An On-Line Real-Time SPC Scheme and Its Performance

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Abstract

This paper considers a recent environment in the manufacturing process in which data in large amounts can be obtained on-line in real-time. Under this environment an on-line real-time Statistical Process Control (SPC) scheme equipped with detection of a process change, change-point estimation, and recognition of the change pattern is proposed. The proposed SPC scheme is composed of a Cusum chart, filtering methods and Akaike Information Criterion (AIC). We examine the performance of this scheme by Monte Carlo simulation and show its usefulness.

Keywords: process change, change-point, change pattern, Cusum chart, filtering, Akaike Information Criterion (AIC)

1. Introduction

Today the environment of obtaining data in the manufacturing process has been changing because of the development of measurement techniques. We can automatically obtain on-line real-time data in large amounts. These data are useful for automatic inspection and process regulation but should also be effectively utilized in process control. However, the traditional Statistical Process Control (SPC) methodologies lag behind the changing environment.

The traditional SPC methodologies are concerned only with the detection of process change. However, further information can be obtained from data analysis, for example, when the process changes, (i.e., the change-point estimation) and what the change pattern is, (i.e., recognition of the change pattern). Combining such information with feed forward data about the process conditions, we can more easily identify the assignable causes. In addition, analysis should be performed on-line in real-time and automatically for the purpose of taking rapid action and reducing costs for operator who gazes intently at the computer display.

In this paper, we propose an on-line real-time SPC scheme. This scheme is equipped with the functions mentioned above: detection of a process change, change-point estimation and recognition of the change pattern. These functions can be automatically performed. This scheme is composed of a Cusum chart, filtering methods and Akaike Information Criterion (AIC). The first and second functions are performed by the Cusum chart and a filtering method, and the third is done by the filtering methods and AIC. We present the algorithm of this scheme and examine its performance by Monte Carlo simulation.

2. Process Models

In this paper, we suppose that observations $X(t)$ possibly have the three fundamental process changes as follows:

Shift model: $X(t) = \mu_0 + \varepsilon(t) \, (t = 1,2,\ldots,\tau)$

$= \mu_0 + \delta\sigma + \varepsilon(t) \;\; (t = \tau + 1,\ldots,T)$

Trend model: $X(t) = \mu_0 + \varepsilon(t) \, (t = 1,2,\ldots,\tau)$

$= \mu_0 + \theta\sigma(t - \tau) + \varepsilon(t) \;\; (t = \tau + 1,\ldots,T)$

Impulse model: $X(t) = \mu_0 + \varepsilon(t) \, (t = 1,2,\ldots,\tau)$

$= \mu_0 + a + \varepsilon(t) \;\; (t = \tau + 1)$

$= \mu_0 + \varepsilon(t) \;\; (t = \tau + 2,\ldots,T)$,

where $\varepsilon(t)$ is independently distributed as $N(0,\sigma^2)$; μ_0 is a known target value; σ^2 is known; τ is an unknown change-point; and T stands for a detection point which is a random variable. Then, without loss of generality, we can assume that μ_0 =0, σ^2 =1, δ >0, θ >0 and a>0. In addition, we suppose that these changes cannot occur simultaneously.

The following functions are installed in the proposed SPC scheme: detection of change (i.e. δ, θ or a $\neq$0), change-point estimation, τ, and recognition of the change pattern (i.e., trend change, shift change or impulse change).

3. Proposed SPC Scheme

3.1 Statistical Tools Installed in the Proposed Scheme

As mentioned earlier, the proposed scheme is composed of a Cusum chart, filtering methods and AIC. The proposed scheme has the following significant features:

(1) Its final purpose is recognition of the change pattern, however, the functions of detection of change and change-point estimation are installed, also.

(2) As it has the function of detection of change, the type error is considered.

(3) Change pattern can be recognized sequentially, however, the number of observations required for the recognition can be kept from inflating.

It is well known that Cusum charts have a high sensitivity to relatively small shifts in the process mean. Cusum charts, however, also show high performance in the estimation of change-point, as shown in Nishina [1].

Some methods for recognition of the change pattern using the neural network (for example, Smith [2]) and the wavelet transform (see Suzuki and Uchida [3]) has been proposed. In this paper, filtering methods are mainly used for recognition of the change pattern in order to install a sequential recognition. Love and Simaan [4] proposed the automatic pattern recognition of fundamental process changes using specific filtering methods. These filtering methods are useful tools in the recognition of a pattern change. However, their analysis is not performed sequentially, but retrospectively. In the proposed scheme, filtering methods are installed following Love et al. [4], but they are modified by setting a sequential recognition rule.

In this sequential recognition analysis, the number of observations required for the recognition of the change pattern is a random variable. Its large variance means increasing the number of observations after the change-point. A stopping rule should be installed in this sequential analysis to avoid the large variance. In this paper, a modified AIC is used as the recognition rule at a stopping point.

3.2 Detection of a Process Change and Estimation of Change-point by a Cusum Chart

The Cusum statistic is given by

$$S(0) = 0,$$

$$S(t) = \max\{S(t-1) + X(t) - k, 0\}. \quad (t = 1,2,\ldots)$$

Here, the process mean is regarded as out of control if

$$S(t) \geq h,$$

where h and K are the predetermined positive parameters. ISO/TR 7871 [5] introduced (h,k)=(5.0, 0.5) as a standard scheme, where the values of (h,k) are the unit of σ. Its average run length, where the process mean is maintained at a target μ_0, is 930.

A change-point estimator proposed by Page [6] (hereafter abbreviated as CP estimator) is given by

$$\hat{\tau} = \{t : S(t) = 0, 0 < S(r) < h$$
$$(r = t+1,\dots,T-1), S(T) \geq h\}.$$

The CP estimator is a sort of maximum likelihood estimator under the condition that in the shift model is equal to $2k$. Under a mild condition on τ and δ, its distribution can be approximated by a unimodal distribution with a mode coinciding with the true value τ (see Nishina [1]). However, the distribution of the CP estimator in the trend model has a mode greater than the true value τ. Fig. 1 shows the distributions of the CP estimator of Cusum chart (h,k) =(5.0,0.5) in cases where τ=10, δ=1.0 in the shift model and τ=10, θ=0.1 in the trend model (see Appendix), thus illustrating the above characteristics of the distribution.

3.3 Automatic Pattern Recognition of Process Change Using Filtering Methods

The median filter, the moving average filter and the slope filter are used in the proposed scheme. Hereafter, these filters are abbreviated by ME filter, MA filter and SL filter, respectively. In this scheme, $X(t)$ is replaced by the median $Y_1(t)$ using the ME filter. The impulse change can be detected and recognized at this step. If a process change is detected by the Cusum chart, $X(t)$ is replaced by the moving average $Y_2(t)$ using the MA filter, and the SL filter is applied to $Y_2(t)$; then $Y_2(t)$ is replaced by $Y_3(t)$.

The ME filter is a very effective

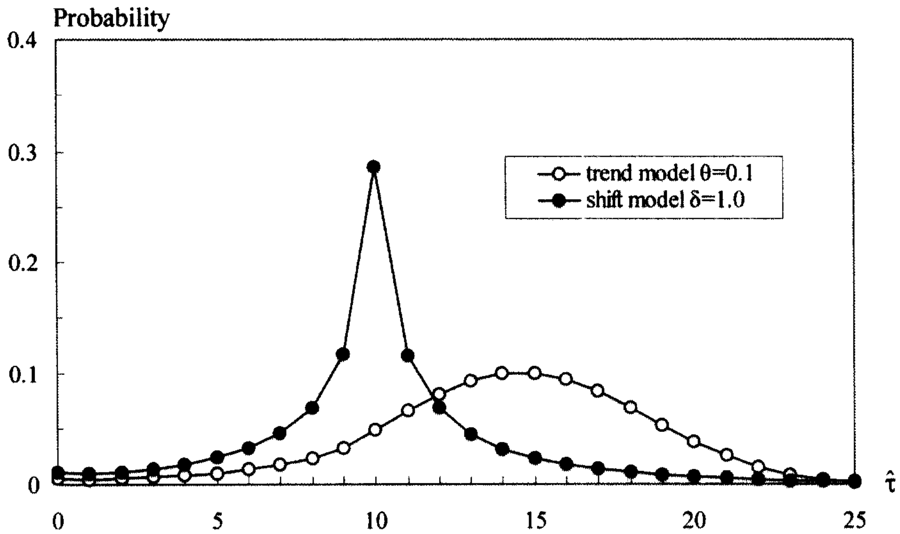


Fig. 1 Distribution of the change-point estimator of the Cusum chart (h, k) = (5.0, 0.5) in the case of the change-point τ.

technique for suppressing the impulse content of a signal (see Love et al. [4]). It is well known that the MA filter is a simple and useful smoother. The SL filter is a derivative filter. Its function is that the shift change can be transformed into a mild impulse pattern, and the trend change can be done into a shift pattern (see Love et al. [4]).

The ME filter applied to $X(t)$ produces the output $Y_1(t)$:

$$Y_1(t) = X(t) - median\{X(i);\ i = t - W_1, \ldots, t + W_1\}$$

where the observation set $X(t)$ $(t = t - W_1, \ldots, t + W_1)$ is a nearest symmetric neighborhood with the window W_1. This is to pick $X(t)$ itself, the W_1 observations to the left and the W_1 observations to the right to $X(t)$. Love et al. [4] have proposed that the impulse change is recognized by setting a threshold. In this scheme threshold D is set as in Love et al. 's scheme. The amplitude of the threshold D is determined to ensure that the probability of committing a type error is an assignable constant. Supposing that the process is in control, we generate 10,000 standard normal random numbers, $X(t)(=\varepsilon(t))$. $X(t)$s are transferred to $Y_1(t)$ through the ME filter. As a result of the simulations, for example, when $W_1 = 8$, it is determined that D = 3.19 such that the probability of a type error is 0.001. If $Y_1(t) \geq D$ and $Y_1(t+1) < D$, then the impulse change is recognized.

The MA filter applied to $X(t)$ produces the output $Y_2(t)$:

$$Y_2(t) = [\frac{\Sigma X(i)}{2W_2 + 1}; i = t - W_2, \ldots, t + W_2]$$

Similarly, the SL filter applied to $Y_2(t)$ produces the output :

$$Y_3(t) = [\frac{\Sigma i Y_2(i) - \Sigma i \Sigma Y_2(i)/(2W_3 + 1)}{\Sigma i^2 - (\Sigma i)^2/(2W_3 + 1)};$$
$$i = t - W_3, \ldots, t + W_3]$$

where W_2, W_3 are the windows of the MA filter and the SL filter, respectively. The SL filter is the regression coefficient of a linear fit applied to the nearest symmetric neighborhood with the window W_3 of $Y_2(t)$.

Fig. 2a shows a result of the filtering method being applied to the shift model in the case where τ=50, δ=1.0, W_2=8, W_3 =8. It appears that $Y_3(t)$ indicates a mild impulse like a "hill" near the CP estimate. On the other hand, Fig. 2b shows a result of the trend model under the same condition as Fig. 2a except that θ=0.1. It should be noted that there is quite a difference in the output $Y_3(t)$ between Fig. 2a and Fig. 2b. The difference is the behaviour of $Y_3(t)$ after the CP estimator. The behaviour shown in Fig. 2a indicates a downhill trend and then decreases to zero. On the other hand, the behaviour shown in Fig. 2b indicates keeping to the positive side. In this scheme the recognition of the change pattern, that is, the trend change or the shift change, is performed by observing indications of $Y_3(t)$ after the change-point. From Fig. 2a and 2b, if the change is the shift model, the behavior $Y_3(t)\ (t > \tau)$ tends to indicate a decreasing pattern; if the change is the trend model, they tend to be in the positive side. As the change-point is unknown in practice, we take notice of the behavior of $Y_3(t)\ (t > \tau)$. That is, the CP estimator indicates the set of $Y_3(t)$ necessary for the recognition. From the above consideration, the distribution of the CP estimator has an effect on the recognition performance. In the shift model, the probability of underestimation on the CP estimator increases as δ increases (see Nishina [1]). In the trend model, the CP estimator has a positive bias (see Fig. 1). Consequently, it is better that the $Y_3(t)$ necessary for the recognition, say Θ_Y, is set as

$$\Theta_Y = \{Y_3(t);\ t \geq \hat{\tau} + T_s (T_s > 0)\},$$

where T_s is considered as a parameter for recognition. T_s stands for a lag between the CP estimate and the starting point of recognition.

Next, some specified patterns in the set Θ_Y are found to identify the change pattern. As the result of empirical trials, some rules shown in Fig. 3 can be established to distinguish between indications of set Θ_Y decreasing to zero (Rule 1, 2 and 3) and still showing on the positive side (Rule 4, 5 and 6). In these rules shown in Fig. 3, the threshold $C(>0)$ and the run length of a monotone increase are other parameters of recognition. If $Y_3(t) \leq C$, the change pattern is regarded as a shift without observing the behaviour of $Y_3(i)$ $(i = t+1, \cdots)$ because this behaviour indicates the right tail of a mild impulse (see Rule 1, 2 and 3 in Fig. 3). On the other hand, if the behaviour of $Y_3(t)$ indicates an increasing trend in the zone more than C, even if it indicates a decreasing trend before the increasing trend, the change pattern is regarded as a trend (see Rule 4, 5 and 6 in Fig. 3).

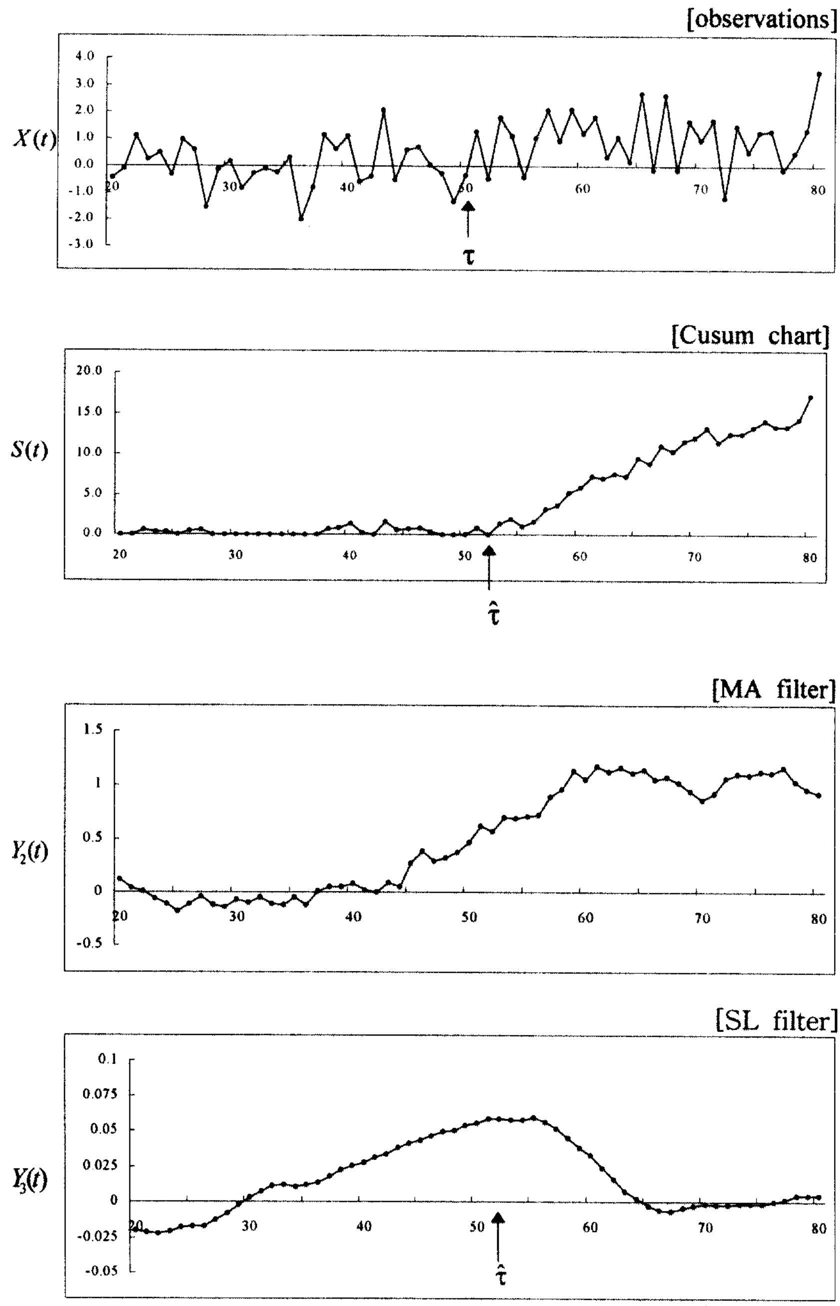


Fig. 2a A result of the filtering methods being applied to the shift model in the case of $\tau=50$, $\delta=1.0$, $W_2=8$, $W_3=8$.

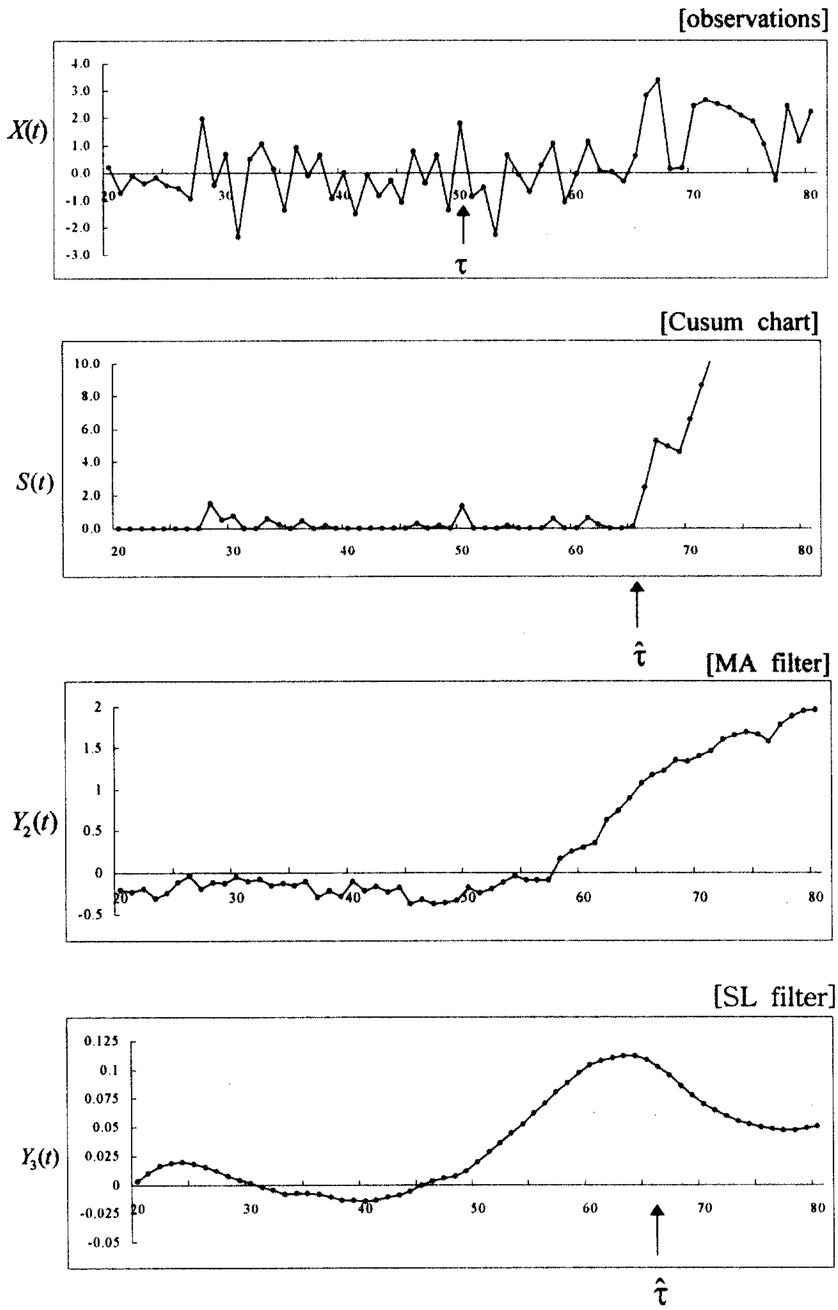


Fig. 2b A result of the filtering methods being applied to the shift model in the case of τ =50, θ =0.1, W_2 =8, W_3 =8.

3.4 Pattern Recognition Rules at Stopping Point Using AIC

As described in 3.1, the number of observations required for recognition using the filtering methods is a random variable. A stopping rule should be installed to avoid an extreme number of observations for recognition. In this paper, a modified AIC is used when time t for recognizing the change pattern reaches a time point $\hat{\tau}+R$, where R is the predetermined point. Then AIC is given by using the set of observations $X(t)$ $(t=\hat{\tau}+T_s,\ldots,\hat{\tau}+R)$ R is set as $W_2+W_3+T_s+L$, which is the largest constant number required for the pattern recognition in Fig. 3. Either the shift model or the trend model below is selected depending on the AIC values:

$$X(t)=\delta+\varepsilon(t),$$

$$X(t)=b+\theta(t-\hat{\tau})+\varepsilon(t), \quad (t=\hat{\tau}+T_s,\ldots,\hat{\tau}+R)$$

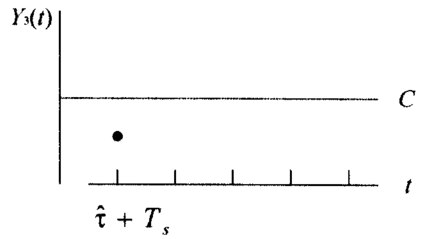
where b is not necessarily equal to zero.

Let p and $SS_e(p)$ denote the number of parameters and the residual sum of squares of the above models with the p parameters, respectively. Then AIC is given by

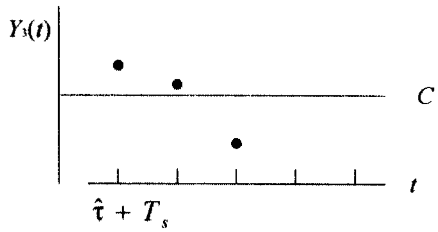
shift model: $AIC(1)=(R-T_s)\log SS_e(1)+2$,

trend model: $AIC(2)=(R-T_s)\log SS_e(2)+4$.

However, it should be noted that if δ is small (e.g. $\delta \cong k$) in the shift model, then the set of observations $X(t)$ $(t=\hat{\tau}+T_s,\ldots,T)$ can have a positive bias because in this case $\hat{\tau}$ can be overestimated (see Nishina [1]). On the other hand, the supplemental observations $X(t)$ $(T+1 \le t \le \hat{\tau}+R)$ required for recognition can not be taken with the positive bias. Consequently, the estimate of θ can be negative, and $AIC(1)$ can then be larger than $AIC(2)$. This is an erroneous recognition. In order to avoid this error, the shift mode is selected if the estimate of θ $AIC(1) > AIC(2)$ is negative; even if.



Rule 1



Rule 2

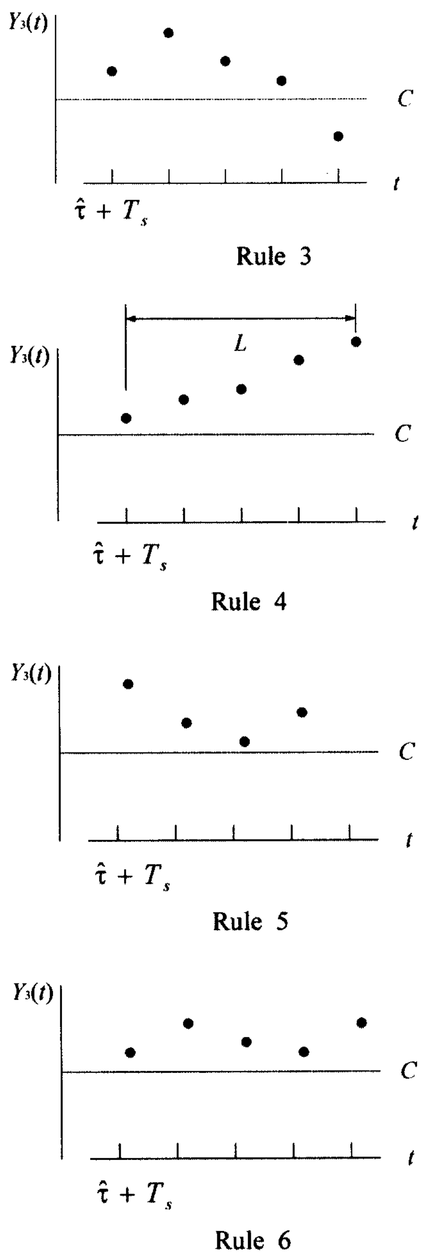


Fig. 3 Rules of pattern recognition for discrimination between the shift model and trend model.

4. Algorithm of Proposed SPC Scheme

Fig. 4 shows the algorithm of the proposed SPC scheme. Summarizing the above discussions, this scheme has the following parameters: (h,k) in the Cusum chart; the window of the ME filter W_1 , the window of the MA filter W_2 and the window of the SL filter W_3 in the filtering methods; the threshold D for the detection of the impulse change, the starting point T_s, the run length of a monotone increase L, the stopping point R and the threshold C for pattern recognition between the shift change and the trend change.

The flow of the algorithm is as follows: at first a shift change or a trend change is detected by the Cusum chart, and then the change-point is estimated. Next, the change pattern is recognized according to the rules shown in Fig. 3 with respect to the behavior of $Y_3(t)$. Then if the change pattern can not be identified until the detection point T, sequential recognition is carried out by supplemental observation until the stopping point $\hat{\tau}+R$. At the stopping point $\hat{\tau}+R$, recognition is performed using the modified AIC rule. Simultaneously, the ME filter is applied to detect an impulse change.

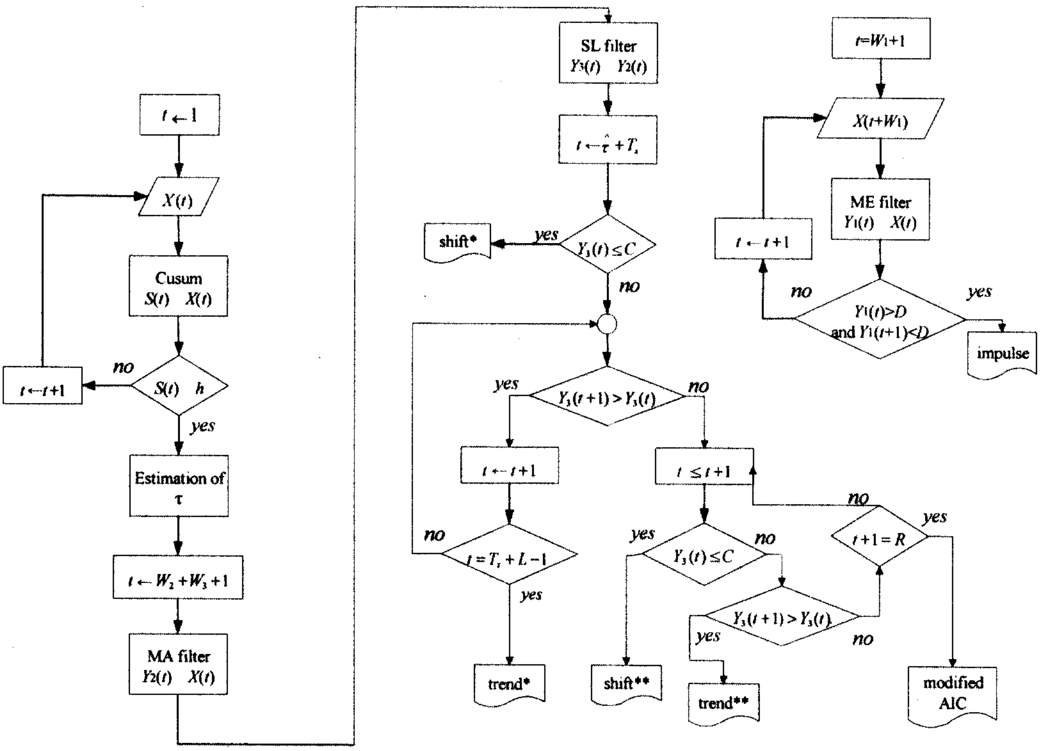


Fig. 4 Flow chart of the proposed SPC scheme.

Note 1: "shift*" stands for application of Rule 1 shown in Fig. 3, and "shift**" indicates Rule 2 or Rule 3. "trend*" stands for application of Rule 4, and "trend**" indicates Rule 5 or Rule 6.

Note 2: As the need arises, a supplemental observation is sequentially taken.

5. Performance of Proposed SPC Scheme

First, setting the parameters of the proposed scheme is discussed. Regarding the Cusum chart, (h,k) are set as (5.0, 0.5) following the ISO/TR 7871 [5] as described earlier. The other parameters should be set so that the SPC scheme is optimal in the sense of recognition of the change pattern. However, there is no scheme which is uniformly optimal for the process parameters a, δ and θ. In particular, there is no uniformly optimal scheme concerning the discrimination between the shift change and trend change. Therefore, we cannot help setting the parameters of recognition so that the recognition performance is as high as possible for the region of δ and θ. The performance index is defined as the proportion of the correct recognition.

We determine the performance index of recognition (abbreviated by PIR) by Monte Carlo simulation, which is undertaken using random normal variables under the process change models generated by the Box-Muller method. We evaluate 2000 simulations on the condition of no type error. The PIR is given by

$$\text{PIR} = \frac{\text{[number of trials which indicate correct recognition]}}{\text{[number of simulation trials without type error]}}$$

Fig. 5 shows the PIR as simulation results where the parameters of the proposed scheme are set as follows:

Cusum: (h,k) = (5.0, 0.5)

Filter: W_1=8 (for the ME filter), W_2=8 (for the MA filter),

$W_3 = 8$ (for the SL filter)

Threshold: D =3.19 (for the ME filter),

C =0.023 (for the SL filter)

Others: $T_s = 3$ (the lag from the change-point estimate),

L=6 (the increasing run of the SL filter),

R=25 ($= W_2 + W_3 + T_s + L$)

(the stopping point)

where the values of h, k, D and C are set in units of σ. The value of C is the standard deviation of $Y_3(t)$ under the process in control, which is determined by the simulation similarly to the case of the determination of D. These values are determined such that PIRs are as large as possible for the region of δ and θ shown in Fig. 5. In addition, Fig.5 includes an average sample number required for recognition after the change-point (abbreviated by ASNR). The process parameters in Fig. 5 are set as δ=0.75 (0.25)2.0, θ=0.06(0.01)0.11, a = 3.5(0.5)6.0 and τ=40. The reason why the change-point τ is set to 40 is that the Cusum statistic in control state can be regarded to have already reached the stationary state at t=40. It is confirmed that these simulation results of the PIR are identical to two decimal places, from altering the initial value of the random number.

From Fig. 5 we can see the following:

(1) The PIR is larger than 0.8 and stable for δ (0.75 2.0),

(2) The PIR is increasing for θ (0.06 0.11).

These high performances shown in Fig. 5 are due not only to the filtering methods but also to the change-point estimation. Particularly, significance is the effect of setting the lag between the CP estimate and the starting point of recognition, which is determined from the property of the distribution of the CP estimator as shown in Fig. 1.

(3) But in cases where θ=0.06, 0.07, there is a relative high probability that the trend model will be confused with the shift model.

This is caused by the behaviour of $Y_3(t)$. As shown in Fig. 2a and Fig. 2b, the discrimination between the shift model and trend model mainly depends on the behaviour of $Y_3(t)$ after its peak. Owing to random error, the typical patterns shown in Fig. 3 do not necessarily occur.

(4) In the case of the trend model, ASNR is decreasing as θ increases but these values are larger than those for the shift model.

This is caused by the bias and variance of the CP estimator in the trend model. As shown in Fig. 1, the variance in the trend model is greater than that in the shift model, and the bias in the trend model is relatively large and positive.

Next we examine how much the performance is affected by the variation of the parameters of the proposed scheme. This is a sort of the sensitivity analysis. We conduct an experimental design as shown in Table 1 for discriminating between the shift change and trend change. We set up the $L_{16} \times L_4$ product orthogonal array. The inner and outer factors and their levels are shown in Table 2a. The inner factors are the parameters of the proposed SPC scheme, the outer factors (P_1: change pattern, P_2: change amount) are related to the process change. Therefore, the inner and outer factors can be regarded as the control factors and the environmental factors, respectively. In the outer array, the interaction $P_1 \times P_2$ has a meaning as four process change models (see Table 2b). It should be noted that the factor P_2 has a meaning as the process change level only when the interaction $P_1 \times P_2$ is considered. The thresholds C and D are determined by depending upon W_1 and (W_2 ,W_3) as in the case of $W_1 = W_2 = W_3 = 8$ (see Table 2c). The outputs of this experiment are the PIR and the ASNR calculated by computer simulation.

The ANOVA tables are shown in Table 3a (PIR) and Table 3b (ASNR). From the results of ANOVA of the PIR, it should be noted that the interactions between the change pattern, P_1, P_2, $P_1 \times P_2$, and the control factors, $W_2 \times W_3$, T_S, L are significantly active, and that the factor W_1 and its related interactions are not active. These active effects are shown in Fig. 6, from which it follows that:

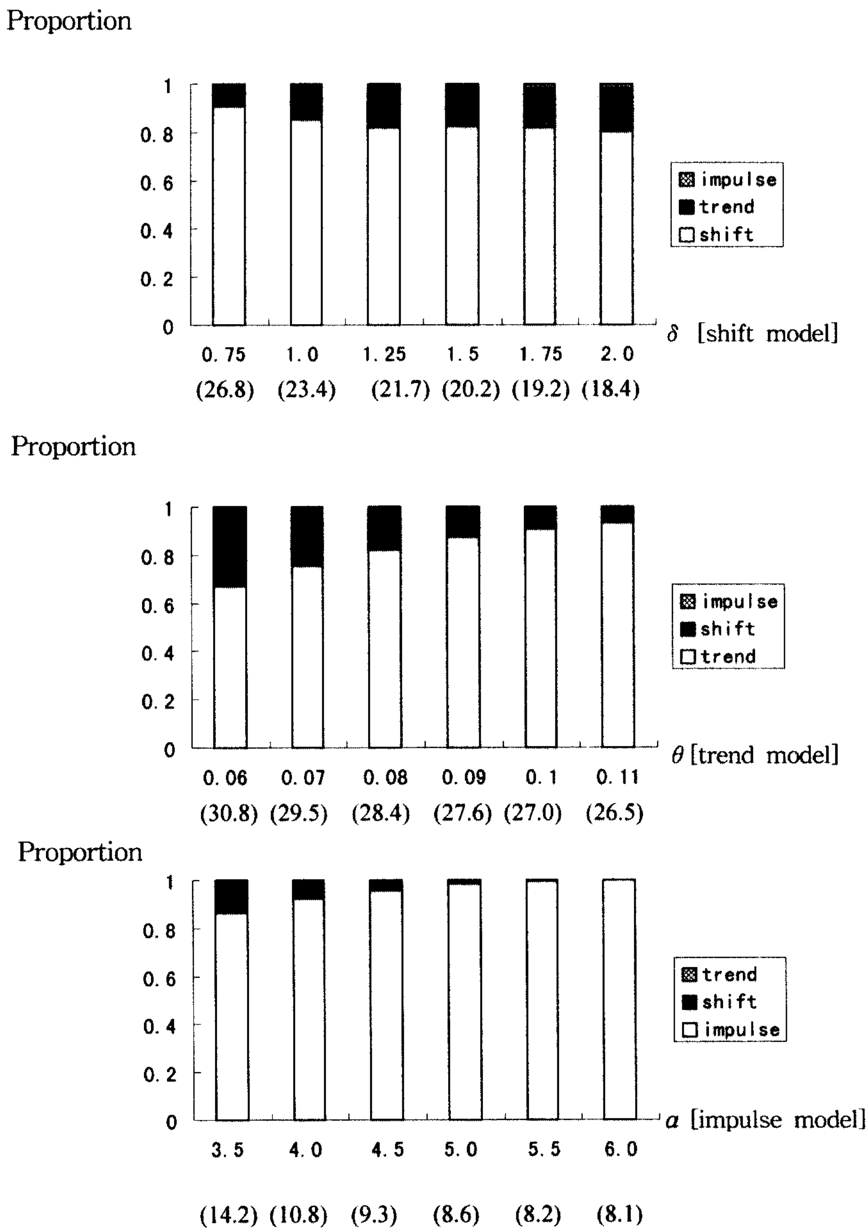


Fig. 5 Performance Index of Recognition (PIR).

Note: Proportion shown by the white portion is PIR.
Numbers in parentheses stand for ASNR.

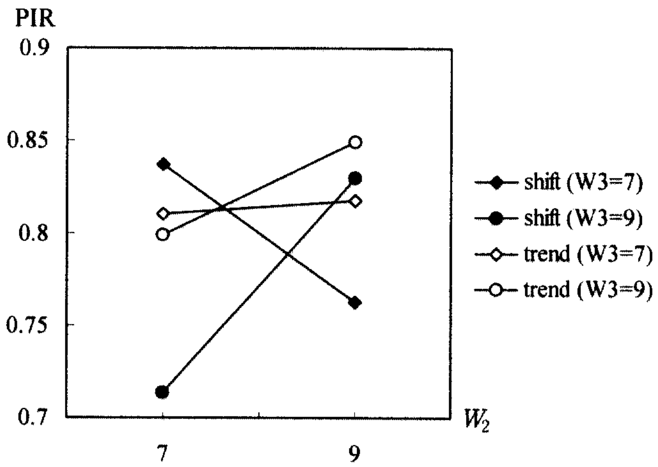


Fig. 6a Pattern of the interaction $W_2 \times W_3$.

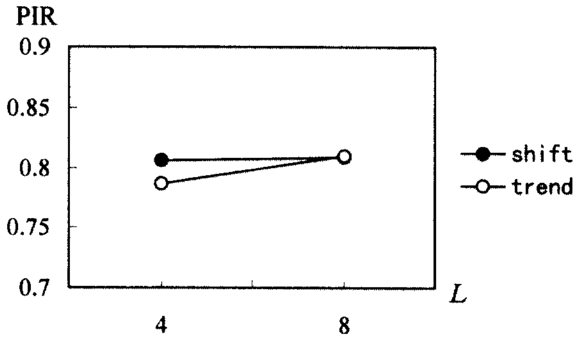


Fig. 6b Pattern of the interaction of $L \times P_I$.

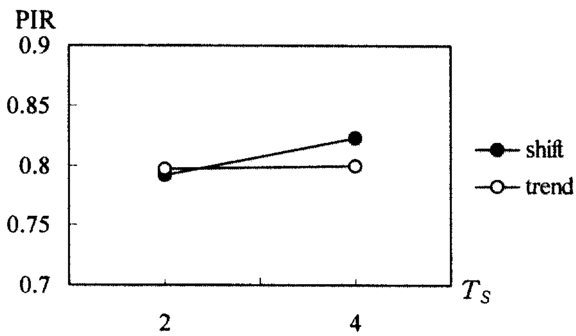


Fig. 6c Pattern of the interaction of $T_S \times P_I$.

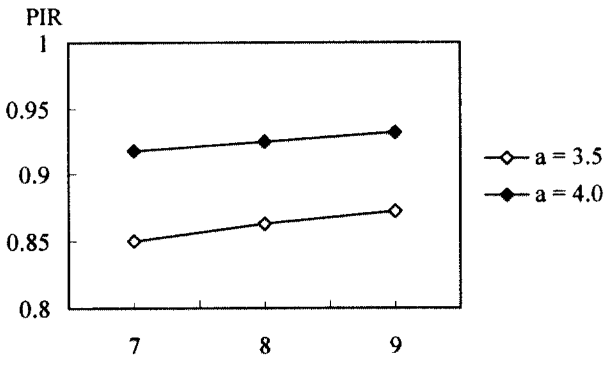


Fig. 7 Effect of W1 in the impulse model.

(1) $W_2=W_3=7$ is best for the shift change, whereas, $W_2=W_3=9$ is best for the trend change. This means that this activity is a trade-off between the shift change and trend change (see Fig. 6a).

(2) the effects of L are active only in the case of the trend change (see Fig. 6b).

(3) the effects of T_S are active only in the case of the shift change (see Fig. 6c).

From the results of ANOVA of the ASNR, every main effect except W_1 is significantly active (see Table 3b). On the other hand, F ratio of every interaction is smaller than that of the significant main effects. This means that the SPC scheme with the larger parameters W_2, W_3, T_S and L requires more observations, no matter what the process condition.

The result in the case of the impulse change is shown in Fig. 7. This experiment or the impulse change a = 3.5, 4.0 is conducted differently from the experiment above. The factor is W_1, of which the levels W_2 = 7 and W_1 = 9 are added to W_1 = 8. From Fig. 7 it can be seen that the PIR improves as W_1 increases. However, as discussed above, it should be noted that the larger W_1 increases ASNR.

Now we consider the region of the process parameters, $3.5 \leq a \leq 60$, $0.75 \leq \delta \leq 2.0$ and $0.06 \leq \theta \leq 0.11$. If a different region of δ, θ and a from this situation is considered, the SPC parameters must be determined again. Then the results of this sensitivity analysis should be referred to.

Table 1 $L_{16} \times L_4$ product orthogonal array for the simulation study and its results, PIR and ASNR

						1	1	2	2	P_1
						1	2	1	2	P_2
						1	2	2	1	$P_1 \times P_2$
W_2	W_3	$W_2 \times W_3$	W_1	T_S	L	$\delta = 1.0$	$\delta = 2.0$	$\theta = 0.06$	$\theta = 0.1$	
1	1	1	1	1	1	0.845 (20.8)	0.797 (16.2)	0.577 (27.8)	0.828 (23.5)	
1	1	1	1	1	2	0.860 (20.8)	0.801 (17.5)	0.584 (28.4)	0.856 (24.8)	
1	1	1	2	2	2	0.892 (21.5)	0.807 (19.8)	0.591 (29.6)	0.883 (26.1)	
1	1	1	2	2	1	0.882 (21.0)	0.810 (16.5)	0.559 (28.8)	0.834 (24.7)	
1	2	2	1	2	2	0.844 (23.9)	0.794 (20.7)	0.710 (31.7)	0.932 (28.3)	
1	2	2	1	2	1	0.848 (23.2)	0.808 (18.5)	0.662 (30.6)	0.904 (26.7)	
1	2	2	2	1	1	0.811 (22.8)	0.791 (18.2)	0.665 (29.5)	0.892 (25.2)	
1	2	2	2	1	2	0.799 (23.0)	0.792 (18.8)	0.710 (30.2)	0.918 (26.8)	
2	1	2	1	2	2	0.800 (24.4)	0.769 (20.0)	0.759 (31.7)	0.938 (28.5)	
2	1	2	1	2	1	0.810 (23.5)	0.778 (18.5)	0.714 (31.0)	0.918 (26.9)	
2	1	2	2	1	1	0.738 (22.8)	0.708 (18.2)	0.731 (30.0)	0.923 (25.5)	
2	1	2	2	1	2	0.768 (23.2)	0.731 (18.5)	0.734 (30.8)	0.924 (27.2)	
2	2	1	1	1	1	0.807 (25.5)	0.791 (20.2)	0.747 (31.7)	0.944 (27.3)	
2	2	1	1	1	2	0.823 (25.6)	0.801 (20.4)	0.763 (33.0)	0.952 (29.6)	
2	2	1	2	2	2	0.854 (26.8)	0.807 (21.2)	0.744 (34.0)	0.960 (30.7)	
2	2	1	2	2	1	0.853 (25.8)	0.808 (20.4)	0.736 (32.9)	0.946 (28.7)	

Note: Numbers in parentheses stand for ASNR.

Table 2a Factors and their levels

	Factor	Level 1	Level 2
Inner factors			
W_2	window of MA filter	7	9
W_3	window of SL filter	7	9
W_1	window of ME filter	7	9
T_S	lag from change-point estimate	2	4
L	increasing run of SL filter	4	8
Outer factors			
P_1	change pattern	shift model	trend model
P_2	change amount	small	large

Table 2b Levels of outer array

P_2 \ P_1	shift model	trend model
small	$\delta = 1.0$	$\theta = 0.06$
large	$\delta = 2.0$	$\theta = 0.1$

Table 2c Thresholds D and C

D

W_3 \ W_2	7	9
7	0.029	0.022
9	0.024	0.019

C

W_1	7	9
	3.22	3.17

6. Conclusions

We have proposed an on-line, real-time and automatic SPC scheme incorporating detection of a process change, change-point estimation and recognition of the change pattern. It can be concluded that this scheme is a useful method for the recent manufacturing process in which data in large amounts can be obtained on-line in real-time.

Appendix: Derivation of Distribution of CP Estimator in the Trend Model

The distribution of the CP estimator in the shift model has been derived in Nishina [1]. Following Nishina [1], we shall derive the distribution of the CP estimator, $\hat{\tau}$, in the trend model.

At first, we shall derive some characteristics of the Cusum statistic $S(t)$ in the trend model. Let us denote them as follows:

$\varphi(t,z)$: the p.d.f. of the distribution function $\Pr\{S(t) \le z | S(0) = 0\}$ under $\mu_0 = 0$; however, $\varphi(t,0)$ is a probability mass.

$\Psi(t)$: the probability for the Cusum statistic not to reach at time t under $\mu_0 = 0$.

$g(t,s,z;\theta)$: the p.d.f. of the distribution function

$$\Pr\left\{\begin{matrix} S(t) \le z, 0 < S(i) < h(z > 0, 0 < i < t) \\ |S(0) = s(0 \le s < h) \end{matrix}\right\}.$$

$g_0(t,s;\theta)$: the probability

$$\Pr\{S(t) = 0 | S(0) = s(0 \le s < h)\}.$$

The term "conditional" above means the condition in which type I error does not occur.

The Cusum statistic can be regarded as a

Markov process in which continuous states have an absorbing state $(S(t) \geq h)$ and a reflecting state $(S(t) = 0)$ in discrete time (see Brook and Evans [7]). Since the Markov process has a reflecting state, the distribution of the Cusum statistic has a probability mass for $S(t) = 0$. From the Markov process analysis, $\varphi(t,z)$, $g(t,s,z;\theta)$ and $g_0(t,s;\theta)$ satisfy the following recurrence equations, respectively.

$$\varphi(0,z) = \begin{cases} 1 & (z=0) \\ 0 & (z>0) \end{cases}$$

$$\varphi(t,z) = \begin{cases} f^+(0,z;0) & (t=1) \\ \int_0^h f^+(y,z;0)\varphi(t-1,y)dy \\ +f^+(0,z;0)\varphi(t-1,0) \\ (t=2,3,\ldots) \end{cases} \quad \text{(A1)}$$

where

$$f^+(s,z;\mu) = \begin{cases} f(z+k-s-\mu) & (z>0) \\ \int_{-\infty}^{k-s} f(x-\mu)dx & (z=0) \end{cases}$$

$$f(x) = \exp[-x^2/2]/\sqrt{2\pi}.$$

The function $\varphi(t,z)$ has a probability at z = 0 ; therefore, the second term on the right side in the equation (A1) is needed. However, for the convenience of description the transition of which the integration is represented by the equation (A1) is rewritten hereafter as follows:

$$\varphi(t,z) = \int_{0^+}^h f^+(y,z;\delta)\varphi(t-1,y)dy.$$

Similarly,

$$g(t,s,z;\theta) = \begin{cases} f(z+k-s-\theta) & (t=1) \\ \int_0^h f(z+k-y-t\theta)g(t-1,s,y;\theta)dy. \end{cases}$$

$$(t=1,2,3\ldots)$$

Similarly,

$$g_0(t,s;\theta) = \begin{cases} f^+(s,0;\theta) & (t=1) \\ \int_{0^+}^h f^+(s,y;t\theta)g_0(t-1,y;\theta)dy. \end{cases}$$

$$(t=1,2,3\ldots)$$

From the definition of $\varphi(t,z)$, we have

$$\Psi(t) = \int_{0^+}^h \varphi(t,z)dz.$$

Let $p(t)$ $(t=1,2,3\ldots)$ denote the distribution of $\hat{\tau}$ in the trend model. Then we can obtain $P(t)$ by using the characteristics of the Cusum statistic above as follows:

$$p(t)=\begin{cases}\{\varphi(t,0)/\Psi(\tau)\}\sum_{l=1}^{\infty}\int_h^{\infty}\int_0^h g(\tau-t,0,y;0)g(l,y,z;\theta)dydz & (t=1,2,\ldots,\tau-1)\\ \{\varphi(\tau,0)/\Psi(\tau)\}\sum_{l=1}^{\infty}\int_h^{\infty} g(t,0,z;\theta)dz & (t=\tau)\\ \int_{0^+}^h g_0(t-\tau,y;\theta)\{\varphi(\tau,y)/\Psi(\tau)\}dy\sum_{l=1}^{\infty}\int_h^{\infty} g(l,s,z;\theta)dz & (t=\tau+1,\tau+2,\ldots)\end{cases}$$

In the case of the shift model, the distribution of $\hat{\tau}$ can be driven similarly by replacing $t\theta$ with δ.

The distributions of the CP estimator shown in Fig. 1 can be calculated by the numerical integrations. It is performed by the Legendre-Gauss quadrature formula (15 Gaussian quadrature points).

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