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Analysis of Incidence Angle Using Total Station in Leveling

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Abstract

Total Station (TS) is currently widespread and used in many fields of surveying projects. However, its application functions are not perfectly understood and so insufficiently used. One of them is indirect leveling method using TS. Because we can reach a considerable accuracy level with this method, it is gradually expanding for public surveying works such as construction of roads, airports and harbors. This paper gives results of an experiment to increase accuracy of indirect leveling by TS without direct leveling, which is more comfortable and quick to determine elevation.

Keywords : total station, indirect leveling method, incidence angle, regression analysis

1. Introduction

At present, Total Station has been widely spread and used in many survey sites, and sometimes it is not fully used since users misunderstand the principles of this unit. One of them is leveling, and in case we use Total Station for leveling, this is classified as the indirect leveling method, and since it is judged that this method can maintain the considerable accuracy, now it has been increasingly used for many public works such as road, airport and city etc.

This study empirically made a research in the improvement of accuracy of precise leveling by using the indirect leveling method, Total Station that can more simply and quickly find elevation by replacing the direct leveling.

2. Measurement Principle & Error of EDM

2.1 Measurement Principle of EDM

The principle of the measurement device, EDM, which is currently used, is that it calculates the distance by measuring the phase shift during the radiated light wave from EDM's main unit returns by being reflected through the reflector, which is positioned at measurement point.

This phase shift can be regarded as a part of frequency that appears as the unit of time or length under a specific condition.

When the slope distance L and slope angle φ is measured by EDM, if the elevation of point A is the reference point, we can find the elevation of point B by the following formula (1).

$$\text{Elevation of Point } B = \text{Elevation of Point } A + \text{HI} \pm L\sin\varphi - \text{HR} \quad (1)$$

2.2 Error in EDM

The distance measured by EDM is expressed as the formula (2).

$$S = U + \frac{m\lambda}{2} \quad (2)$$

Here, we have

U : Phase shift of the reflected light wave

λ : Wavelength

m : Number of transmitted wavelength

If we measure repeatedly 2 or 3 respectively different wavelengths in order to check value m, we can know wavelength λ is the function of frequency f and electric wave's velocity v.

$$\lambda = \frac{v}{f} \quad (3)$$

Though electronic wave's velocity v is 299792.5 km/sec

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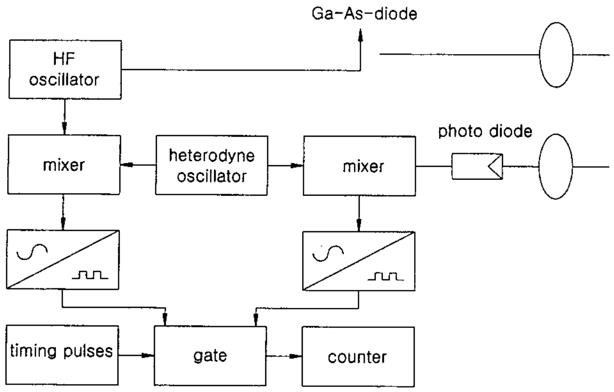

Fig. 1. Structure of EDM.

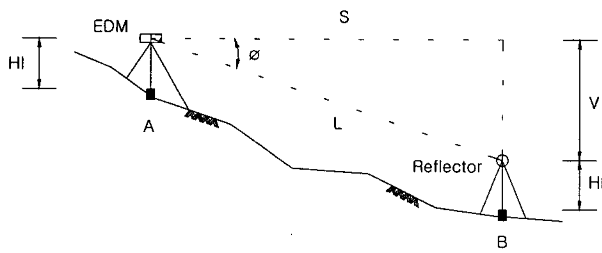

Fig. 2. Geometry of an EDM Measurement.

same as light velocity c under vacuum, but since it always slower than light velocity in the atmosphere, we can correct the influence by the atmosphere and calculate it by the following formula.

$$v = \frac{c}{n} \tag{4}$$

Here, we have
n : air's refractive index.

We should measure the temperature, pressure and humidity in the air according to measurement line if we try to find the exact value n.

If we substitute formula (4) for formula (3), the Value λ of the transmitted signal becomes,

$$\lambda = \frac{c}{nf} \tag{5}$$

and if we assume the wavelength under a specific atmosphere condition is λ_1, since it is described as

$$\lambda_1 = \frac{c}{n_1 f} \tag{6}$$

We can express EDM's distance S_1 is $U_1 + m\frac{\lambda_1}{2}$, and U_1 can be expressed by the phase shift of $1/2\lambda_1$.

If it is $n = n_2 \neq n_1$ during the measurement, the corrected value of λ is,

$$\lambda_2 = \frac{c}{n_2 f} \tag{7}$$

and at this time, the real distance S is

$$S = U_2 + m\frac{\lambda_2}{2} \tag{8}$$

From the formula (7) and formula (8), we can get

$$\lambda_2 = \lambda_1 \frac{n_1}{n_2} \tag{9}$$

and the corrected distance, that is, the correction formula of the measurement distance under a specific atmosphere condition is expressed as the following formula (10).

$$S = U_1\frac{n_1}{n_2} + \frac{m\lambda_1 n_1}{2n_2} = S_1\frac{n_1}{n_2} \tag{10}$$

In order to find the final corrected distance, we should correct the error of EDM's zero point Z_0 and add earth's curvature, slope and the corrected error of average sea level ΔS, and consequently the formula for the final corrected distanceas it is shown in the following formula (11).

$$S_0 = S_1\frac{n_1}{n_2} + Z_0 + \Delta S \tag{11}$$

Here, we have
S_1 : Measured distance
n_1 : Refractive index when correcting in Lab
n_2 : Refractive index at the moment of measuring

If we substitute the formula (6) and (10) for (11), the corrected distance is as follows.

$$S_0 = U_1\frac{n_1}{n_2} + m\frac{c}{2n_2 f} + Z_0 + \Delta S \tag{12}$$

The variance of the distance S is as follows.

$$\sigma_{S_0}^2 = \sigma_u^2 + \left(\frac{m}{2nf}\right)^2 \sigma_c^2 + \left(\frac{m}{2nf^2}\right)^2 \sigma_f^2 + \left(\frac{m}{2n^2 f}\right)^2 \sigma_n^2 + \sigma_{z_0}^2 + \sigma_{\Delta}^2 \tag{13}$$

And if we simply express the formula (13) by $2S = m\lambda = mc/(nf)$, it is described as follows.

$$\sigma_{S_0}^2 = \sigma U^2 + S^2\left[\left(\frac{\sigma_c}{c}\right)^2 + \left(\frac{\sigma_f}{f}\right)^2 + \left(\frac{\sigma_n}{n}\right)^2\right] + \sigma_{Z_0}^2 + \sigma_{\Delta S}^2 \tag{14}$$

Here, we have
σ_u: $U_1(n_1/n_2)$, the standard deviation of the total value

EDM's accuracy is expressed as the following general formula.

$$\sigma_s^2 = a^2 + b^2 S^2 \quad \text{or} \quad \sigma_s = \pm a \pm bS \tag{15}$$

And if we express the formula (14) by the form of formula (15), a^2, b^2 is expressed as follows.[1)]

$$\sigma_u^2 + \sigma_{z0}^2 = a^2 \tag{16}$$

$$\left(\frac{\sigma_c}{c}\right)^2 + \left(\frac{\sigma_f}{f}\right)^2 + \left(\frac{\sigma_n}{n}\right)^2 = b^2 \tag{17}$$

Here, we have

σ_c : Error of electric wave's velocity same as light velocity under vacuum
σ_f : Error of modulated frequency
σ_n : Error of refraction coefficient
σ_u : Error of phase shift measurement
σ_{uo} : Error of zero point
σ_{AS} : Geometric error not included in the formula (15)

3. Error Correction in EDM

3.1 Weather Correction

The device for measuring distance by light wave always should have the correction for the measured value. Every kind of distance measurement device is the same, but they are generally influenced by the following factors. Since the weather condition already exists, the difference of influence made by measurement devices is combined with the observation variable that we can directly observe and the non-observation variable that we can not directly observe.

The variables that are possible to observe and determine a specific condition are temperature, atmospheric pressure and steam pressure, and the effect of them is various.

The refraction index is calculated by the variables of temperature, atmospheric pressure and steam pressure, and it determines the ratio between the velocities of electromagnetic wave's electric wave under vacuum (C_v) and electric wave under the regular atmospheric condition (C_A).

Therefore, it is expressed as $n = C_v/C_A$, and if we know the refraction index, which is always less than 1, we can know the temporary spread velocity of electromagnetic wave and this equals to the spread velocity of measurement signal.

Though the measurement signal progresses the same distance with the various velocities according to the weather condition, since the computer installed at the measurement device is programmed in advance to use a certain fixed wave velocity, this velocity deviation comes to generate the signals in proportion to the measured distance.[1)]

$$n_L = \frac{n_{gr} - 1}{(1 + \alpha t)} \cdot \frac{p}{760} \tag{18}$$

Here, we have

t : Temperature (°C)
p : Pressure (mmHg)
n_{gr} : Refraction index to the avelength of the reflected frequency under °C and 760 mmHg
α : 0.003661(1/°C), Expansion oefficient of air per 1°C

The variables that are impossible to directly observe and influence on the measurement process are rapid weather change, rainfall, heat haze and fog etc.

Therefore, it is desirable necessarily to correct the factors that largely influence on the measurement result, as much as possible. The variables to be corrected are refraction of atmosphere, height of sea level, refraction by projection method and difference of scale coefficient and so on.[2)]

$$D = S_m \cdot \sin Z$$
$$\Delta H = S_m \cdot \cos Z + E - R = i - s \tag{19}$$

Here, we have

D : Horizontal distance
ΔH : Difference of elevation
S_m : Slope distance
Z : Ceiling angle
E : Earth Refraction index
R : Influence by refraction
i : Height of horizontal, central axis of device reference point
r : Earth radius (6,377 km)

The distance correction value, D_0 in sea level is as follows.

$$D_0 = D\left(1 - \frac{h}{r}\right) \tag{20}$$

Here, we have

D : Horizontal distance from sea level to reflector's height
h : Reflector's height in sea level

3.2 Zero Correction

Since a prism reflector decreases if its slope angle is getting bigger to prism face, and it has much dispersion of light if measurement distance is getting longer, we should install a reflector uprightly facing to a measurement device when we install it.[3)]

3.2.1 Prism integer

The light radiated from the distance measurement device by light wave is reflected through the inside of prism, and if we convert the inside length into the distance in the air, it becomes longer two times.

It is shown in Fig. 3 as follows.

$$B = \frac{a + b + c}{2}$$

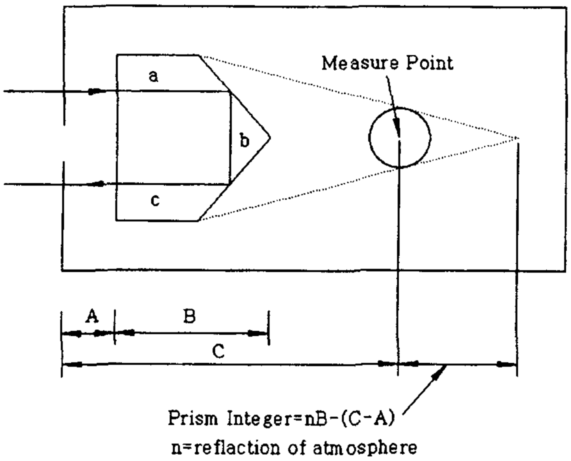

Fig. 3. Prism integer.

Since a prism has its own integer, it is better to use a prism that exactly fits to a distance measurement device by light wave), and if we use it under the condition that a prism's integer is wrong or it is not exactly adjusted, we may get large measurement error. In Fig. 3, the distance we actually measure is Line B, but what we try to find is Line (C-A). The difference between Line B and Line (C-A) becomes the prism's integer.

Since the refraction index of prism is higher than the atmosphere, we should consider this refraction index and decide it, and generally the prism's integer C is expressed as follows.

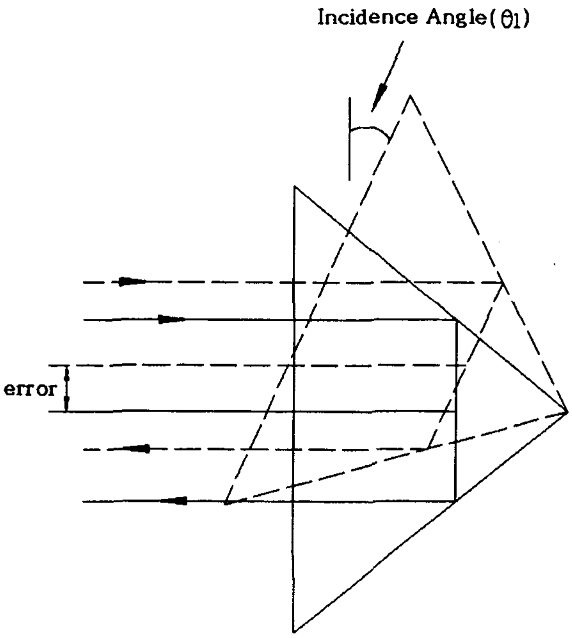

Fig. 4. Error by Incidence Angle.

Table 1. Error by Incidence Angle

Incidence angle(θ_1)	Error (mm) (per 1 km)
0°	0
5°	0.2
10°	0.6
20°	2.5
30°	5.7
40°	10.3

$$C = nl' - 1 \tag{21}$$

Here, we have

n : Refraction index of the atmosphere

l' : B in Fig. 3

l : (C-A) in Fig. 3

3.2.2 Error by Incidence angle

Since the error happens according to the light angle that enters into a prism, it is desirable to set up a prism side vertically and exactly to the measuring point when we set up a prism.

The following Table 1 shows the experimental value that TOPCON Co. executed at 1km section by prism's incidence angle.

4. Measurement

4.1 Specifications of Measurement Field and Measurement Device

We selected the asphalt-paved road equivalent to total 650 m near Namchun-Dong, Sooyoung-Ku, Busan, Korea as the measurement field.

We used TOPCON's GTS-701, Total Station as the measurement device, and in order to compare the accuracy, we also used TOPCON's first class leveling device as the leveling device, and specifications for each device is shown in

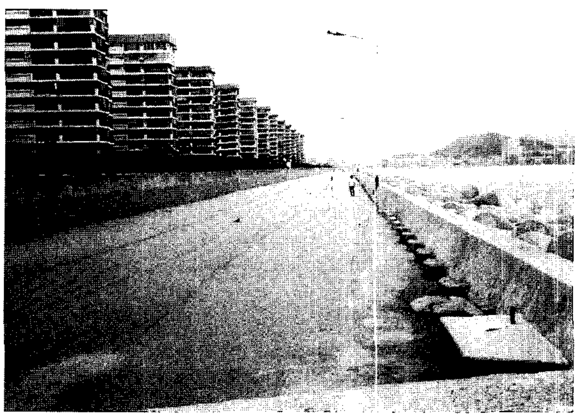

Fig. 5. Measurement Field.

Table 2. Measurement Device(Level)[4]

Device Name	TOPCON TS-E1 (Level)	
Telescope Part	Magnification	42X
	Objcet Lens Diamete r(mm)	50
	Visibility	1°
	Resolving	2"
	The shortest focal distance	2
Bubble Sensitivity	Horizontal Bubble pipe (/2 mm)	10"
	Round Bubble pipe (/2 mm)	4'
1 km round trip measurement degree (mm)		±0.2

Table 3. Measurement Device (TS)[5]

Device Name	TOPCON GTS-701 (Total Station)	
Telescope Part	Magnification	30X
	Objcet Lens Diameter (mm)	45
	Visibility	1° 30'
	Resolving	2.5"
	The shortest focal distance	1.3
Bubble Sensitivity	Horizontal Bubble pipe (/2 mm)	30"
	Round Bubble pipe (/2 mm)	10'
Angle	Minimum recognition	1"
	Degree	2"
Distance	Measurement Range	2.4~3.1km
	Degree	(2mm+2ppm)

the following Table 2 and Table 3.

4.2 Measurement Method

We selected a random point (No. 0) in the measurement object area, and we set out total 15 measuring points at the same 40 m intervals from that point as reference. The distance between a target staff and a device is decided by several conditions, and since the collimation distance of leveling requires 40 m of high precision, this study also set the same interval of the target staff as 40 m and measurede each measuring point by the first class leveling device.

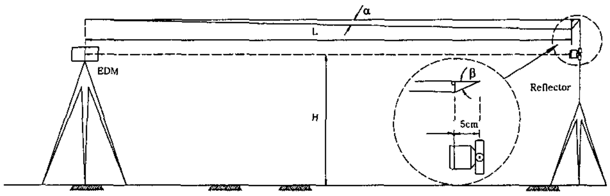

Fig. 6. Incidence Angle Measurement Method.

And when we measured it by Total Station, we fixed the device at the reference point(No. 0), and made heights of the device and reflector same, and we selected the direction of the incidence angle that faced downward, and we measured and compared it with the value that was measured by the first class leveling device and analyzed it.

$$\tan\beta = \frac{h}{5\ \text{cm}} \quad \therefore h = 5\ \text{cm} \times \tan\beta \tag{22}$$

$$\tan\alpha = \frac{h}{L} = \frac{5\ \text{cm} \times \tan\beta}{L} \quad \therefore \alpha = \tan^{-1}\left(\frac{5\ \text{cm} \times \tan\beta}{L}\right) \tag{23}$$

4.3 Measurement and Analysis

The result that we measured respectively three times by the first class leveling device, Total Station and calculated the average value is shown in Table 4 and Fig. 7.

As a measurement result, it showed that the rearranged error to the distance of 600 m measured by the first class leveling device was 0.6 mm, and it satisfied the allowable error of the first class leveling, 1.9 mm.

And in case of measuring by Total Station, it showed that the maximum distance to discriminate was 280 m, and if we see the measured value without a prism's incidence angle, the error was 0.2~23.3 mm.

Table 4. Measurement of Data

Station	Distance (m)	Level (m)	TS (about 0°)		TS (about 10°)		TS (about 20°)		TS (about 30°)	
			G.H (m)	error (mm)	G.H (m)	error (mm)	G.H (m)	error (mm)	G.H (m)	error (mm)
No. 1	40	100.0000	100.0014	1.4	99.9986	−2.8	99.9868	−14.6	99.9750	−26.4
No. 2	80	100.0084	100.0070	−1.4	100.0038	−3.2	99.9930	−14.0	99.9770	−30.0
No. 3	120	99.9924	99.9922	−0.2	99.9894	−2.8	99.9782	−14.0	99.9656	−26.6
No. 4	160	99.9943	99.9916	−2.7	99.9799	−11.7	99.9729	−18.7	99.9625	−29.1
No. 5	200	100.0158	100.0148	−1.0	100.0028	−12.0	99.9962	−18.6	99.9866	−28.2
No. 6	240	100.0150	100.0152	0.2	100.0012	−14.0	99.9964	−18.8	99.9832	−32.0
No. 7	280	100.0041	100.0018	−2.3	99.9880	−13.8	99.9798	−22.0	99.9654	−36.4
No. 8	320	100.1017	100.0978	−3.9	100.0836	−14.2	100.0694	−28.4	100.0594	−38.4
No. 9	360	100.1537	100.1508	−2.9	100.1338	−17.0	100.1200	−30.8	100.1154	−35.4
No. 10	400	100.0838	100.0720	−1.8	100.0870	15.0	100.0434	−28.6	100.0200	−52.0
No. 11	440	100.0661	100.0530	−13.1	100.0748	21.8	100.0354	−17.6	100.0222	−30.8
No. 12	480	100.1038	100.0876	−16.2	100.1136	26.0	100.0662	−21.4	100.0550	−32.6
No. 13	520	100.0960	100.1076	11.6	100.0808	−26.8	100.0706	−37.0	100.0372	−70.4
No. 14	560	100.0999	100.0766	−23.3	100.0472	−29.4	100.0474	−29.2	100.0338	−42.8
No. 15	600	100.1288	100.1072	−21.6	100.0708	−36.4	100.0843	−22.9	100.0768	−30.4

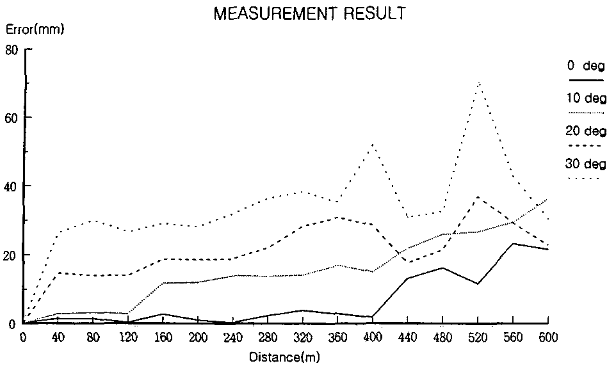

Fig. 7. Analysis of Data.

These are the values appeared by each distance for No. 0 point, and it is considered that they appeared because of the observer's collimation error, light wave's dispersion coming from Total Station's main unit and since it was not vertically positioned to the exact central point of a prism.

And when we measured by Total Station, we could calculate the correlation between the incidence angle and the error as the form of rm $y = a + bx$ (y : error, x : distance) in the section where the measurement distance was 0~400 m.

5. Conclusion

1. As a result that we measured the indirect leveling by Total Station, it was observed that the maximum distance we could discriminate the prism's central point from the telescope lens was 280 m, and its error was 2.3 mm that satisfies the second class allowable error, 2.6 mm. Therefore, it is judged that if we apply the distance, we can discriminate the prism's central point, it can satisfy the second class leveling.

2. From the result, we made the regression analysis on the correlation of prism's incidence angle when we measured an indirect leveling by Total Station, the following correlation was formed at the distance of 0~400 m.

$y = a + bx$

y : error (±mm), x : distance (m)

	About 10°	About 20°	About 30°
a	1.60	10.0	21.9
b	0.04	0.05	0.05
r^2	0.8144	0.8266	0.6697

3. It is judged that if we calculate the correction value between the distance and incidence angle when measuring an indirect leveling by Total Station and apply it, we can find the more exact leveling value.

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