The Same Distribution Principle and Its Applications*

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Abstract

In this paper, the same distribution principle is developed based on the studies with many years for short run processes. The quality control philosophy and background of the principle are introduced. Some implementing techniques for the principle are presented, and application cases both in industry and evaluation & decision-making areas are shown.

Key words: Quality control, Short run , Evaluation & decision-making

1. Introduction

Process is a very popular concept in modern management fields, as we know, many problems must be considered as some processes, which are great improvement for effectiveness and efficiency with the problem-solving. Two branches of current management systems, ISO9000 standard systems and National Quality Award models are very strongly emphasized on process management, the process model is employed even in description of new version for ISO9000: 2000, as shown in Figure 1.

There are various kinds of processes in real world from engineering, science areas to social systems, such as design & development process, production process, business process, evaluating and decision making process, medical diagonal process, financial report process as well as a number of different sorts of management processes. From this meaning, we can say that our society is formed by various of processes. Especially, an enterprise can consist of many processes, all of which can also be divided

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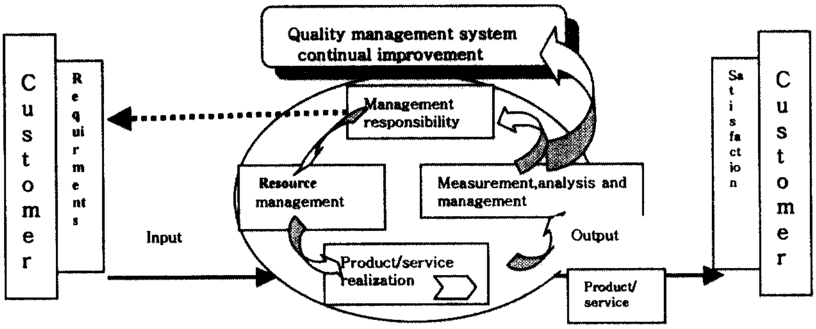
engineering processes and management processes. Usually, a process is depicted asUsually, a process is depicted as an activity with both an input and output, seen as Fig. 2.

production process produce products or parts as output when materials are inputted. If the process is a comprehensive management activity, for instance, in Fig.1, the input is customer requirement, the output is customer

Fig. 1 Process description for quality systems



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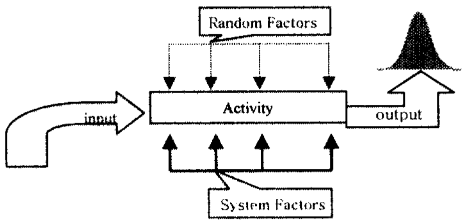

Fig. 2 Process model

satisfaction and the activity is the performance for quality systems to be utilized. In evaluating and decision-making areas, the activity is experts evaluating, the evaluation results, output, can be obtained and the input is evaluation objects. Many backgrounds for processes may be discussed in different fields.

In process approaches, a kind of processes has aroused the interesting of researchers and practitioners. With some production activities, a process can be used in manufacturing different types of parts. As we known, different types of parts are of their different distribution, the data coming from different type parts could not be put together simply to reveal statistical law of the process performance. The problem is how to use different type data generated with the process to depict performance of the process. In evaluation and decision-making process, for instance, a expert group is rating projects, each expert in the group may result in different distribution data sets, could we directly employ the data sets to perform evaluation and comparison for the rated objects? This kind of process can be encountered from many other practical cases. In terms of these processes, the key methodology for solving problems is to merge reasonably different data sets into big one, which can be used to demonstrate its distribution law, and make use of that for control and design as well as evaluation & decision-making.

In this paper, the concept of a general process is described and its model is given. The same distribution principle based on the above problem is developed. According to the principle, the implementing methods for improving process are studied. Finally, several application cases, such as a target process, proportional process, general process for small batch, expert base evaluation process, and etc. are discussed.

2. The Same Distribution Principle

As discussed above, a short run process may produce several different types of parts, which may have different distributions for different types of parts, that is, different moments, typically different means and variances, see Fig. 3. In order to get the enough data, to reveal the performance or statistical law of the short run process as well as to monitor and improve the process, we must collect all data coming from different type of parts with small batch to be got together and become an enough larger data set. In this way, a mistake will happen to the collection, because there are different distributions for different types, the data of each type could not be put together simply. We must transform the each type of data to be of the identical distribution by some ways or transformations, which may be called as the same distribution transformation.

In a real assessment and decision-making process, the evaluating values coming from different experts in a expert base will be put together, and the final decision is made by means of comparison based on the data. We may know that there are different moments including the means and variances for different expert evaluation data. It is the differences among the experts that result in the unreasonableness of the comparison. The main problem is to eliminate the difference in various moments of evaluation results among experts so that the data from different experts are of equivalence. In principle, only the data that are of the same distribution can be reasonably compared, and the data can be obtained by the same distribution transformation.

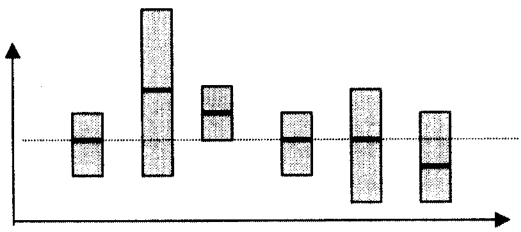

Fig 3 Boxplot chart

It seems that some common things have been discovered from the above discussion, which can be summarized the same distribution principle as follows.

The same distribution principle Suppose S = {1, 2,...., n}. F_k is a distribution function of the process output with $k \in S$, define F_S as the distribution function of the process, then F_S can be obtained from the same distribution transformation for F_1, F_2,..., F_n.

Here, the distribution function of processs output, F_k, can be replaced with density function, f_k, or data set, ξ_k, of process output, in which ξ_k can also be considered as random variable. Therefore, the same distribution principle can be described by different ways.

Any process must be affected by two kinds of factors, one is random factors and the other is system factors. Random factors cause variation of the process. The variation must be of the statistical distribution law because of the domination of the system factors. For a process, once its system factors are set up, objectively, there must exist the determined distribution law for the process whether or not the process is operated and the data is outputted. When none of data or parts is produced, no one knows the distribution law of the process without any information of process performance. Only does the god know that. However, we cant deny the existence of the distribution. The crucial problem is for human beings how to reveal the distribution law by much more advanced techniques. Similarly, if the process does not output enough parts or data, then there is no sufficient data and information for the process performance. So it must be difficult to find out the statistical law for the process. However, people have to believe in the existence of the distribution law. Fortunately, in some occasions, a process often produces different types of parts with different data sets. In terms of each one, the data and information included in the process are not enough to demonstrate the distribution law of the process. Each type of data will contain the equivalent information of the process performance, which offers a great opportunity to merge all types of data sets into a big one, so there may be enough data and information to indicate the distribution law in the merged data set. The critical problem is to transfer different type data into an identical distribution just before being merged. Its based on the ideology that we develop the same distribution principle and the methodology for this kind of process. The principle is very useful in relative to areas such as industry and evaluation & decision-making. In the following sections, some applications in these areas are studied, in which the obtained results are exciting.

3. Target Process

Some kinds of processes output different type parts with the corresponding measurements denoted by ξ_k, in which each type parts has its own working target, T_k, k=1,2,..., n, say nominal, and there is the same variability, σ, from part type to part type. The process with the above characters is defined as a target process.

To the target process, the same distribution transformation may be built as follow

$$\xi_k^* = \xi_k - T_k \qquad k = 1, 2, \ldots, n$$

$$\xi = \bigcup_{k=1}^{n} \xi_k^*$$

Usually, ξ_1^*, ξ_2^*, ξ_3^*,..., ξ_n^* are of the same distribution, and the merged data set, ξ, can be used in revealing statistical law of the process performance. It is easy to show that the above conclusion is true. Assume that the ξ_k is normally distributed, $N(\mu_k, \sigma^2)$, with the same variance σ, and usually the target T_k of nominal is close to the mean μ_k, k=1, 2, 3,..., n. It is very easy to prove that the ξ has a normal distribution $N(0, \sigma^2)$ approximately. The control chart and robust design or any statistical control techniques for the process can be performed based on the same distribution $N(0, \sigma^2)$.

Case 1 for target process

As an example, lets examine the process for cutting bar stock to length. We cut it to three types and each type only has few pieces. The customers want to see the control charts for their own order types. The details for three types can be shown in Table 1.

Table 1 The data for three orders

Order type	Length	Number of pieces
1	2.25 in	6
2	4.50 in	12
3	6.75 in	18

The data for three types of bar stock length are plotted in the tier chart, see Figure 4. The data are grouped in threes and plotted over time. Notice that the data for the three different part types are centered on their respective nominals. In addition, the variability from part type to part type is roughly the same.

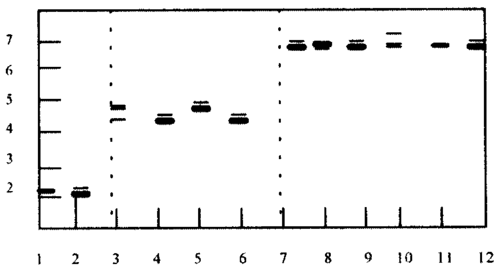

Fig. 4 Tier chart

As shown in Fig.5, three types of parts, with targets 2.25, 4.50, 6.75 respectively, is to be processed. Obviously, there are three kinds of data with different means which could be 2.25, 4.50, 6.75. In order to get the data coming from the same population, the transformation that the observations subtract their targets is performed. The transformed data have the same distribution, that is, coming from the same population with the zero means and the determined variation. Based on the transformed data, the statistical methodology can be easily used, for example, getting the distribution law, setting up the control charts, doing robust design with variation reduction and so on, which is the philosophy and technology of quality control for target process.

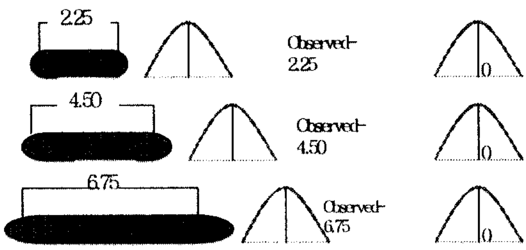

Fig. 5 The same distribution transformation

4. Proportional Process

Some kinds of processes may produce different type parts with the corresponding measurements denoted by ξ_k, in which each type parts has its own working target, T_k, say nominal, and variability, σ_k, where k=1, 2,..., n.

If the change in variability is proportional to the change in target from part type to part type, the process is defined as a proportional process.

In terms of the proportional process, we may have the same distribution transformation as follows.

$\xi_k^* = \xi_k / T_k \qquad k= 1, 2,..., n$

$$\xi = \sum_{k=1}^{n} \xi_k^*$$

Usually, $\xi_1^*, \xi_2^*, \xi_3^*,..., \xi_n^*$ are of the same distribution, and the merged data set, ξ, can be used in revealing statistical law of the process performance.

In order to show that the above conclusion is true, namely, ξ_k^* is of the identical distribution for any $k \in \{1, 2,..., n\}$, we assume that the ξ_k is normally distributed, $N(\mu_k, \sigma_k^2)$, and usually the target T_k of nominal is close to the mean μ_k, so it is considered that the $T_k = \mu_k$ for any $k = 1,2,...., n$. Since the change in variability is proportional to the change in target from part type to part type, it is reasonable to suppose that $T_k/\sigma_k = c$, where the c is a constant.

Therefore,

$E(\xi_k^*) = E(\xi_k/T_k) = \mu_k/T_k = 1$

Variance

$D(\xi_k^*)=D(\xi_k/T_k) = T_k^{-2}\sigma_k^2 = c^2, k=1, 2,..., n$

So the $\xi_k^* \sim N(1, c^2)$ approximately, the ξ has a normal distribution, and the control chart and robust design or any statistical control techniques for the process can be performed based on the same distribution $N(1, c^2)$.

Case 2 for proportional process

Now consider a paint process involved in four types of paint thicknesses, namely, four target values or nominals are 7.50 mils, 0.55 mils, 3.25 mils, 1.10 mils respectively. The data for the thicknesses in this process are plotted in tier chart as shown in Fig.6. In Fig.6, we notice that the variability in the thicknesses changes from part type to part type as the nominal changes. For example, as nominal changes from 0.55 to 1.10 mils, the variability double. As nominal changes from 1.10 to 3.25 mils, the variability triples. In other words, the change in variability is proportional to change in nominal.

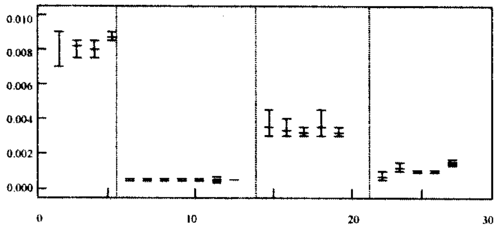

Fig. 6 Tier chart

In order to get the data with the same

distribution, the above proportional transformations, $\xi_k^* = \xi_k/T_k$, k = 1,2,3,4, are performed. The transformed data have the same distribution. Based on the merged transformed data, we can use the statistical methodology and techniques to monitor and improve the proportional process.

5. Evaluation & Decision-Making Process

In evaluating and decision-making process, different experts in an expert base may result in their different rating values denoted by ξ_k, in which each experts has his own rating expectation value, $E(\xi_k)$, and variation, σ_k, where expert base consists of n experts, so k=1,2,...., n.

In order to eliminate the error caused by the different expectation values of experts in an expert base, it is necessary to work out the expectation value of each expert by the corresponding data, even we need to get the various moments of each expert sometimes. In this situation, the expectation values of each expert in the expert group should be same, otherwise, it is not reasonable for all evaluation results or data to be equally compared, and further obtained the final decision of the expert group. We must deal with all data coming from different experts to have the same distribution. The transformed data can be put together to get the statistical law of the expert base, to set up quality monitor tool such as control charts, and to indicate the performance of evaluation of each expert or the expert base.

According to concrete case, we need to study the same distribution transformation. For some experts, they may have very closed characters and preferences, perhaps there are a difference only in expectation value, which will be very easy to carry out the same distribution transformation by subtracting their expectation value, just as does in target process.

If both expectation value and variation are different, the standard transformation skill can be employed. Sometimes, we may encounter the different distribution types although the case is very few. In this case, we need to find out main various moments of the experts. Based on the various moments, the same distribution transformation can be performed. When the same distribution data are processed ready, the surveying for evaluation results of different experts in an expert base will be in a right way, and the high quality evaluation or decision could be obtained.

On the above discussion, it is better way to correct various moments of different experts so as to implement the same distribution principle, that is, transferring evaluating values from different experts so that various kinds of data have the identical moments. In practice, it is common to calibrate the expectation value and conformability or variation of expert evaluation by the standard transformation in order to implement the same distribution principle. The standard transformation for the evaluating values is the following.

$$\xi_k^* = \frac{\xi_k - E(\xi_k^*)}{\sigma_k}$$

$$\xi = \mathrm{Y}_{k=1}^{n} \xi_k^*$$

Usually, after transformed, the data ξ_1*, ξ_2*, ξ_3*,..., ξ_n* have the identical moments of one dimension and two dimensions, and also they are regarded as the same probability distribution approximately. These evaluating values can be combined together and then the priority of evaluation results is ranked by comparison, finally, the high quality evaluation and decision can be made. The merged data set, ξ, can reveal the statistical law of the expert base performance, and can be used that to monitor, control and improve the quality of the expert base.

6. Conclusion

This paper studies the same distribution principle, based on that, some typical processes with small batch such as target process and proportional process are investigated. The principle is also used in evaluation decision-making process. In industries, we may encounter some cases that both target and variability are changed without special regularity from part type to part type. SIf the distribution types are the same, the standardization transformation can be suggested, otherwise the various moments would be used for the same distribution transformation. In evaluation and decision-making area, the research in this paper is quite basic, the ideology of the same distribution can be widely employed in quality control area for evaluation and decision-making.

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