A Study of Predicting Method of Residual Stress Using Artificial Neural Network in CO_2 Arc Welding

Y. Cho, S. Rhee and J. H. Kim

Abstract

A prediction method for determining the welding residual stress by artificial neural network is proposed. A three-dimensional transient thermo-mechanical analysis has been performed for the CO_2 arc welding using the finite element method. The first part of numerical analysis performs a three-dimensional transient heat transfer analysis, and the second part then uses the results of the first part and performs a three-dimensional transient thermo-elastic-plastic analysis to compute transient and residual stresses in the weld. Data from the finite element method are used to train a back propagation neural network to predict the residual stress. Architecturally, the fully interconnected network consists of an input layer for the voltage and current, a hidden layer to accommodate the failure mechanism mapping, and an output layer for the residual stress. The trained network is then applied to the prediction of residual stress in the four specimens. It is concluded that the accuracy of the neural network predicting method is fully comparable with the accuracy achieved by the traditional predicting method.

Key Words : CO_2 arc welding, Residual stress, Artificial neural network, Heat input, FEM, Thermo-elastic-plastic analysis, Current, Voltage predicting method

1. Introduction

In arc welding, the welding area becomes partially heated according to the arc heat, and thus unequal amounts of temperature distribution generate excess heat stress and inelastic change. With these changes, stress remains in the welding area, which has been through a cooling constrainer after a welding process. These stresses affect the brittleness, destructive strength, fatigue strength, vibration characteristics, corrosion resistance, etc. The above phenomenon are common issues in most welding process, which has led to many studies in this field[1-2]. In order to solve these welding residual stress issues, first the distribution of residual stress on the welds must by understood and analyzed precisely, and further more a reliable and economical prediction method is required.

In order to propose an effective prediction method of residual stress, a multi-layered artificial neural network, based on an error back propagation algorithm, is introduced to apply to the residual stress prediction of the welds. A computer simulation is performed to produce the data needed for the artificial neural network learning. The simulation results are also verified by experiment of Hole-Drilling Method, which shows that learned data set is quite reliable.

The proposed method is used to predict residual stress more accurately and quickly than earlier methods. It is predicted that this study will not only help tremendously in precisely understanding the cause of residual stress from complex welding phenomena, but will also help to improve and to control the quality of welding structures, by achieving real time output of residual stress in welding structures.

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2. Artificial neural network

2.1 Background

In some case residual stress of welded structure is a serious problem that cannot be avoided. It is also a well-known fact that it affects the performance and stability of the welds. The most essential point to see in welding residual stress is that it cannot be easily measured. Therefore, in order to study the influence of residual stress, it is important to predict precisely the size and the distribution range of the welding residual stress. It is very difficult to measure the residual stress of the structure in a general experimental process. Because, in order to measure the hidden stress of the welds, the residual stress must be exposed in any form and monitored with sensors, however, this will cause the destruction of the welds. In order to make up for this defects, numerical analysis techniques can be introduced. However, the calculation time will be another factor for delaying the time, so in a real systems there will be limitations. Therefore, a new residual stress predicting method is needed to cover both the above problems; destruction and time consuming. In this study, in order to reduce defects and increase the merits, the residual stress prediction method using artificial neural network is studied and applied to welding

2.2 Structure and characteristics of neural network

Neural network is a data process system that imitates the nervous system of living creatures. It has inter-connected simple elements, and activate a dynamic state response from external input to create a resulting output. This network consists of the PDP (Parallel Distributed Processing) system or neuro-morphic systems, which sequentially calculates the saved information in the memory equipment from the coded algorithm, and overcomes the technical limitations of memorizing the results using the current digital computer's sequential calculation methods[3]. Generally, neural network has a CAM (Content Addressable Memory) ability, which can detect information from partial or even similar contents, and also a learning ability, which can generalize fault-tolerance. This network system is currently being studied not only in computer fields, but also in engineering, biology, medical fields, etc. As it is difficult to extract a specific rule for all variables in welding systems, which include many difficult and complex variables, a neural network that can be applied to a real time control is used to find nonlinear relationship between welding parameters and residual stress in this study.

2.3 Theory of neural network

Artificial Neural Network is based on the simple model of nervous system, constructed by neuron and synapse. The neuron receives many input signals from many neighboring neurons and outputs one result, and then the synapse receives the output of neurons and multiplies it by the weight and gives it to another neuron. These basic elements are divided into input layer, output layer and hidden layer, to form the entire nervous system. The basic elements, that have simple calculating abilities, are connected together systematically and produces complex mapping between input and output, and if these mapping relations are severely nonlinear, it can be applied effectively. The standard model of the neural network system is shown in Fig. 1.

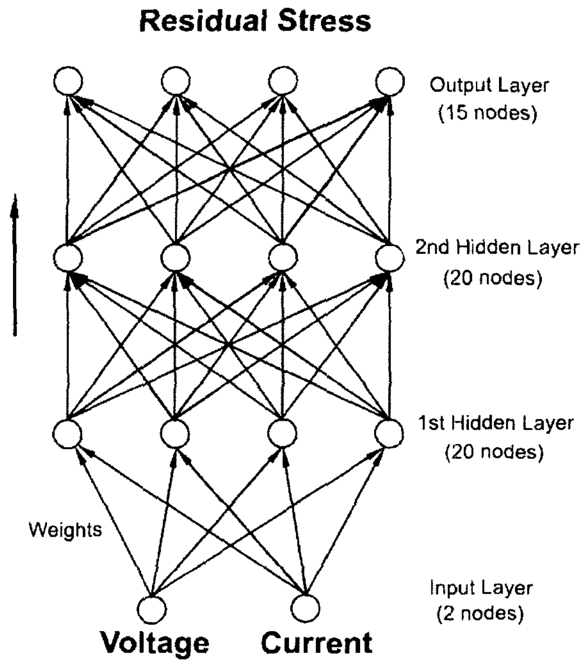


Fig. 1 Schematic diagram of neural network

3. Finite element method

3.1 Finite element method in welding

As mentioned before, the basic theory about mathematical models of the welding, have been accomplished by Rosenthal[4] in the 1940s. However, these calculation methods were performed in ideal conditions, which made it unsuitable to find the exact solution, but instead it explains the nonlinearity of the welding process, which gave a stable foundation for numerical analysis. After the 1970s, with the help of computers and finite element method, many researchers could effectively manage the differential equation and complex forms of transient heat transfer problems. From this, a more precise welding simulation was possible. The study researched by Hibbitt and Marcal[5], mentioned a new predicting method of welding residual stress by using finite element method. They were able to carryout a effective heat transfer calculation with proper boundary conditions and nonlinearities of the materials, and used this result to understand residual stress. In this research, the residual stress was predicted, considering the elastic and plastic behavior of the material, according to temperature dependent material properties and time dependent creep effect. Friedman[6] predicted residual stress by calculating a more accurate plastic deformation, in considering the transient and residual stress of the welding. By the end of the 1980's, with the basis of earlier studies, commercial finite element method package was developed, and Tekriwal and Mazumda[7] used this package to effectively calculate the 3 dimensional heat transfer problem of GTA welding. They also used a technique to gradually add elements indirectly, according to the welding process, to understand GMA welding[8]. In 1991, Tekriwal at al.[9] continued his studies and tried to understand transient GMA welding and residual stress, however considering the restrictions of the specimen size, there seemed to be a few problems.

In this study, the problems of the above studies are strengthened and the analysis of CO_2 arc welding heat transfer and residual stress is carried out. The analysis of CO_2 gas arc welding could be handled in the same way as the GMA welding. In this study, the thermal analysis and stress analysis are divided into two stages considering the accuracy of the results. This method can be performed, by applying the results from thermal analysis to stress analysis.

3.2 Analysis and assumption of welding process

In order to analyze the CO_2 arc welding, welding arc with distributed heat source is moving across the plate at v speed to x direction, shown in Fig. 2.

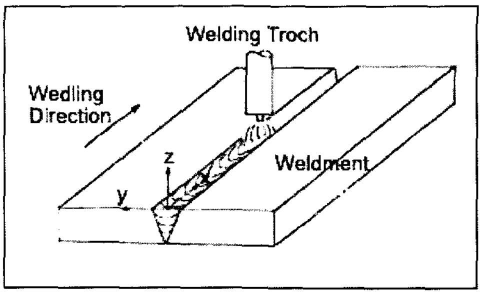


Fig.2 CO_2 arc welding procedure with coordinate system

Active shielding gas, with steady flow from the welding torch, is used to protect the welding part from air pollution. The welding wire is melted by the arc heat and the other parts of the heat input are absorbed into the welds. Eventually, the heat flow spreads into all 3 dimensional directions and the 2 parts are welded with the melted metal. In the assumption that heat input has the Gaussian distribution, which is symmetrical, only half of the part is analyzed. After the welding is finished, constrain force takes place near the welded part, due to sudden heating and cooling. In this temperature changing process, and elasto-plastic deformation and resulting stress occurs near the welded joint, and after cooling, it becomes residual stress. This stress changes can be analyzed by using the results from stress analysis. The

Table 1 Material properties of mild steel for FEM

Ambient Temperature :	293K
Solidus Temperature :	1700K
Liquidus Temperature :	1755K
Latent Heat of Fusion :	273790 J/kg
Density of Mild Steel :	7870 kg/m^3
Poisson Ratio :	0.3

	Thermal Conductivity of Mild Steel	Specific Heat of Mild Steel	Thermal Expansion Coefficient	Young's Modulus	Yield Strength
T [K]	K [W/m/K]	Cp [J/kg/K]	[m/m/K]	[kgf/mm^2]	[kgf/mm^2]
293	73	675	1.2 E-5	2.1 E4	27
750	59	675	1.35 E-5	2.0 E4	24
1000	52	675	1.42 E-5	1.0 E4	0.12
1250	45	675	-	-	-
1500	38	675	1.51 E-5	1.0 E-3	0.001
1600	36	675	-	-	-
1699	42	675	-	-	-
1700	48	5668	-	-	-
1755	48	5668	-	-	-
1756	61	695	-	-	-
2000	61	695	1.51 E-5	1.0 E-3	0.001

process here after idealized by using several assumption from earlier studies. The material properties of the specimen are shown in Table 1.

3.3 Analysis procedure

3.3.1 Arc heat input model

The procedure of heat input modeling is extremely important, as it has direct effects on the temperature distribution, cooling speed, size of the melting area and heat affected zone. Welding conditions, such as shielding gas, contact tip to workpiece distance, groove effect, etc, also have important effects on the heat input rate, but in this study, the change of these values are ignored and a constant value of 67% of arc is used as a arc efficiency. Temperature dependent nonlinear thermal conduction rate and specific heat are used with latent heat of 2300K capacity. The arc heat input is assumed to be symmetrical Gaussian distribution. The model for this distribution is shown in equation (1).

$$q(x,y,t) = \frac{3\eta_{eff} VI}{\pi r_b^2} \exp\left[-\frac{3}{r_b^2}[(x-vt)^2 + y^2]\right] \qquad (1)$$

3.3.2 Surface heat loss

Natural convection loss occurs on all surfaces, apart from the symmetrical surface of $y = 0$. On the basis of the earlier models, $h = 10\ W/m^2K$ is used on all surfaces, regardless of the shielding gad, and equation (2) is used as the forced convection value of the surface area under the welding torch area.

$$h = 13 \mathrm{Re}^{1/2} \mathrm{Pr}^{1/3} k_{gas} / D \qquad (2)$$

Considering CO_2 gas characteristics, Re and Pr is calculated, and as the heat source moves along the welding direction, the appropriate heat flux is added to the nodal point of each time-step, regarding speed, and as a new mesh is activated a new boundary conditions are decided. As a boundary condition, constant h value is used in all areas near the nozzle, and it does not change its radius direction.

3.3.3. Mesh generation

Mesh generation is one of the important issues in the accuracy and economical efficiency of numerical analysis results. As high temperature suddenly changes near the welding centerline, more precise elements are needed. Approximately 1300 nodes and 800 elements are used for the model. Element of minute size, near the welding line, guarantees an accurate numerical analysis, however, as mesh is generated when the size of the element increases as the distance becomes further from the welding centerline, a more economical numerical analysis is possible. In CO_2 arc welding, before welding in completed, melted metal is continually added, thus, to obtain a more accurate result, mesh must continuously be activated with nodes when modeling a V groove. 195 nodes are arranged in the V groove. 150mm is divided into 64 parts and is analyzed through 32 stages, and in each stage an element of 4.68mm is activated. After the first pass is completely finished, it passes through a cooling process for 30 minutes in the air and then the second pass is preceded. This model is shown in Fig. 3.

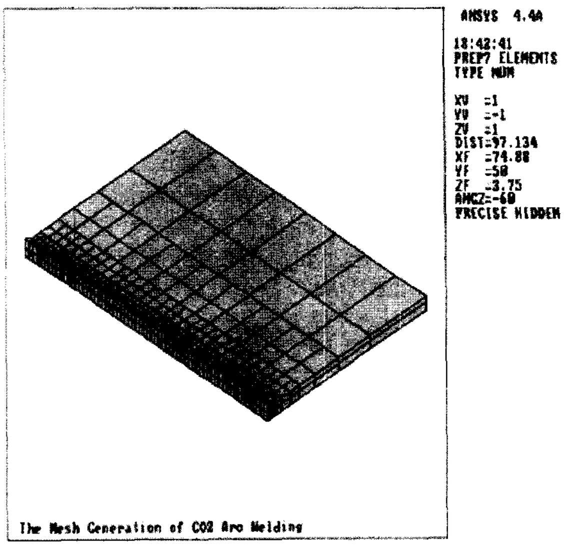


Fig. 3 CO_2 arc welding FEM mesh model

3.3.4 Cooling procedure

After the welding, the cooling process starts. The cooling speed is an important factor in deciding the characteristics of the welds. In the cooling analysis process, a sub-routine is made and used in the program. The ambient temperature is fixed at 293K, and a new boundary condition is given on the entire surface, except $y = 0$ surface. Right after the welding, the time-step is small and susceptible to a sudden temperature change. Considering the economical efficiency, time-step is increased as time passes. When calculating repeatedly each load-step, a ramp shaped load is applied and errors between the temperature differences of each step are reduced. An optimized time-step method is used to divide the repeated calculations in appropriate sizes. With the mesh shape, the size of the cooling time-step is an important problem in finding an accurate and economical analysis, however for a reliable result a small time steps are needed. The cooling process is divided into a 60 time-step, and is carried out until 17200 seconds.

3.3.5 Residual stress calculation

After the above processes, including the cooling process, are finished, the results are used to calculate the residual stress of the welds repeating similar calculations in stress analysis module. In the stress analysis stage, the nodal solutions of thermal analysis are transferred to 8-node isoparametric elements, which makes stress analysis possible. The material of the welds is assumed as isotopic and homogeneous material, which is regarded as being the same material as the deposit metal. Depending on the cooling temperature of the welds; each different plastic deformation rate of the bilinear structure, due to kinematic hardening model, is applied and an appropriate plastic deformation is considered, including Bausinger effect in each temperature. The constraint of the welds restricted the displacement in all directions of the $y = 0$ surface, and other nodes are changed freely. Following the above conditions, the thermal analysis results of each step are used and carried out in 90 stages of stress calculation. This process is executed by using the thermal analysis results, and it needs large memory storage and a lot of time for transaction.

3.4 Analysis results and discussion

Fig. 4 shows the second pass of the welding process. Next, in order to compare the resulting data obtained from the experimental result, the stress distributions of the nodes of the welds surface ($z = 6$ mm) at $x = 75$ mm are observed. These results are shown in Fig. 5 and Fig. 6. Longitudinal stress gradually decreases from the center line and compression stress is effected near the end part. The largest value of longitudinal stress is slightly larger than the yielding stress value.

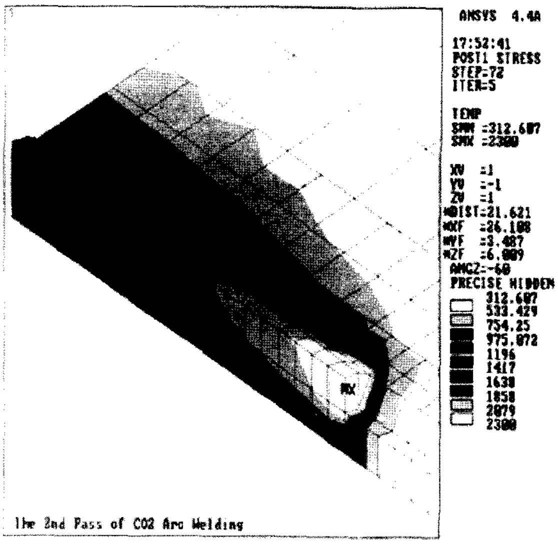


Fig. 4 2nd pass of welding procedure

The transverse stress shows a relatively small value compared to the longitudinal stress, and also maximum longitudinal stress is shown around the welding centerline. Due to the thickness of the specimen (6mm), stress of the thickness direction can be excluded from consideration. As the welding heat input increases, the region of maximum longitudinal stress moves further away from the centerline, and the compression stress of the end region also gradually increases. The traverse stress also increases as the heat input increases, showing that high heat input make it more defective in terms of residual stress. Depending on the size of the specimen, it can be seen that the size of the compression stress generally becomes larger as it becomes nearer to the end part. Among the numerical analysis results of the 10 welding conditions, the 4 results are shown in Fig. 5 and Fig. 6 as longitudinal and transverse residual stress. Results other than these are discussed in later section.

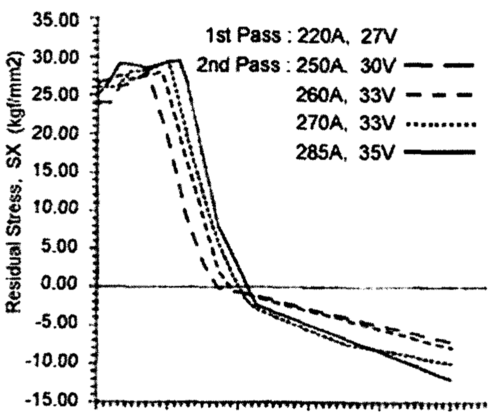


Fig. 5 Longitudinal residual stress distribution

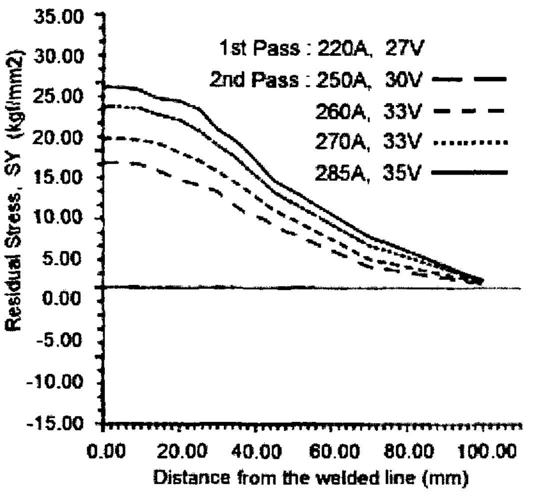


Fig. 6 Transverse residual stress distribution

4. Residual stress measuring: hole-drilling method

4.1 Hole-drilling method

Before the numerically analyzed welding residual stress is used as data for artificial neural network studies, the results have to be verified for its reliability. As an experimental verification method, a hole-drilling method is used in this study. As the stress of free surface has to be zero, if an extremely small hole is drilled through the residual-stressed material, the stress in that area will become exposed. The relief of the radial stress, which exists in the hole surface, changes the stress distribution around the hole area, and thus, localized stress changes on the measured workpiece surface. By measuring the strain of the hole area, the hidden residual stress of the material can be monitored. In real application, a blind-hole, which the depth is similar to the diameter of the hole and the depth of smaller than the measured workpiece, is used, not a through-hole. The structure of a blind-hole is more complex than the through-hole, thus, the elasticity theory, which calculates the residual stress from the strain, is difficult to use directly. In this study, blind-hole drilling method is applied under the ASTM standard base upon the theoretical foundation of through-hole theory.

4.2 Experiment specimen

In order to measure the residual stress of welding structures, the size of the welded specimen have to be large enough to include residual stress that exists in the real structure. The length and the width effects of the welding structure are taken under consideration, and the following welding specimen is produced. SS400, which is the most widely used welding structure, is used as the basic material of the welding. Four specimens, with a 100mm width, 50mm length, 6mm thickness, are welded, while changing the voltage and current. In the first pass, the welding is performed under conditions of welding speed 9 mm/sec, 200 A and 27 V. In the second pass, 9.4 mm/sec welding speed, current of 250~285 A and voltage of 30~35 V are used. The welding is performed using a welding robot and CO_2 arc welding machine controller. CO_2 shielding gas is used with flow rate of 15 l/min. The specimen used in the experiment is shown in Fig. 7.

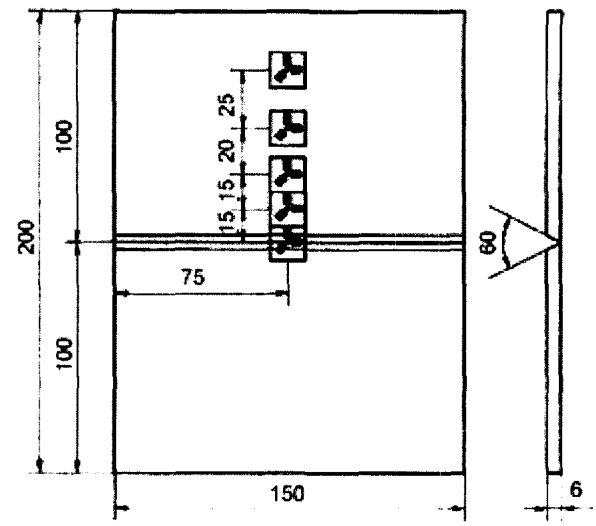


Fig. 7 Schematic draft of weldment

4.3 Experiment result and discussion

For a more precise experiment result, stress measuring devices such as strain gauge, strain meter and hole-drilling machine are decided carefully. It is extremely difficult to attach three different stress gauges, with a fixed degree and distance. Therefore, a rosette strain gage, which is a gage that can contain three grids, is used. A high-precision digital strain indicator is used, as the measuring device for the change of strain. And by using a switch and balance unit, which can obtain more than three outputs simultaneously, three strain are obtained during the hole-drilling process. A milling guide that supports drill shaft, is used, as the hole must be processed very accuracy in the center of the gage, regardless of the drilling vibration. As shown in Fig. 7, with the welding centerline as the standard line, five strain gauges are attached on one side of the specimen. The diameter of the hole is decided as 1.5875mm, considering the characteristics of the gage, and the depth of the hole, 1.5875mm, is drilled in 10 steps, using the milling guide. The principal stress is calculated, according to the measured strain, and with this the longitudinal residual stress and the transverse residual stress are calculated. This process is repeated and carried out four times, on four different specimens. The residual stress shows a reducing patter from the centerline to the end. The maximum value is slightly larger than the yield stress. By comparing this result with the numerically calculated result, Fig. 8, a figure of comparing experimental data and analysis data of each welding condition, can be drawn. In general the tendency of those two stress distributions are similar, there is a slight error but quantitatively. This is considered to be the cause of error during the experiment process or several assumption of the analysis condition. Especially in the experiment process of drilling a small hole has be drilled in the exact center of the gauge. This procedure can include errors to come extent. When comparing the analysis results with the experimental results, it is concluded to be reliable, thus these numerical results can be used properly in neural network.

5. Residual stress prediction using artificial neural network

5.1 Learning and data process

Artificial neural network can be used effectively in processes with many variables that are extremely nonlinear; therefore, it is a new alternative way to predict

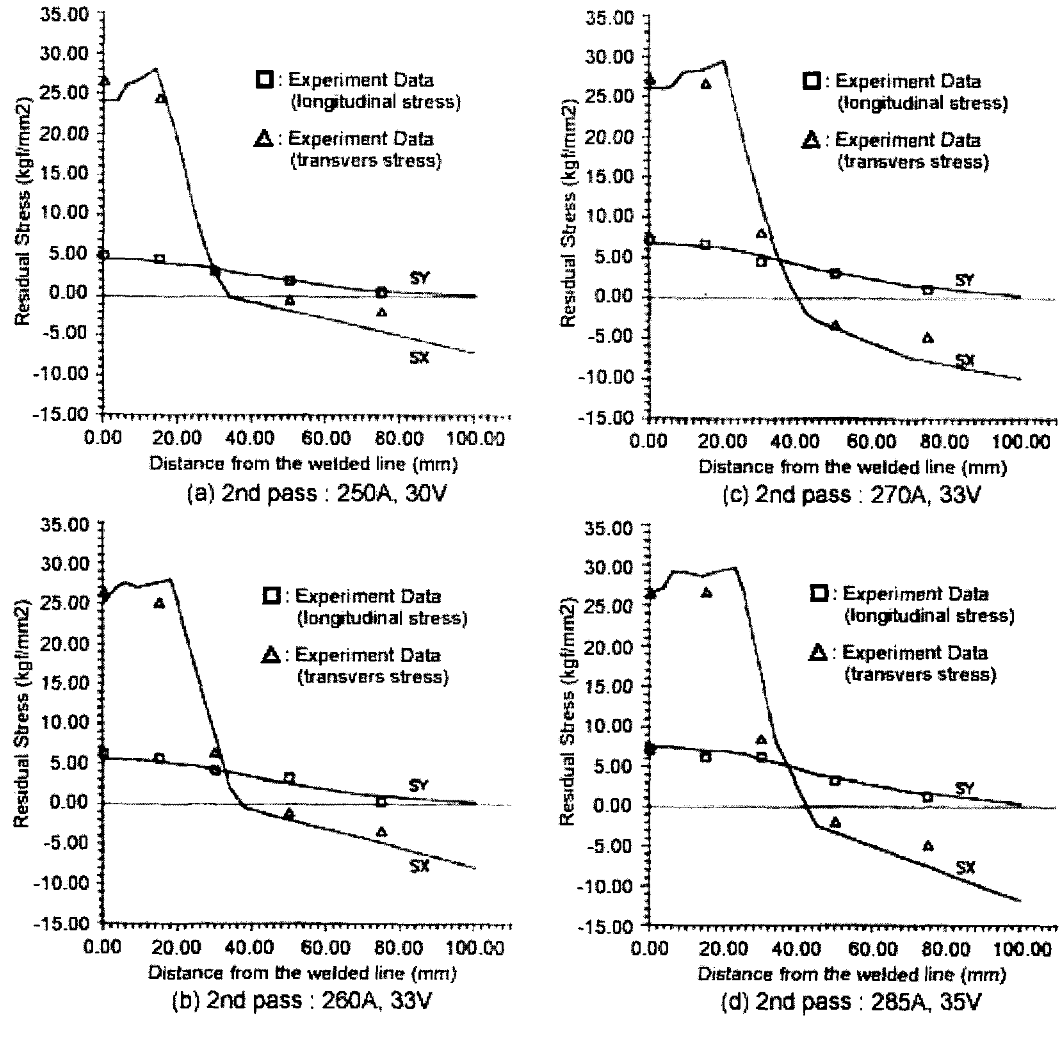


Fig. 8 Comparison of experimental data with analysis data (1st pass : 220 A, 27 V)

residual stress of CO_2 arc welding. Welding is performed while the current and voltage changes; thus, the input variable is decided as the current and voltage, and due to the characteristics of sigmoid function, used as the activation function, each values are normalized to 0.1 ~ 0.9. The longitudinal stress and the transverse stress are also normalized to 0.1 ~ 0.9. The stress change between each node is almost linear, therefore a similar proportional nodal solutions are applied to corresponding output. Equation (3) is used for normalization, and equation (4) is used to transform neural network output into residual stress value.

$$\frac{(0.9-0.1)}{x_{max}-x_{min}}\times(x-x_{min})+0.1 \tag{3}$$

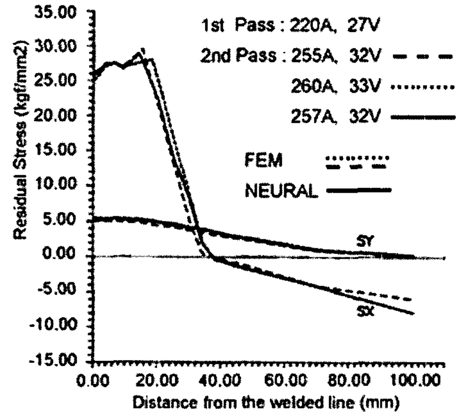
$$\frac{x_{max}-x_{min}}{(0.9-0.1)}\times(x-0.1)+x_{min} \tag{4}$$

In order to decide the hidden layer, single-layered models with the 10 and 20 nodes, and multi-layered models of 10×10, 15×15, 20×20, and 25×25, are applied in this study, and the learning rate of the models show to be 79.2%, 82.5%, 91.7%, 94.1%, 95.3% and 95.1%, respectively. The hidden layer with the form of 20×20, which have the least error and a high learning rate is finally decided. The data need for the study is based on the numerically analyzed data when the current and voltage is 200~285 A, 27~35 V, respectively. As each residual stress distribution change is not large, the number of output nodes are limited to 14. Also, bias of 0.7 and momentum rate of 0.9 are used in the neural network, and the maximum iteration number is decided as 10,000.

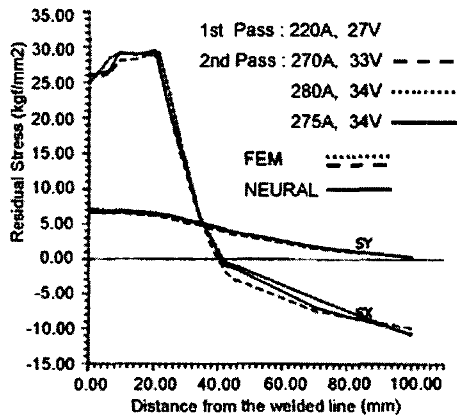
5.2 Residual stress prediction

Learning rate of 95.3% is obtained using the artificial neural network, following the above method, and the largest residual stress error is 6.7%. This can be compared to the output when random current and voltage values are inputted into the neural network, and this result can be then shown as longitudinal direction and transverse direction in Table 2 and Table 3. When applying the same set of learned data into the prediction model, there is hardly any difference in the results. And when inputting the value in between the learned data, the value in between the learned stress data can be outputted.

Fig. 9 shows the residual stress prediction results from random voltage and current. Fig. 9 is verified using the data from numerical analysis method, by applying the longitudinal residual stress prediction result of artificial neural network to the finite element method. Fig. 9(a) shows the comparison of the calculated results of 255× 32 Watts heat input and 260×33 Watt heat input, and the residual stress of 257×32 Watt heat input using artificial neural network. Fig. 9(b) shows the comparison of the calculated results of 270×33 Watts heat input and 280× 34 Watt heat input, and the residual stress of 257×34 Watt heat input using artificial neural network. As shown in the figures, stability exists in between the boundary of numerical result. However, near the welding centerline, the results of the new prediction method are slightly differing from the numerical. This is due to the fact that, only 14 numbers of nodes are used as input nodes, thus, the node value of the peak point is not considered. When comparing the residual stress prediction method presented in this study to the existing prediction method, not only is there no large difference from the numerical prediction method and it's reliability, but also it is much more effective than the experiment method or the numerically calculating method. This 'welding process, studies with the appropriate data, can find welding residual stress quickly and accurately; therefore, it can be applied in the industrial environment, as well as improve quality and production.



(a) 2nd pass : 257 A, 32 V by neural network



(b) 2nd pass : 275 A, 34 V by neural network

Fig 9 Verification of predicted data

Table 2 Error for the FEM & neural network (longitudinal residual stress, 1st pass : 220 A, 27 V)

Residual Stress of Longitudinal Direction (kgf/mm²)						
No.	2nd Pass	255A	32V	2nd Pass	260A	33V
Node	FEM	Neural	Error %	FEM	Neural	Error %
1	26.17	26.32	0.57	25.04	25.12	0.32
2	27.54	27.58	0.15	27.03	27.31	1.04
3	27.53	27.83	1.09	27.52	27.68	0.58
4	27.16	27.36	0.74	27.19	27.56	1.36
5	29.09	29.11	0.07	27.51	27.60	0.33
6	21.52	21.63	0.51	25.85	25.87	0.08
7	13.19	13.23	0.30	16.50	16.79	1.76
8	5.01	5.12	2.20	8.60	8.71	1.28
9	0.43	0.45	4.65	2.01	2.02	0.50
10	-0.11	-0.11	0.00	-0.51	-0.52	1.96
11	-0.65	-0.63	3.08	-0.98	-0.99	1.02
12	-1.00	-1.01	1.00	-1.58	-1.59	0.63
13	-4.21	-4.23	0.18	-4.31	-4.33	0.46
14	-0.04	-6.09	0.83	-8.03	-8.06	0.37

Table 3 Error for the FEM & neural network (transvers residual stress, 1st pass : 220 A, 27 V)

Residual Stress of Longitudinal Direction (kgf/mm²)						
No.	2nd Pass	255A	32V	2nd Pass	260A	33V
Node	FEM	Neural	Error %	FEM	Neural	Error %
1	5.01	5.06	1.00	5.40	5.42	0.37
2	4.98	5.01	0.60	5.39	5.41	0.37
3	4.97	5.00	0.60	5.38	5.40	0.37
4	4.94	4.99	1.01	5.35	5.36	0.19
5	4.75	4.93	3.79	5.25	5.27	0.38
6	4.35	4.56	4.83	4.91	4.98	1.43
7	4.11	4.11	0.00	4.60	4.63	0.65
8	3.70	3.71	0.27	4.23	4.25	0.47
9	3.39	3.51	3.54	3.85	3.86	0.26
10	3.05	3.04	0.33	3.51	3.55	1.14
11	2.73	2.77	1.47	3.13	3.17	1.28
12	2.49	2.48	0.40	2.84	2.85	0.35
13	0.75	0.76	1.33	1.05	1.09	3.81
14	0.15	0.14	6.67	0.17	0.17	0.00

6. Conclusion

The residual stress prediction method, using finite element method and artificial neural network, of CO_2 arc welding is examined and the following conclusions are obtained.

1. Three dimensional heat transfer analysis of CO_2 arc welding using finite element method is carried out. These results are applied to stress analysis to interpret welding residual stress.
2. After performing CO_2 arc welding, under different welding conditions, the residual stresses are measured using the hole-drilling method, and the residual stress distribution and characteristics are analyzed by the location.
3. Various numerical analysis results are compared with the experimental result, and the reliability of the numerical analysis results, using finite element method are proved.
4. A new prediction method, which can obtain residual stress, is introduced.
5. The predicting method introduced in this study is more quick and accurate than the existing method.

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