Jour. Korean For. Soc. 90(2) : 176 - 183. 2001
韓國林學會誌 90(2) : 176 - 183. 2001

Weibull Diameter Distribution Yield Prediction System for Loblolly Pine Plantations[1]*

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테다소나무 造林地에 대한 Weibull 直徑分布 收穫豫測 시스템에 關한 研究[1]*

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ABSTRACT

Loblolly pine (*Pinus taeda* L.) is the most economically important timber producing species in the southern United States. Much attention has been given to predicting diameter distributions for the solution of multiple-product yield estimates. The three-parameter Weibull diameter distribution yield prediction systems were developed for loblolly pine plantations.

A parameter recovery procedure for the Weibull distribution function based on four percentile equations was applied to develop diameter distribution yield prediction models. Four percentiles (0th, 25th, 50th, 95th) of the cumulative diameter distribution were predicted as a function of quadratic mean diameter. Individual tree height prediction equations were developed for the calculation of yields by diameter class. By using individual tree content prediction equations, expected yield by diameter class can be computed.

To reduce rounding-off errors, the Weibull cumulative upper bound limit difference procedure applied in this study shows slightly better results compared with upper and lower bound procedure applied in the past studies. To evaluate this system, the predicted diameter distributions were tested against the observed diameter distributions using the Kolmogorov-Smirnov two sample test at the α=0.05 level to check if any significant differences existed. Statistically, no significant differences were detected based on the data from 516 evaluation data sets. This diameter distribution yield prediction system will be useful in loblolly pine stand structure modeling, in updating forest inventories, and in evaluating investment opportunities.

Key words: Pinus taeda, *Weibull distribution, percentiles, parameter recovery*

要 約

本 研究에서는 木材의 多目的 生産量 (multiple-product yield) 豫測에 대한 解決策으로서 테다소나무 (*Pinus taeda* L.) 造林地를 대상으로 하여 Weibull 直徑分布 收穫豫測 시스템을 開發하였다. 直徑分布 收穫豫測 模型을 開發하기 위하여, 4개의 百分位數(percentiles) 式들을 根據로 한 母數 回復 (parameter recovery) 節次法을 適用하였다. 또한 直徑級에 대한 收穫量 計算을 위하여 單木 樹高 豫

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[1] Received on August 25, 2000.
Accepted on October 12, 2000.

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* This work was supported by the grant of Post-Doc. Program, Kyungpook National University (2000), Korea.

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測式을 開發하였으며, 그리고 單木 材積 豫測式을 利用함으로써 直徑級에 대해 기대되는 材積量을 計算할 수가 있다. 本 研究에서 使用된 直徑級에 대한 Weibull 累積函數의 上限線 差異 方法이 既存의 上限線과 下限線의 節次法 보다도 括約誤差를 줄 일수 있는 보다 나은 節次法이였다. 本 研究에서 提示된 Weibull 直徑分布 收穫豫測 시스템에 대한 妥當性 檢定의 한 方法으로서 Kolmogorov-Smirnov test 結果, 各 plot당 豫測된 直徑分布와 觀測된 直徑 分布級 사이에서 統計的 有意性이 없는 것으로 나타났다. 이와 같은 直徑分布 收穫豫測 시스템은 多目的 木材 生産量 豫測과 林分 構造模型 및 林分의 經營에 有用한 情報를 提供할 것이다.

INTRODUCTION

Forest managers often require information concerning the size-class distribution of a forest stand such as the tabulation of numbers of trees by diameter class. Size-class distribution information is important because it affects the type and timing of management strategies for merchantability standards. Stand diameter distributions are particularly important as volume distribution which is closely related to diameter distribution.

A number of methods have been proposed to model diameter distributions in forest stands. Many statistical distribution functions such as normal, lognormal, exponential, Beta, Weibull, the Johnson's SB, and Bivariate distribution have been used to describe diameter distributions in forest stands (Lenhart 1968, Hafley and Schreuder 1977, Clutter *et al.* 1983, Knoebel and Burkhart 1991). However, nearly most of the recent work has been used the Weibull distribution to model diameter distributions since the early applications by Bailey and Dell (1973). Because a closed-form, expression of the cumulative distribution function exists and it is relatively simple in form. Burkhart and Strub (1974) compared the Weibull function with the Beta density function in stands of loblolly pine and found better fits with the Weibull function.

Following the introduction of the Weibull function to the forestry (Bailey and Dell 1973), Weibull parameters were predicted by empirical functions of whole stand characteristics such as age, site index, and density (Smally and Bailey 1974). Subsequently, parameter recovery techniques replaced the parameter prediction approach (Bailey *et al.* 1981, Hyink and Moser 1983, Cao and Burkhart 1984, Borders *et al.* 1987, Lenhart 1988, Bailey *et al.* 1989). There were several different methods for estimating the parameters of the Weibull distribution (Clutter *et al.* 1983). The parameter recovery technique employed in this work was first presented by Bailey *et al.* (1989). This parameter recovery procedure utilizes the expected value of the minimum observation from a sample size n from the Weibull distribution, four percentiles, and the second moment of the Weibull distribution to estimate the a, b, and c parameters.

The objective of this study was to develop percentile-based Weibull diameter distribution yield prediction system using the long-term repeated measurement data sets in the south-western United States and an example is given for the practical application of the results.

MATERIALS AND METHODS

1. Loblolly pine measurements

The study area consists of 22 counties in East Texas, USA. Generally, the counties are located within the rectangle from 30° - 35° north latitude and 93° - 96° west longitude.

The East Texas Pine Plantation Research Project (ETPPRP) was initiated in 1982. Measurements are on a 3 year cycle because it takes 3 years to measure all plots. Each plot is located in a different plantation and consists of two adjacent subplots separated by a 60 ft buffer zone. Latitude and longitude coordinates are known for each plot. One subplot is designated for model development and the other for model evaluation. A subplpot is

100 x 100 ft in size, and all planted loblolly pines within a subplot are tagged and measured. Typical site preparation methods for establishing the plantations in which ETPPRP plots are involved various combinations of shearing, pushing down, piling and or chopping, plus burning. The summary statistics of the observed variables are depicted in Table 1 for loblolly pine plantations.

Table 1. Observed descriptive statistics for unthinned loblolly pine stand data sets.

	Mean	Std Dev.	Min.	Max.
AGE	11	5.2	1	27
S	69	15	18	117
TPA	467	149	87	1,002
HT	36.4	18	2	100
D_0	1.6	1.3	0	6.9
D_{25}	3.9	2.2	0	10
D_{50}	4.8	2.5	0	11.2
D_{95}	6.6	3.2	0	15.1
DQMEAN	4.8	2.5	0	11.2
BA	69.3	48.6	0.5	192.7
RS	0.5	0.6	0.1	7.7

Where: AGE= plantation age (yrs), S= site index (base age 25; (ft)), TPA= total trees per acre, HT= average height of ten tallest trees (ft), D_0= 0th diameter percentile (in.), D_{25}= 25th diameter percentile (in.), D_{50}= 50th diameter percentile (in.), D_{95}= 95th diameter percentile (in.), DQMEAN= quadratic mean diameter (in.), BA= basal area per acre (ft^2), RS= relative spacing.

A total of 1,597 subplot observations from loblolly pine plantations were utilized for model fitting. Average age of loblolly pine was almost 11 years. Average site index of loblolly pine was about 70 ft (base age : 25 yrs, Lee and Hong 1999). The average number of trees per acre was 467.

2. Model development

1) Prediction of diameter distribution percentile equations

The Weibull parameter recovery method was applied in this study that required use of the 0th, 25th, 50th, and 95th diameter percentiles. By using a FORTRAN program, the 0th(D_0), 25th(D_{25}), 50th(D_{50}), and 95th(D_{95}) percentiles were obtained for each subplot. It was assumed that these percentiles were located sufficiently far apart in the diameter distribution to represent the spread of the distribution. Separate regression equations for the percentiles were developed for loblolly pines based on the model selection criteria.

2) Weibull parameter recovery methods

The Weibull distribution parameter recovery procedure developed by Da Silva (1986) and subsequently utilized by Bailey *et al.* (1989) and Brooks *et al.* (1992), first determines the predicted location parameter 'a' using the predicted values for D_0 and D_{50}, and an initial assumption that the shape parameter 'c' is 3.0. Shape 'c' and scale 'b' parameters are then recovered using location parameter 'a' plus the other predicted percentiles. The expected value of the first order statistic (D_0) of the sample of size n taken from Weibull distribution is :

$$E(D_0) = a + (b/n^{1/c})\Gamma(1 + 1/c). \qquad (1)$$

The minimum D_0 has a Weibull distribution with parameters 'a' and 'c' unchanged but with b replaced by $b/n^{1/c}$. The equation for the 50th percentile is :

$$D_{50} = a + b\,[-\ln(0.5)]^{1/c}. \qquad (2)$$

When c=3.0, to one decimal place (approximation), equations (1) and (2) become

$$E(D_0) = a + (b/n^{1/3})(0.9) \text{ and} \qquad (3)$$

$$D_{50} = a + b(0.9), \text{ respectively.} \qquad (4)$$

By approximation, replace (0.9) with 1.0, and combining equations (3) and (4), an initial estimate for 'a' results as :

$$\hat{a} = (n^{1/3} D_0 - D_{50})/(n^{1/3} - 1), \text{if } a<0.0 \text{ then } a=0. \qquad (5)$$

The sample size (n) is the number of trees on the plot. The shape parameter is estimated by

$$\hat{c} = \ln\left[\frac{\ln(1-0.95)}{\ln(1-0.25)}\right] / \ln\left[\frac{D_{95}-\hat{a}}{D_{25}-\hat{a}}\right], \quad (6)$$

and the scale parameter, 'b', was obtained by solving the second moment of the Weibull distribution for the positive root using the following equation :

$$\hat{b} = -\frac{\hat{a}\Gamma_1}{\Gamma_2} + \sqrt{\left(\frac{\hat{a}}{\Gamma_2}\right)^2 (\Gamma_1^2 - \Gamma_2) + \frac{D_q^2}{\Gamma_2}} \quad (7)$$

where :

Γ = the gamma function,
$\Gamma_1 = \Gamma(1+1/c)$,
$\Gamma_2 = \Gamma(1+2/c)$,
D_q = quadratic mean diameter.

Two favorable advantages of this percentile-based parameter recovery procedure rather than other recovery procedure were : the location parameter was obtained by using of two percentile equations rather than arbitrary proportion of the minimum diameter, and the shape parameter was obtained by using of two percentile prediction equations.

3) Individual tree height-diameter relationship model

The estimation of tree volume, site index and growth depend on height prediction models. The most widely used height prediction models are the 'height-diameter' equations, which predict tree height as function of tree diameter at breast height. The individual tree height model used in this study was originally developed by Lenhart (1968). The basic model was :

$$\ln(h_{i,t}/HT_t) = f(\ln(d_{i,t}/DMAX_t), T_t)), \quad (8)$$

where :

$h_{i,t}$ = predicted height of the ith tree at age t,
HT_t = average height of dominant and codominant at age t,
$d_{i,t}$ = dbh of the ith tree at age t,
$DMAX_t$ = midpoint value of the largest diameter class at least one tree,
T_t = surviving number of trees at age t.

A property of the equation (8) is that as d approaches DMAX, h approaches H. This property has been attractive to several researchers working with other planted species (Amateis *et al.* 1984, Zhang *et al.* 1997). Other variants of this type of tree height prediction model for even-aged stands have been developed, which relate tree height to dbh and variety of stand attributes (Clutter *et al.* 1983).

4) Application of the stand table calculations

If a population of values has a Weibull distribution, the proportion of the population with values greater than L and less than U is given by the following equation (Clutter *et al.* 1983)

$$P_i(L_i < X < U_i) = \exp\left[-\left(\frac{L_i-a}{b}\right)^c\right] - \exp\left[-\left(\frac{U_i-a}{b}\right)^c\right] \quad (9)$$

where :

P_i = proportion of trees in diameter class i,
L_i = lower limit of diameter class i,
U_i = upper limit of diameter class i.

The location parameter 'a' which gives the minimum value of the distribution (minimum diameter values is ≥ 0), and the scale parameter 'b' which is related to the range of the diameter distribution, and the shape parameter 'c' which determines the skewness of the distribution. This equation was widely used for calculating diameter class frequencies with all Weibull diameter distribution models.

(1) Application of Weibull cumulative probability function

For the Weibull distribution, the cumulative distribution function is

$$F(X)=1-\exp\left[-\left(\frac{X-a}{b}\right)^c\right] \qquad (10)$$

$(a \leq X < \infty)$, 0 otherwise.

The method applied in this study is application of Weibull cumulative probability difference procedure (Avery and Burkhart 1994).

$$P_i=\left(1-\exp\left[-\left(\frac{U_i-a}{b}\right)^c\right]\right).$$
$$\left(1-\exp\left[-\left(\frac{U_{(i-1)}-a}{b}\right)^c\right]\right) \qquad (11)$$

First, calculate proportion (P_i) for every dbh class upper bound (U_i), then, subtract successive previous proportion of $U_{(i-1)}$ for every dbh class upper bound and multiply by trees per acre. This procedure is more appropriate than equation (9) to calculate diameter distribution computations in terms of reducing rounding-off errors. Predicted diameter distributions were calculated and compared with observed diameter distributions. Comparison of predicted versus observed total volumes was also done. Although this is not an exact test, trees per each evaluation plot from observed and predicted diameter distributions were tested by using the Kolmogorov-Smirnov two-sample test at the 0.05 level of significant.

RESULTS AND DISCUSSION

1. Diameter percentile prediction equations

The components of a diameter distribution yield prediction system are equations to estimate certain diameter percentiles and quadratic mean diameter. A total of 1,597 observations from loblolly pine plantations were utilized for model fitting. Separate regression equations for the minimum dbh (D_0) on the plot and 25th, 50th, and 95th percentiles were developed for loblolly pines based on the model selection criteria. The prediction equations for the 0th, 25th, 50th and 95th percentiles plus quadratic mean diameter (DQMEAN) are presented in equations (12) to (16). The R^2 variation explained by these equations ranged from 65.0% for equation (12) to 99.2% for equation (14).

- Estimating D_0 as :

$$D_0=\exp(-2.76552+1.88503\ln(DQMEAN))$$
$$(R^2=0.650 \qquad \text{RMSE}=0.443) \qquad (12)$$

- Estimating D_{25} as :

$$D_{25}=\exp(-0.56174+1.20622\ln(DQMEAN))$$
$$(R^2=0.962\text{RMSE}=0.125) \qquad (13)$$

- Estimating D_{50} as :

$$D_{50}=\exp(-0.14455+1.07649\ln(DQMEAN))$$
$$(R^2=0.992 \qquad \text{RMSE}=0.057) \qquad (14)$$

- Estimating D_{95} as :

$$D_{95}=\exp(0.52745+0.86842\ln(DQMEAN))$$
$$(R^2=0.983 \qquad \text{RMSE}=0.068) \qquad (15)$$

Quadratic mean diameter (DQMEAN) is the most important independent variable in predicting percentile-based diameter prediction equations.

- Estimating DQMEAN as :

$$DQMEAN=\exp\left(3.72533-27.19575\left(\frac{1}{HT}\right)\right.$$
$$(+0.31755\ln(AGE)-0.24421\ln(AGE*TPA))$$
$$(R^2=0.957 \qquad \text{RMSE}=0.089) \qquad (16)$$

By the second moment estimate for Weibull probability density function, we can derive some useful stand-level variable DQMEAN information. The next step in developing a diameter distribution yield prediction model for planted loblolly trees was needed to develop individual tree height prediction model.

2. Individual loblolly pine tree height prediction equations

After a study of the literature and an examination of the reasoning, it was concluded that a variation of the Lenhart (1968) function was suitable approach for individual tree height prediction modeling. A total of 80,322 individual loblolly pine tree data sets from five repeated measurement cycles were used for model fitting. The following individual tree height prediction equation was selected based on the model selection criteria.

$$\ln(h_i) = \ln(HT) + 0.02199 + (\ln(d_i) - \ln(DMAX))(0.36343 + 0.04493\ln(DQMEAN)) \quad (R^2 = 0.656 \quad \text{RMSE}=0.117) \qquad (17)$$

3. Illustrations of the Weibull diameter distribution prediction system

To illustrate the use of this system, we can consider a 20-year-old loblolly pine plantation with 600 trees per acre and average height of dominants and co-dominants equal to 50 ft. The solution of above equations gives the following Weibull diameter distribution prediction computations. The tree volume prediction equation was treated as external equation. Previously published parameter estimates from Lenhart *et al.* (1987) were used to estimate tree content volume.

4. Validations

Data from 516 evaluation subplots, which were separated from developments by a 60 ft wide buffer zone, provide an opportunity to analyze the accuracy of the diameter distribution yield prediction system. The predicted diameter distributions were tested against the observed diameter distributions using the Kolmogorov-Smirnov two-sample test at the 0.05 probability level. Statistically, no significant differences at the 0.05 probability level were detected for any of the predicted diameter distributions. Plottings of predicted versus observed yields for each evaluation subplots were checked. After the diameter distribution yield prediction system was used to calculate the predicted volumes (ft^3) in the total stem for each evaluation subplot, the estimated yields were compared to the observed actual yields. The average differences of total volume were nonsignificant from zero (P>0.05).

SUMMARY

The objective of this study was to develop the three parameter percentile-based Weibull diameter

Table 2. Predicted stand structure for a 20-year-old loblolly pine plantations with height of 50 ft and 600 trees/acre.

DBH Class (in.)	Upper Limit (in.)	Weibull cumulative upper limit difference probability for each dbh class	No. of Trees/ acre	Avg. Height (ft)	Basal Area (ft^2)	Wood & Bark Volume (ft^3)
2	2.5	0.007283	4	23.0	0.1	1.1
3	3.5	0.031350	19	27.5	0.9	13.9
4	4.5	0.090835	55	31.3	4.8	82.4
5	5.5	0.185751	111	34.6	15.1	290.3
6	6.5	0.267473	160	37.5	31.4	659.8
7	7.5	0.250192	150	40.2	40.1	909.1
8	8.5	0.131959	79	42.7	27.6	668.3
9	9.5	0.032257	19	45.0	8.4	215.7
10	10.5	0.002836	2	47.1	1.1	29.5
11	11.5	0.000065	-	-	-	-
12	12.5	0.000000	-	-	-	-
Total		1.000000	599		130	2,870.1

distribution yield prediction system for loblolly pine plantations.

A parameter recovery procedure for the Weibull distribution function based on four percentile equations was applied to develop diameter distribution yield prediction models. Individual tree height prediction equations were developed for the calculation of yields by diameter class. The Weibull cumulative upper bound limit difference procedure applied in this study shows slightly better results compared with upper and lower bound procedure applied in the past studies This diameter distribution yield model could provide estimates of the number of trees per acre by dbh classes. By estimating the content of a tree with dbh equal to the dbh class midpoint, multiplying the number of trees per unit area in that class, and then summing these values over expected dbh classes, an estimate of total stand yield per acre can be obtained. If restrictions are imposed on the tree merchantability standards and on dbh values, multi-product yield estimates can be readily obtained and aid the foresters in evaluating investment opportunities.

ACKNOWLEDGEMENTS

This project was supported by industrial forest companies in the East Texas Pine Plantation Research Project-Champion International Corporation, International Paper Company, Louisiana-Pacific Corp., Resource Management Services, Inc., and Temple-Inland Forest Products Corp. - are appreciated.

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