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Optical Principles of Beam Splitters

Chang-Kyung Lee*

Abstract

In conventional photogrammetry, three-dimensional coordinates are obtained from two consecutive images of a stationary object photographed from two exposure stations, separated by a certain distance. However, it is impossible to photograph moving objects from two stations with one camera at the same time. Various methods to overcome this obstacle were devised, e. g. taking the left and right scenes simultaneously with one camera using a beam splitter attached to the front, thus creating a stereo scene in one image. A beam splitter consists of two outer mirrors and two inner mirrors. This paper deals with research where the optical principles of the beam splitter were evaluated based on light path phenomena between the outer mirrors and the inner mirrors. A mathematical model of the geometric configuration was derived for the beam splitter. This allows us to design and control a beam splitter to obtain maximum scale and maximum base-height ratio by stepwise application of the mathematical model. The results show that the beam splitter is a very useful tool for stereophotography with one camera. The optimum geometric configurations ensuring maximum scale and base-height ratio are closely related to inner and outer reflector sizes, their inclination angles and the offsets between the outer mirrors.

Keywords : Stereophotogrammetry with one objective, Beam splitter optical principles, Beam splitter optimum design

1. Introduction

Stereoscopic Photogrammetry first appeared almost at the same time as normal photography, about one hundred years ago. Initially it was widely used as a new discovery in various branches of science and engineering, in cartography, for aerial photography, micrography and for X-ray work (Valyus, 1966).

Numerous specialized stereoscopic cameras are available, but the photograph may be made by means of a standard camera. When a single camera is used, the simplest method of stereoscopic photography is first to photograph the object on one plate from one position. Then to move the camera a certain distance horizontally to the left or right of the optical axis, and to take a second picture on a second plate from this position. Another convenient method is to rotate the object in front of the objective instead of moving the camera. The two pictures represent component images of the stereogram of the objects. This method can be used only for motionless objects, building, sculptures, etc. It is impossible to photograph moving objects from two stations with one camera at the same time. Various methods to overcome this obstacle were devised. The following major photogrammetric techniques measuring three dimensional coordinates of moving objects can be distinguished.

- Deriving 3-dimensional coordinates of objects just from sequential single images instead of stereo images (Huang & Tsai, 1981)
- Taking right and left scene in a image simultaneously with a single-lens camera.
  - Single (or double) mirror stereo-attachment (Gruner (1955), Faig (1972), Valyus (1966))
  - Beam splitter attachment (van Wijk & Ziemann (1976), Ivar (1992), Lu *et al.* (1992), Lee (1999))
- Taking left and right image using two cameras as the same time, separated by some base length(s)
  - Synchronizing by code imagined on left and right images (Baltsavias & Stallmann, 1991)
  - Synchronizing by video slow motion function (Lee & Faig, 1996)
  - Synchronizing by external devices flashing visible light.
  - Exposure control by internal electrical signal

Although stereo pairs photographed by two cameras are useful in some applications, it is much more convenient to

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*Faculty of Civil & Environmental Engineering, Kunsan National University, Kunsan, Chonbuk 573-701, Korea (E-mail: leeck@kunsan.ac.kr)

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use a single-lens camera equipped with special attachments which enables the two component images of the stereogram to be recorded in a picture simultaneously. The simplest stereoscopic attachment consists of one or two mirrors in front of the lens. The drawback was that they gave a somewhat distorted image. As an alternative, a four mirror symmetrical stereoscopic attachment may be used. In this paper, the beam splitter attachment is reviewed analytically. The objective of this research is to derive the optical principles of a beam splitter and suggest its optimum design policies for stereo images.

2. Principles of Beam Splitter

2.1 The Structure of a Beam Splitter

A beam splitter generally consists of two large externally silvered mirrors lying in vertical planes and inclined at 45° to the optical axis of the lens; between them there are two smaller mirrors; the plane of one of them is parallel to the large right-hand mirror, and that of the second is parallel to the plane of the left large mirror. The path of the rays from the object, to the mirror and through the lens is indicated by arrows as illustrated in Fig. 1.

When mounted in front of the camera lens, two separate bundles of rays are formed so that a photographed object is recorded in two different locations on the photograph. This photograph contains, therefore, the same information as offered by two photographs taken by simultaneously operated, perfectly synchronized cameras, which is of particular interest in photogrammetric studies of high speed process. Besides combining two images into one, a beam splitter divides a beam into two beams.

The base length for the stereo photographs, obtained by this beam splitter attachment, is approximately the distance between the two outside mirrors. The use of the system is consequently limited to close-range applications involving reasonably short base lengths.

2.2 Structural Conditions for Simple Interpretation

When deriving optical principles of a beam splitter, it is desirable to prescribe some conditions for simpler interpretation of its ray paths. Derived optical principles are based on the following conditions which are not only ideal but also possible prerequisites, as illustrated in Fig. 1.

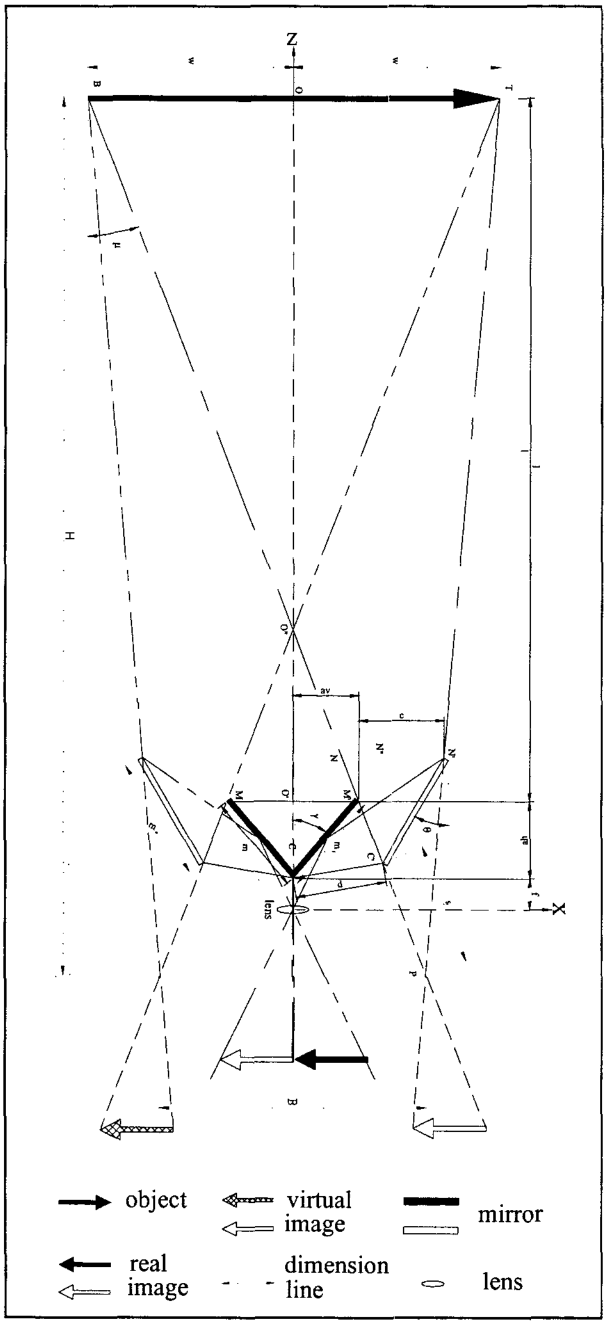

Fig. 1. General Model of Beam Splitter.

(a) The origin of coordinates system is at the principal point of camera lens. The positive z axis is the direction from the camera to the object; positive y axis is the gravity direction, and the positive x axis is based on the right-hand rule.

(b) The inner and outer mirrors, and the principal plane of the camera lens lie in vertical planes.

(c) The center of the object, and the middle point of the edges of inner and outer mirrors are in the optical axis of the camera lens (z axis).

(d) The inner mirrors are of equal size, so are the outer mirrors.

(e) The left-hand mirror and right-hand mirror of both inner and outer mirrors are symmetric about the optical axis of the camera lens.

(f) The inner mirrors incline at γ° to the optical axis of the camera lens. The outer mirrors incline at θ° to the optical axis of the camera lens.

2.3 Observed Optical Phenomena of a Beam Splitter

(a) Convergence of outer and inner mirrors

* If the outer mirror(▯) and inner mirror(▮) are parallel, the path of rays incident on the outer mirror is parallel to that of rays reflected by the inner mirror (Fig. 2(a)).

* If the outer mirror inclines at δ° to inner mirror, the path of rays incident on the outer mirror inclines at $2\delta^\circ$ to that of rays reflected by the inner mirror (Fig. 2(b)).

(b) Offset between inner and outer mirrors

* Offsets between the path of rays incident on the outer mirror and reflected by the inner mirror, l_1 and l_2, is proportional to the offsets between inner and outer mirrors, d_1 and d_2 (Fig. 3).

(c) Direction of the rays forming an image

* Any rays blanketed by the inner mirror before they reach the outer mirror could not form an image (Fig. 4(a)).

* If rays which are reflected by the outer mirror could not reach the inner mirror, they are not able to form an image (Fig. 4(b)).

* If rays are reflected by the inner mirror, only the rays that exit through the space between the outer mirrors could form an image (Fig. 4(c))

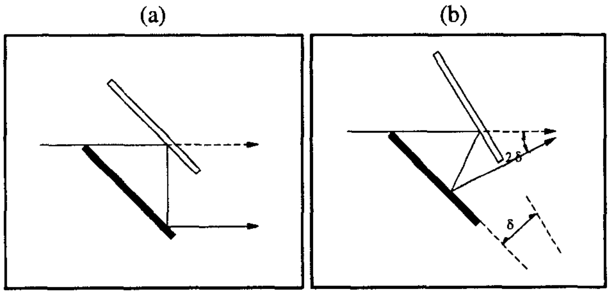

Fig. 2. Inclination Effect between Inner and Outer Mirrors.

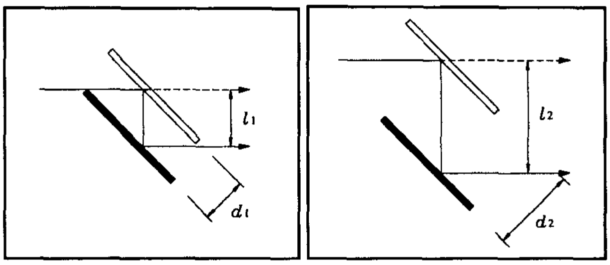

Fig. 3. Gap Effect between the Inner and Outer Mirrors.

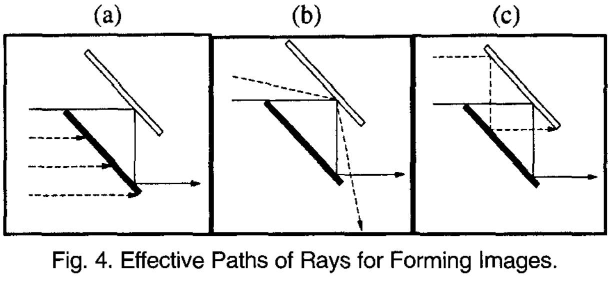

Fig. 4. Effective Paths of Rays for Forming Images.

2.4 Conditions for Optimum Stereo Image

Based on structural conditions and observed optical phenomena, it is possible to derive conditions for capturing optimum stereo images with a camera with a beam splitter. Optimum conditions mean a compact configuration of the beam splitter which enables taking the stereo photograph of a given object with a scale as large as possible with one camera. These conditions can be achieved by adjusting the mirror sizes, the offset between inner and outer mirrors, inclination angle between inner and outer mirrors, and the object distance. Referring to Fig. 1, the following 3 conditions are suggested for an optimum beam splitter.

- Rays emitted from the left end of the object(|) must be reflected by the right-hand outer mirror after passing the front edge of the right-hand inner mirror. The rays must reach the rear edge of the inner mirror. After being reflected by the right-hand inner mirror, the rays must go along the optical axis of the lens.
- Rays emitted from the right end of the object(↑) must reach the rear edge of the inner mirrors after being reflected by the right-hand outer mirror.
- The camera must be placed at the optical axis towards the mirrors. The smaller the distance between the rear edges of the inner mirrors and the interception point of rays with optical axis, the bigger the magnification of the stereo image we get.

2.5 Mathematical Model of a Beam Splitter

There are 8 parameters to be determined for optimum design of a beam splitter.

· Object half size	: w
· Object distance	: L
· Size of inner mirror	: m_i
· Size of outer mirror	: m_o
· Inclination angle of inner mirror	: γ
· Inclination angle of outer mirror	: ϕ
· Coordinates of rear edge of outer mirror	: X_o, Z_o
· Pseudo focal length	: F_p

When capturing stereo images using a camera with a beam splitter, we can consider two cases. The first case is that we build an optimum beam splitter to photograph a given object. In this case only the object size (w) is fixed, while the remaining 7 parameters must be determined. There are a lot of solutions. The solution depends upon parameters acting as constraints.

The other case is that we are planning to photograph an object using a beam splitter already made. In this case, m_i, m_o, and w are fixed parameters. For operating beam splitter with easy, we can fix γ. Consequently, the remaining 4 parameters must be determined. Based on a given beam splitter and object, the mathematical model for optimum parameters of the beam splitter are derived in this study.

(a) Horizontal component (c_h) and vertical component (c_v) of the inner mirror

$$c_v = m_i \times \sin\gamma \tag{1}$$

$$c_h = m_i \times \cos\gamma \tag{2}$$

(b) Angle between optical axis and rays from the left end of the object to the inner mirror front edge

$$\mu = \mathrm{Tan}^{-1}\left(\frac{w + C_v}{L}\right) \tag{3}$$

(c) Inclination angle of the outer mirrors

$$\theta = \gamma - \frac{\mu}{2} \tag{4}$$

(d) Distance from the rear edge of the inner mirror to the rear edge of the outer mirror

$$d = \frac{\sin(\mu + \gamma)}{\sin(2\gamma + \mu)} \times m_i \tag{5}$$

(e) Coordinates of the rear edge of the outer mirror

$$X_o = d \times \sin 2\gamma \tag{6}$$

$$Z_o = d \times \cos 2\gamma \tag{7}$$

(f) Distance from the rear edge of the outer mirror to the pseudo focal point

$$s = \frac{(\sin\theta + \cos\theta\tan\mu)\left\{\dfrac{w + c_v}{\sin\mu} + m_i\dfrac{\sin\gamma}{\sin(2\gamma + \mu)}\right\} - 2w\dfrac{\cos\theta}{\cos\mu}}{\dfrac{2w}{m_o} - \{\sin\theta + \cos\theta\tan\mu\}} \tag{8}$$

(g) Pseudo focal length

$$F_p = s - d \tag{9}$$

(h) Base height ratio

$$\frac{B}{H} = \frac{2(d \times \sin 2\gamma + s \times \sin\mu)}{L + C_h - d \times \cos 2\gamma + s \times \cos\mu} \times 100 \tag{10}$$

3. Implementation

3.1 Configuration of a Pilot Beam Splitter

A pilot beam splitter was manufactured at the University of New Brunswick, Canada(Fig. 5). The skeleton was made of aluminum panel, and internally silvered mirrors were used for reflectors. Its dimensions are 400 mm(L) x 100 mm (W) x 80 mm(H), excluding legs. The beam splitter has the following 4 functions in order to use it for various situations.

- Adjustable inclination angle of inner and outer mirrors.
- Adjustable offset between left-hand and right-hand outside mirrors.
- Movable location of inner mirrors along the optical axis.
- Changeable inner and outer mirrors sizes.

3.2 Parameters of the Beam Splitter

When photographing an object with the beam splitter, we must consider the size of the object first. Even if the size of the object is big, it is possible to photograph the object with the beam splitter. In this case the stereo image scale is small so the overall accuracy of measurement is decreased. In this study, two table tennis balls bouncing on a plane table (40 cm(L) x 30 cm(W)) were selected as the object. Therefore the half size of the object (w) is 20 cm.

(a) Size and inclination angle of inner mirrors

Using the big inner and outer mirrors of the beam splitter, we can photograph an object at a large scale within a short object distance. The size is related to the base line. Though the length of mirror stand of the beam splitter is 10 cm(W), we used 15 cm(W) x 20 cm(L) mirrors in order to get larger scale image.

The inclination angle of the inner mirror of the beam splitter influences the object distance and inclination angle of the outer mirrors. If the inclination angle becomes wider,

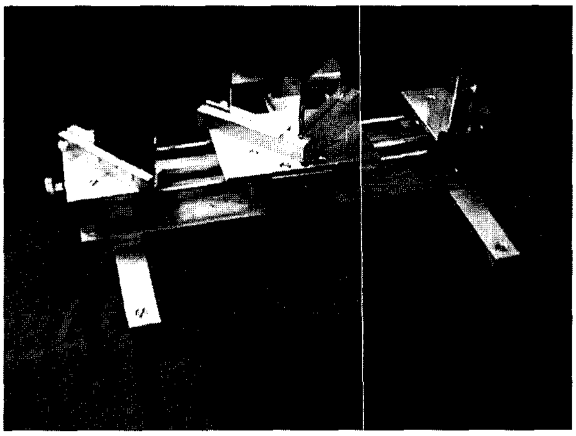

Fig. 5. Beam Splitter Manufactured for Tests.

the base height ratio(B/H) increases from equation (10). In this experiment, the inclination angle of the inner mirror was fixed at 45° for easy setting.

(b) Object distance

After the size and inclination angle of the inner mirrors are fixed, the object distance is related to the size and inclination angle of the outer mirrors. The relationship between the object distance and offset of outer mirrors is shown in Table 1. Considering the maximum adjustable gap between the outer mirrors is 280 mm, it is impossible to place the object closer than 100 cm from the beam splitter. We placed the object at 110 cm from beam splitter. In this case B/H is 30.25%. Now, the inclination angle of the outer mirrors (θ) and the coordinates of the rear edge of the outer mirror (X_o, Z_o) are determined from equations (6) and (7).

(c) Size of the outer mirrors

The size of the outer mirrors can be set smaller or larger than that of the inner mirrors. When the size of the outer mirrors is smaller than a certain limit, the pseudo focal length becomes negative from equation (9). In this case the camera with the beam splitter cannot take stereo images. When the size of the outer mirrors is between 10 cm and 25 cm, the beam splitter can form stereo images as shown in Table 2. The bigger the size of the outer mirrors, the larger the B/H ratio becomes. However, if the rays emitted from the right-hand end of the object arrive at a point farther than T_3 from the rear edge of the outer mirror, the rays reflected by the outer mirror cannot reach the inner mirror as shown in Fig. 6. Therefore, the beam splitter cannot form stereo images. Based on parameters fixed previously, the distance between the rear edge of the outer mirror (C') and T_3 is 17.12 cm. We use 20 cm x 20 cm mirrors(m_o) as outer mirrors in order to reserve some surplus.

(d) Final configuration of object and beam splitter

The final configuration of object and beam splitter was

Table 1. Relationship between Object Distance and Outer Mirror Gap

(m_o=15 cm, γ=45°, w=20 cm)

Object Distance (cm)	90	100	110	120
Gap of Outer Mirrors (cm)	28.4	27.6	27.1	26.6
B/H (%)	37.3	33.4	30.3	27.6

Table 2. Relationship between Outer Mirror Size(M_o) and Pseudo Focal Length(F_p)

(m_o=15 cm, γ=45°, w=20 cm, L=110 cm)

Outer Mirrors Size (cm)	10	15	20	25
Pseudo Focal Length (cm)	8.6	24.7	46.4	77.5
B/H(%)	27.5	30.3	33.2	36.5

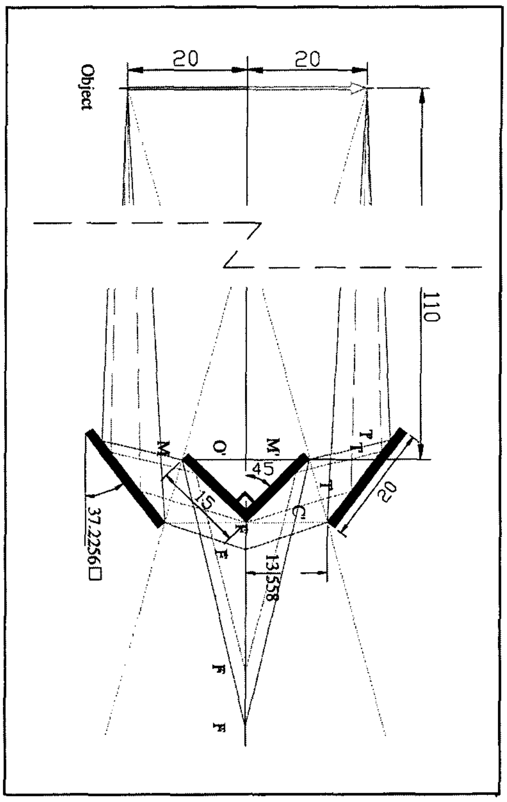

Fig. 6. Final Configuration of Beam Splitter.

Table 3. Parameters of Beam Splitter Final Setup

Given Parameters
- Half size of object (w) = 20.0 cm
- Size of inner mirror (m_i) = 15.0 cm
- Inclination angle of inner mirror (γ) = 45°

Determined Parameters
- Object distance (L) = 110.0 cm
- Inclination angle of outer mirror (ϕ) = 37.2256°
- Coordinates of rear edge of outer mirror
  X_o = 0.0 cm, Z_o = 13.55 cm
- Size of outer mirror (m_o) = 20.0 cm
- Pseudo focal length (F_p) = 46.44 cm

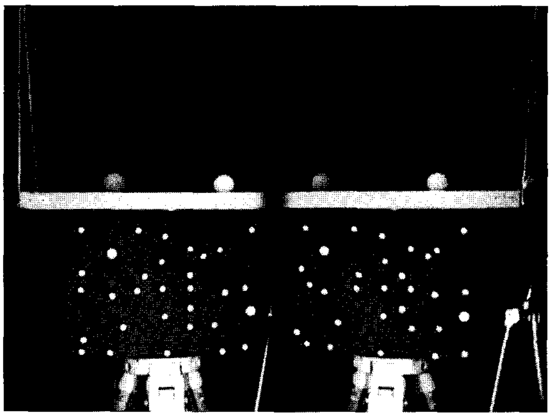

Fig. 7. Stereo Image(640 x 480) Photographed using a Camera with a Beam Splitter Attachment.

determined, with the parameters as shown in Fig. 6 and Table 3. To prove the above mathematical model and derivations, two table tennis balls on a plane table were recorded using a video camera with a beam splitter configured with the parameters determined above. It works well as shown in Fig. 7.

4. Conclusion

The optical principles of the beam splitter were evaluated based on light path phenomena between the outer mirrors and the inner mirrors. A mathematical model of the beam splitter was derived for capturing stereo images with one camera. This allows us to design and control a beam splitter to obtain maximum scale and maximum base-height ratio by stepwise application of the mathematical model.

The results showed that the beam splitter is a very useful tool for stereophotography with one camera. The optimum geometric configurations ensuring maximum scale and base-height ratio are closely related to inner and outer reflector sizes, their inclination angles and the offset between the outer mirrors.

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