Economic Design of the Specification for Geometrical Quality Characteristic

Ma Yizhong

Northwest Polytechnic University Xian,
710000, P. R. China

Zhao Fengyu Xu Jichao

Zhengzhou Institute of Aeronautics
Zhengzhou, Henan, 450005 ,P.R.China.

Abstract

The economic design of specification limits must be determined on an economic basis where we minimize total loss to society, which consists of both the producer and the consumer. Economic specification limits have been developed based on the assumption that the quality characteristic is normally distributed. Unfortunately, the assumption is not to meet some practical cases. In this paper, some non-normal distributions are considered for quality characteristic with geometrical features. An economic model for selecting the optimum specification limits on the basis of minimizing total cost is introduced. A case study is presented to illustrate the application in practice.

Key words: Quality improvement; Optimization; Geometrical features; Quality loss function

1. Introduction

The Taguchi loss function has been widely used in quality engineering problems. When a product or its production process deviates from a customer-identified target value, quality loss function(QLF) is used as a measurement system to quantify the quality loss on a monetary scale, that is, this loss function assigns a monetary loss when the quality characteristic of a product or its production process deviates from its target value. The loss function may be used in three types of quality characteristic: 'nominal the best', 'type(N-type)', 'smaller the better' type (S-type), and larger the better type (L-type). For any type characteristic, it is necessary to reduce variability in order to decrease quality loss for a product. However, it is sometimes difficult to decrease the variability. One possible way to do this is to develop specification limits for the process and truncate the distribution of the quality characteristic by inspection using these limits. How to develop these specification limits, the key question is to minimize total loss to the customer as well as to the producer.

Most of the recent works on the development of specification limits are based on assumption that quality characteristic has a normal distribution. For example, Kapur considered a normal distribution for a quality characteristic and presented the optimization model for the economic specification limits by truncating the distribution by inspection. This problem is further extended by Chung-Ho Chen, M.D. Phillips and B.R.Cho. presented economic design of the specification with circular and spherical specification regions, and so on. However, in mechanical industry, the geometrical features, such as straightness, flatness, parallelism, roundness and concentricity, etc, are very common and important non-normal quality characteristic and have very strong impact on the characters of machines. It is generally considered that the geometrical features have two distribution types: one is the folded-normal distribution; the other is Rayleigh distribution.

cAs advanced types of computerized inspection equipment become an integral part of modern manufacturing systems, screening (or 100% inspection) is becoming an attractive means of maintaining a consistently high quality in finished products. This article addresses the problem of selecting specification limits of geometrical feature for screening purposes, such that the total costs associated with rejection, inspection, and quality loss are minimized.

This article is arranged as follows: first, two kinds of non-normal truncated distributions with geometrical features are introduced, Then, the characters of geometrical features are discussed. On the basis of these, an economic model for selecting the optimum specification limits associated with rejection, inspection, and quality loss are presented. Finally, a case study is also presented to illustrate the application in practice.

2. Non-normal Truncated Distributions with Geometrical Features and Their Properties.

In industrial areas, practitioners frequently use a certain distribution to model a quality characteristic of a product. When a specification limit is used for screening purposes, a truncated distribution is used for the truncated quality characteristic.

Let y be a continuous random variable of a quality characteristic, $f(y)$ be the probability density function (p.d.f.) of the random variable y, $\tilde{y}$ be the truncated quality characteristic, then the truncated p.d.f $f_T(\tilde{y})$ can be defined by

$$f_T(\tilde{y}) = \frac{1}{q} f(y)$$

here$q = \int_A f(y)dy$

and $\tilde{y} \in A$ is defined to be truncated specification of interest. In fact $f_T(\tilde{y})$ is a p.d.f of $\tilde{y}$ because $f_T(\tilde{y}) \geq 0$ and $\int_A f_T(\tilde{y})\,dy = 1$. Now, we discuss the properties of both truncated folded-normal distribution and truncated Ray leigh distribution.

2.1 Truncated folded-normal distribution and its properties.

We know that if random variable y follow the folded-normal distribution and folded at Target value T=0, then the p.d.f. of y can be defined by

$$f(y) = \begin{cases} 0 & y < 0 \\ \dfrac{2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} & y \geq 0 \end{cases}$$

where σ_2 is distribution parameter, which is equal to Ey2. According formula (1), for the specification rejoin A=[o,r], we can easily get

$$q = \int_0^r f(y)dy = \int_0^r \frac{2}{\sqrt{2\pi\sigma}} e^{-\frac{y^2}{2\sigma^2}} dy$$
$$= 2\int_0^r \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{y^2}{2\sigma^2}} dy = Erf[\frac{r}{\sqrt{2}\sigma}]$$

where $Erf[x] = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

and

$$f_T(\tilde{y}) = \begin{cases} 0 \\ \dfrac{2}{Erf[\frac{r}{\sqrt{2}\sigma}] \bullet \sqrt{2\pi}\sigma} e^{-\frac{\tilde{y}^2}{2\sigma^2}} \end{cases}$$
$$\tilde{y} \in (-\infty, 0) \cup (r, +\infty)$$
$$0 \leq \tilde{y} \leq r$$

Thus, the expected square value of the difference between the target value T and the value of quality characteristic $\tilde{y}$ for a product, $E(\tilde{y}-T)^2$, can be given by

$$E(\tilde{y}-T)^2 = E\tilde{y}^2 = \int_0^r \frac{2}{Erf[\frac{r}{\sqrt{2}\sigma}]\sqrt{2\pi}\sigma} \tilde{y}^2 e^{-\frac{\tilde{y}^2}{2\sigma^2}} d\tilde{y}$$
$$= \sigma^2 - \frac{2r}{Erf[\frac{r}{\sqrt{2}\sigma}]\sqrt{2\pi}} \sigma e^{-\frac{r^2}{2\sigma^2}} \quad (2)$$
$$\leq \sigma^2 = Ey^2$$

2.2 Turncated Rayleigh Distribution and Its Properties

If the p.d.f. of Rayleigh distrubution is $\int(y)$ $A=[0,r]$. Then. and we concerned with specification region

$$f(y)=\begin{cases}\frac{y}{\sigma^2}e^{-y^2/(2\sigma^2)} & y\geq 0\\ 0 & y<0\end{cases}$$

According to formula (1). We can get the truncated p.d.f. of Ray leigh distribution, $f(\tilde{y})$ That is

$$f(\tilde{y})=\begin{cases}\frac{\tilde{y}}{q\sigma^2}e^{-\tilde{y}^2/(2\sigma^2)} & 0\leq\tilde{y}\leq r\\ 0 & \tilde{y}\in(-\infty,0)\cup(r,+\infty)\end{cases}$$

where $q=1-e^{-\frac{r^2}{2\sigma^2}}$

In the same way,

$$E(\tilde{y}-T)^2=E\tilde{y}^2$$

$$=\int_0^r\frac{\tilde{y}^3}{q\sigma^2}e^{-\frac{\tilde{y}^2}{2\sigma^2}}d\tilde{y}$$

$$=2\sigma^2[1-\frac{r^2}{2\sigma^2}\bullet\frac{e^{-\frac{r^2}{2\sigma^2}}}{1-e^{-\frac{r^2}{2\sigma^2}}}] \qquad (3)$$

$$\leq 2\sigma^2=EY^2$$

From formula (2) and (3), we can find that the truncated distribution has less variation among the products.

That is, for the products that are actually shipped to customer, the variation in the quality characteristic is reduced, thus resulting in greater consistency in quality.

3. The Development of Optimization Model

3.1 Model Assumptions

For convenience sake, we have the following assumptions:

(1) Products quality characteristics are deviations from the target value T=0, and products are produced continuously.

(2) All finished products are subject to inspection.

(3) The manufacturing process is in a state of statistical control.

(4) 100% screening inspection with no inspection error. This implies that all the products shipped to customer are within specification limits.

3.2 Model Development

In this article, three types of quality costs are considered: loss due to variability from the target value, lost due to inspection, and loss due to rejection. First, when we perform inspection to screen the products, inspection costs are incurred to the producer. We assume that inspection cost for each product is C_2 for the quality characteristic of interest. Further, any product that does not fall in the specification limits is rejected and a rejection cost, C_1 , is also incurred to the producer. Finally, a quality loss due to variability is incurred when the actual measurement of the quality characteristic deviates from its desired target value. Hence, the expect total cost (ECT) per unit product is

$$ETC = E[L(\tilde{y})] + C_1[1-q] + C_2 \qquad (4)$$

Our objective is to find the optimum specification region that minimizes ETC. According to formula (2) or (3), we can find that the equation (4) is a function of *r*. Hence *r* is decision variable. That is, when we minimize *ETC*, the optimal value of the decision variable, *r**, will determine the most economical specification limits for the total manufacturing process.

The model (4) is a simple nonlinear programming problem, which can be solved using numerical optimization techniques.

4. Case study

We consider the geometrical feature problem, where the manufacturing company has presented 2000 data from the production process. The quality characteristic of interest is deviation from the ideal state, represented by zero. By analysis using both histogram and test of goodness-to-fit, we find that the measurement data are folded at zero and follow the folded-normal distribution with parameter, $\hat{\sigma}=1.20$. Since the measurement of interest is the target deviation and its target value T=0, if the target deviation fluctuate substantially, causing excessive defective parts, then there will be losses associated with customer dissatisfaction as well as rejection cost. Based on historical data. A loss coefficient associated with quality loss caused by target deviation is C=80. if the product is rejected in-factory, the loss per product is $C_1=200$, and the cost due to screening inspection for each product is $C_2=3$.

If management decides not to screen products, then neither a inspection cost nor rejection cost would be incurred. The expected total cost per product is due to variability and is calculated by using Taguchis quality loss function.
Where

$$E[L(y)] = C^2\sigma = 80 \times 1.2^2 = 115\text{RMB}$$

If products are inspected, then both inspection and rejection costs would be incurred and include the loss due to variability. Hence, the optimization model is

$$\min ETC = E[L(\tilde{y})] + C_2[1-q] + C$$

where $r>0$ is decision variable. Thus, by formula (2), the expected total cost is given by

$$\min ETC = 80[\sigma^2 - \frac{2r}{Erf[\frac{r}{\sqrt{2}\sigma}]\sqrt{2\pi}}\sigma \exp(-\frac{r^2}{2\sigma^2})]$$

$$+200[1 - Erf[\frac{r}{\sqrt{2}\sigma}]] + 3$$

Applying the one-dimensional search technique, the minimum total expected cost minETC=92.90, when r^*=1.68.

We provide Algorithm with C Language in Appendix.

Figure 1 show the general states of the different expected lost functions as the decision variable r varies. As r increases, the expected rejection cost decreases while the expected quality loss increases due to variability. Therefore the most economical decision variable r is where the sum of all costs ETC is minimum.

For this example, we make a comparison between using an inspection scheme and not using it. The results are listed in Table 1. Under the inspection procedure, the minimum expected total cost per product is 92.90RMB. Without inspection, the expected total cost per product is 115.2RMB.

A savings as high as 19.40% would be realized by implementing 100% inspection. Moreover, a higher-quality product would be provided to the customer because the product variability, $\sigma = 1.2$, would be reduced by 50% to $\sigma_T = 0.85$. Therefore, product quality was improved greatly.

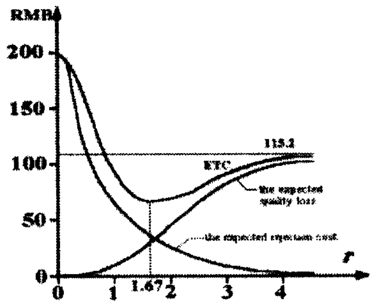

Figure 1. Graphical relationship among the different cost functions as a function of r

5. Conclusion

Quality loss should be evaluated from the viewpoint of both the producer and the customer. The loss to the customer is due to variability from the target value. To the producer, losses are incurred due to inspection and rejection. Sometimes, it is difficult to decrease variability of the quality characteristic. One possible way to overcome this is to develop the specification region

Table 1 Computational Results Under the Two Different Schemes

	Without inspection	With inspection
Variability	1.44	0.719
Fraction passing inspection	1.0	0.838
Expected total cost (per unit product)	115.2	92.90

for the process and truncate the distribution of the quality characteristic. In this article, based on minimizing total cost, we presented an economical model for selecting the optimum specification region about the quality characteristic with geometrical feature, such as folded-normal distribution and Rayleigh distribution, which are very common in mechanical industry. A case study was presented to illustrate its application. The analysis revealed that using a inspection procedure led to substantial cost savings in rejection costs and provided consistently high product quality throughout the manufacturing process.

The Acknowledgement

This work is supported in part by The National Natural Science Foundation of P.R.China, grant member 79900018 and in part by the Education Commission of Henan Province. Their Support is greatly acknowledged.

Appendix

```
#include <math.h>
float erf(float x)
{float s=0;
 float h;
 int i;
 x=x/sqrt(2)/1.2;
 h=x/100;
 s=0.5*h*(1+exp(-x*x));
 for (i=1;i<=99;i++);
 s=s+h*exp(-i*h*i*h);
 return (2*s/sqrt(3.14159));
 }
 main()
 {float minetc=80*1.44,temp;
 float r, minr, var;
 float delta=3*1.2/1000;
 for (r=delta; r<3.6; r=r+delta)

{temp=80*(1.44-2*r*1.2/sqrt(2*3.14159)*exp(
-r*r/2.88)/erf(r))+200*(1-erf(r))+3;
  if (temp<minetc)
    {minr=r;
 minetc=temp;
      printf ("\nminr=%f minetc=%f", minr,
minetc);
```

```
  }
}
printf ("\nminr=%f", minr);
var=1.44-2*minr*1.2/sqrt(2*3.14159)*exp(-mi
nr*minr/2.88)/erf(minr);
printf("\nvar=%f", var);
printf("\nprob=%f", 1-erf(minr));
}
```

References

1. Chen Chung-Ho.(2000) "Specification limit under membership function." Quality Engineering. Vol. 12, pp. 519-521.
2. Kapur K.A. and B.R.Cho.(1996) "Economic design of specification region for multiple quality characteristics." IIE Transactions, Vol. 28, pp. 237-248.
3. Kapur, K.A.(1988) "An approach for developing of specifications for quality improvement." Quality Engineering. Vol. 1, pp. 63-77.
4. Phillips M.D. and B.R.Cho.(1998-1999) "Quality Improvement for processes with Circular and spherical specification Regions.", Quality Engineering Vol. 11, pp. 235-243.
5. Pyzdek T. (1991-1992) "Statistical Process Control and ANSI Y14.5." Quality Engineering, Vol. 4, No. 2
6. Woodward P. (1997) "Accommodation of skewed distribution in SPC." Quality world. April. pp. 308-314.