

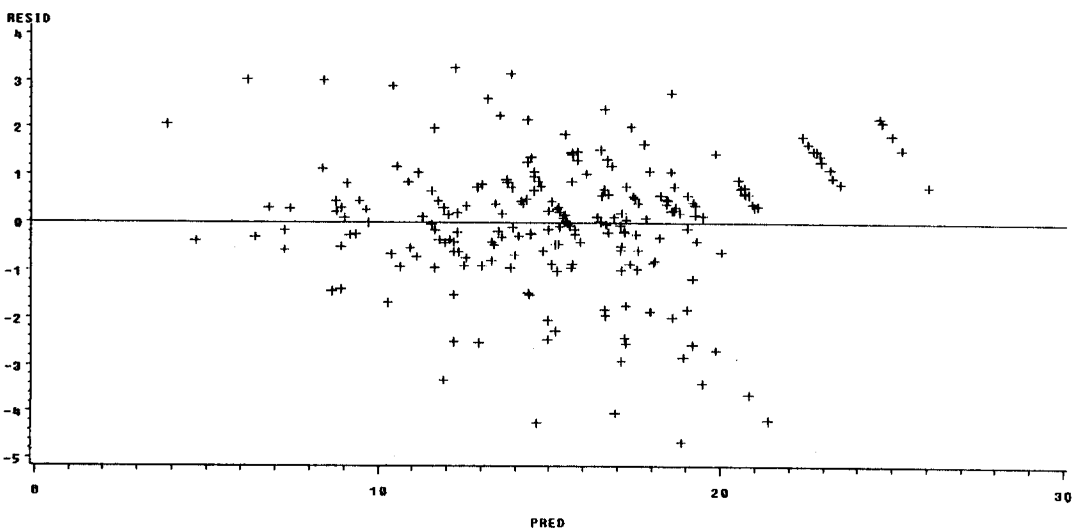

Figure 2. A plot of residuals against the predicted for diameter polymorphic projection equation.

Most anamorphic equations showed to be unsuitable with residuals patterns, while the Scumacher equation had the lowest mean square errors (MSE) value, which has been used as first option for selecting the best fitting model because the equation with the least biased residuals patterns has been found to have the lowest MSE vales, among the anamorphic equations. And had lower mean square errors than the Gompertz and Hossfeld polymorphic functions.

None of the asymptotic 95% confidence intervals of each coefficient contained zero that means the coefficients are significant at the $\alpha = 0.05$ level. However, the confidence interval of the coefficient γ of the Chapman-Richard polymorphic equation was not significant at this level. Therefore, the Gompertz polymorphic function, equation (2), that has the lowest MSE (1. 5958) value was found to represent the best fit.

$$H_2 = \exp(\ln(H_1)\exp(-\beta(T_2-T_1)+\gamma(T_2^2-T_1^2)) + \alpha(1-\exp(-\beta(T_2-T_1)+\gamma(T_2^2-T_1^2)))) \quad (2)$$

The data were evidently well balanced with no apparent bias or systematic patterns and showed goodness of fit as shown in Figure 3.

The PROC UNIVARIATE statistics in Table 8 shows proof that the equation provides an unbiased precise estimate of height as it contained -0.361 value for skewness which indicated little bit long tails to the left and 0.924 value for kurtosis, the heaviness of tails in a distribution. A Shapiro-Wilk test for normality was totally accepted as 0.980 that is very closed to 1 of normal distribution. The mean of the average residuals was 0.03 m, which represents a slight underestimation and an absolute residual of 0.96 m that means that the equation would predict height with an average error of 0.96 m.

Table 8. Summary of statistics of residual values for height projection equation.

Statistics Name	Values
Mean	0.0365
Absolute mean	0.960
Skewness	-0.3621
Kurtosis	0.9236
W : Normal	0.9890

CONCLUSIONS

It showed that the Schumacher and the Gompertz polymorphic projection equations provided satisfac-

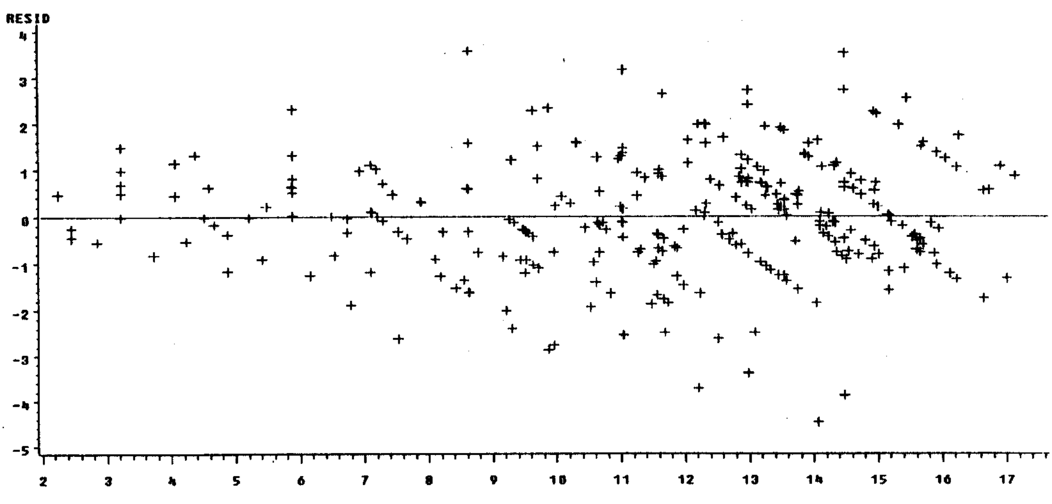

Figure 3. A plot of residuals against the predicted for height polymorphic projection equation.

tory predictions of the diameter and height growth, respectively for *Pinus densiflora* Sieb. et Zucc. grown in Beonsan peninsular of Cheonbuk province. This was ensured by comparing the respective residual mean squares values as well as better residual patterns and residual statistics. It is unrealistic to expect a unique function to perform consistently better than others with forest growth and yield data. Because the best function is changed with what kind of data are being used. And the initial selection of appropriate equations is most important for success of the goodness of fit modes. In order to allow more precise estimations than those of above the basic Schumacher and the Gompertz further study that combines biological realism as input into an empirical equation will be required.

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