Stress Intensity Factor for the Cracked Plate Reinforce with a Plate by Seam Welding

O. W. Kim, S. D. Park and Y. H. Lee

Abstract

The stress intensity factor has been calculated theoretically for the cracked plate subjected to remote normal stress and reinforced with a plate by symmetric seam welding. The singular integral equation was derived based on displacement compatibility condition between the cracked plate and the reinforcement plate, and solved by means of Erdogan and Gupta's method. The results from the derived equation for stress intensity factor were compared with FEM solutions and seems to be reasonable. The reinforcement effect gets better as welding line is closer to the crack and the stiffness ratio of the cracked plate and the reinforcement plate becomes larger.

Key Words: Stress intensity factor, Seam welding, Singular integral equation, Reinforcement plate, Reinforcement effect

1. Introduction

As the damage tolerance design is introduced in many kinds of industries, especially aircraft structures, many researches on crack behavior in reinforced plate have been made[1~3].

Grief and Sanders[4] solved the stresses of the reinforced cracked plate with stringer by integral equation method and Liu and Ekvall[5] investigated the variables to affect a residual strength with fracture mechanics experiment for the reinforced cracked plate with various reinforcement materials. Kan and Ratwani[6] solved the stress intensity factor for the cracked infinite plate bonding infinite composite plate with adhesive showing non-linear behavior by integral equation method of complex function and many other cracked reinforced plates were examined with numerical analysis[7-9].

In case that the thin plate of jet plain have a flaw, it is reinforced by attaching reinforcement plate[10]. In the case of reinforcing a cracked plate, it is necessary to estimate residual life and theoretical evaluation of stress intensity factor is needed for it. The methods to attach thin reinforced plate to the cracked plate are spot welding and seam welding.

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The author solved stress intensity factor for the cracked plate reinforced with a plate by spot welding theoretically[11,12]. In this study, the stress intensity factor is calculated theoretically for the cracked plate subjected to remote normal stress and reinforced with a thin plate by symmetric seam welding.

2. Theory

2.1 Stress intensity factor of the reinforced cracked plate

It is considered that the cracked plate is reinforced with a plate by seam welding symmetrically and applied uniform normal stress σ_0 at infinity as in Fig. 1.

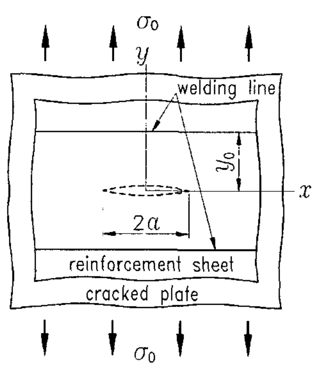
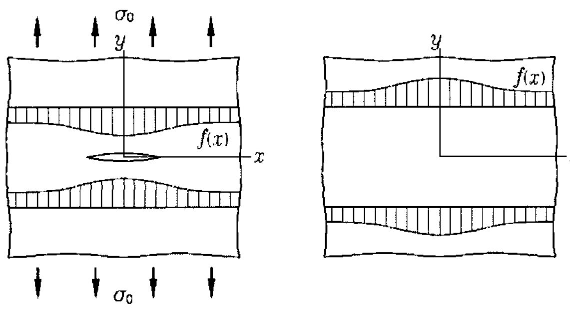


Fig.1 The cracked plate reinforced with a sheet by seam welding.

The cracked plate and the reinforcement plate are assumed to be thin and homogeneous and they are in two dimensional plane stress states and it is also assumed that there is no residual stress by seam welding and the reinforcement forces at bonding lines are applied uniformly through the thickness of two plates.

To simplify the problem, the welding zone is assumed to be a line.

Fig. 2 is free body diagram of Fig. 1



a) Cracked plate b) Reinforcement sheet

Fig. 2 Free body diagram of the reinforced cracked plate.

We consider the unknown function $f(x)$ of reinforcement force distribution in the y-direction, which is generated at welding line. The absolute values of $f(x)$ at both the cracked plate and the reinforcement plate are equal to each other by equilibrium condition. The compatibility equation of displacements at welding line is as follow.

$$v_c(x) - v_r(x) = 0 \tag{1}$$

where $v(x)$ is displacement in y-direction and subscript c and r denote a cracked plate and a reinforcement plate respectively.

The displacement in the y-direction which is generated by applying concentrated forces Y at symmetric 4 points of the cracked plate is given by[12)]

$$v = \frac{Y}{4\pi\mu t}\,\mathrm{H}(x, y\,; x_0, y_0, a) \tag{2a}$$

$$\mathrm{H}(x, y\,; x_0, y_0, a) = \mathrm{I}(x, y\,; x_0, y_0) + \mathrm{J}(x, y\,; x_0, y_0, a) \tag{2b}$$

where μ and t are shear modulus and thickness of the plate respectively, a is half crack length, x_0 and y_0 are the loading points of forces, **I** $(x, y\,; x_0, y_0)$ is the function of displacement in the infinite plate without a crack due to symmetric concentrated forces and it is as follows

$$\mathrm{I}(x, y; x_0, y_0) = \frac{2}{(x+1)}\left\{\frac{x}{2}\log\left|\frac{A_{14}A_{34}}{A_{12}A_{32}}\right| + \frac{a_2^2}{A_{12}} + \frac{a_2^2}{A_{32}} - \frac{a_4^2}{A_{14}} - \frac{a_4^2}{A_{34}}\right\} \tag{3a}$$

$$\begin{aligned}\mathrm{J}(x, y\,; x_0, y_0, a) = \frac{1}{(x+1)}\Bigg[& 2yy_0\Bigg\{-\frac{\alpha_1^2-\alpha_2^2}{A_{12}^2} - \frac{\alpha_1^2-\alpha_4^2}{A_{14}^2} - \frac{\alpha_3^2-\alpha_2^2}{A_{32}^2} - \frac{\alpha_3^2-\alpha_4^2}{A_{34}^2} \\ & + (T_1 - U_1)\frac{1}{rr_0} + (W_1 - V_1)\frac{r_0}{r}\Bigg\} \\ & + (x+1)\left\{\frac{\alpha_{14}^2}{A_{14}} + \frac{\alpha_4^2}{A_{34}} - \frac{\alpha_2^2}{A_{12}} - \frac{\alpha_2^2}{A_{32}} + (Q_1 - P_1)y_0\frac{r_0}{r}\right\} \\ & + \frac{1}{4}(x+1)^2 \log\left|\frac{C(x_0, y_0)\,G(-x_0, y_0)}{B(x_0, -y_0)\,D(-x_0, -y_0)}\right|\Bigg]\end{aligned} \tag{3b}$$

where,

$$\alpha_1 = x - x_0,\ \alpha_2 = y - y_0,\ \alpha_3 = x + x_0,\ \alpha_4 = y + y_0$$

$$A_{ij} = a_i^2 = \alpha_j^2 \qquad (\text{i=1,3 and j=2,4})$$

$$r = \left\{\left[(x-a)^2 + y^2\right]\left[(x+a)^2 + y^2\right]\right\}^{1/4}$$

$$r_0 = \left\{\left[(x_0-a)^2 + y_0^2\right]\left[(x_0+a)^2 + y_0^2\right]\right\}^{1/4}$$

$$\Delta(\xi, \eta) = \left\{\Delta_L(\xi, \eta)^2\right\} + \left\{\Delta_U(\xi, \eta)^2\right\} \qquad (\Delta = B, C, D, G)$$

$$\Delta_L(\xi, \eta) = x\xi - y\eta - sig(\xi)a^2 + r\,r_0\cos\left\{\theta + sig(\xi)sig(\eta)\theta_0\right\},$$

$$\Delta_U(\xi, \eta) = x\eta + y\xi + rr_0\sin\left\{\theta + sig(\xi)sig(\eta)\theta_0\right\}$$

$$sig(x) \equiv \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}$$

$$\Theta\Delta = \tan^{-1}\left(\frac{\Delta_U(\xi, \eta)}{\Delta_L(\xi, \eta)}\right)$$

$$P_k = \left(\frac{\alpha_4}{A_{34}} + (-1)^k\frac{\alpha_2}{A_{32}}\right)\cos\theta_p + \left(\frac{\alpha_3}{A_{32}} + (-1)^k\frac{\alpha_1}{A_{14}}\right)\sin\theta_p,$$

$$Q_k = \left(\frac{\alpha_4}{A_{34}} + (-1)^k\frac{\alpha_2}{A_{12}}\right)\cos\theta_m + \left(\frac{\alpha_1}{A_{12}} + (-1)_k\frac{\alpha_3}{A_{34}}\right)\sin\theta_m, \qquad (k=1,2)$$

$$T_1 = \left(\frac{\alpha_1}{A_{12}} + \frac{\alpha_3}{A_{34}}\right)(x_0\cos\theta_p + y_0\sin\theta_p) + \left(\frac{\alpha_2}{A_{12}} + \frac{\alpha_4}{A_{34}}\right)(y_0\cos\theta_p - x_0\sin\theta_p)$$

$$U_1 = \left(\frac{\alpha_2}{A_{32}} = \frac{\alpha_4}{A_{14}}\right)(y_0\cos\theta_m + x_0\sin\theta_m) + \left(\frac{\alpha_1}{A_{14}} + \frac{\alpha_3}{A_{32}}\right)(x_0\cos\theta_m - y_0\sin\theta_m)$$

$$V_1 = \left(\frac{\alpha_3^2-\alpha_2^2}{A_{32}^2} - \frac{\alpha_1^2-\alpha_4^2}{A_{44}^2}\right)\cos\theta_p + 2\left(\frac{\alpha_1\alpha_4}{A_{14}^2} - \frac{\alpha_3\alpha_2}{A_{32}^2}\right)\sin\theta_p$$

$$W_1 = \left(\frac{\alpha_1^2-\alpha_2^2}{A_{32}^2} - \frac{\alpha_3^2-\alpha_4^2}{A_{44}^2}\right)\cos\theta_m + 2\left(\frac{\alpha_3\alpha_4}{A_{34}^2} - \frac{\alpha_1\alpha_2}{A_{12}^2}\right)\sin\theta_m$$

$$\theta = \frac{1}{2}\left(\tan^{-1}\frac{y}{x-\alpha} + \tan^{-1}\frac{y_0}{x+\alpha}\right)$$

$$\theta_0 = \left(\tan^{-1}\frac{y_0}{x_0-\alpha} + \tan^{-1}\frac{y_0}{x_0+\alpha}\right)$$

$$\theta_p = \theta + \theta_0, \qquad \theta_m = \theta - \theta_0$$

$$x = \begin{cases} (3-v)/(1+v) & \text{(palnce stress)} \\ (3-4v) & \text{(palnce strain)} \end{cases}$$

where, v is Poisson's ratio.

The y-directional displacement of the infinite plate with a crack under remote uniform normal stress σ_0 is given by[12)]

$$v = \frac{\sigma_0}{4\mu}\,\mathrm{F}(x, y, a) \tag{4a}$$

$$\mathbf{F}\,(x, y, a) = \mathbf{L}(x, y) + \mathbf{M}(x, y, a) \tag{4b}$$

where,

$$\mathbf{L}(x,y) = (x+1)y/2 \tag{5a}$$

$$\mathbf{M}(x,y,a) = \{xr - (x^2+y^2-a^2)/r\}\sin\theta - (x-1)y \tag{5b}$$

From Fig. 2, the displacements at the welding lines of the cracked plate and the reinforced plate are presented in the form,

$$v_c(x) = \frac{\sigma_0}{4\mu_c}\mathbf{F}_c(x,y_0,a) - \frac{1}{4\pi\mu_c t_c}\int_0^\infty f(x_0)\,\mathbf{H}_c(x,x_0,y_0,a)dx_0 \tag{6a}$$

$$v_c(x) = \frac{1}{4\pi\mu_r t_r}\int_0^\infty f(x_0)Ir(x,x_0,y_0)dx_0 \tag{6b}$$

Substituting from equation (6) into equation (1), we obtain

$$\mathbf{F}_c(x,y_0,a) - \frac{1}{\pi}\int_0^\infty \frac{f(x_0)}{\sigma_0 t_0}\left[\mathbf{H}_c(x,x_0,y_0,a) + \frac{1}{\gamma}\mathbf{I}_r(x,x_0,y_0)\right]dx_0 = 0 \tag{7}$$

where, the stiffness ratio, γ is multiplication value of shear modulus ratio and thickness ratio for the reinforced plate and cracked plate. The stiffness ratio, γ is presented in the form,

$$\gamma = \frac{\mu_r t_r}{\mu_c t_c} \tag{8}$$

In order to solve the problem, the distributed load equation is divided into two terms, the term by the crack effect and the term without the crack effect as follow,

$$\frac{f(x_0)}{\sigma_0 t_c} = \bar{q}(x_0) + \bar{s}_0 \tag{9}$$

where bar $\bar{s}_0$ is related to the uniform distributed load, which occur in the case of reinforcing the infinite plate without a crack and is presented in the form,

$$\bar{s}_0 = \frac{x_c+1}{8\left[\frac{1}{\gamma}\left(\frac{x_r-1}{x_r+1}\right) + \frac{x_c-1}{x_c+1}\right]} \tag{10}$$

Substituting from equation (7) into equation (9), and introducing the normalized factor, $\bar{x} = x/a$, $\bar{x}_0 = x_0/a$, $\bar{y}_0 = y_0/a$, we obtain the following singular integral equation.

$$(1-\bar{s}_0)\,\mathbf{M}_c(\bar{x},\bar{y}_0,1) = \frac{1}{\pi}\int_0^\infty \bar{q}(\bar{x}_0)\left[\mathbf{H}_c(\bar{x},\bar{x}_0,\bar{y}_0,1) + \frac{1}{\gamma}\mathbf{I}_r(\bar{x},\bar{x}_0,\bar{y}_0)\right]d\bar{x}_0 \tag{11}$$

Solving the singular integral equation for $\bar{q}(\bar{x}_0)$ and substituting from the result into following equation[12], we can obtain the stress intensity factor by reinforcement force due to crack effect.

$$\lambda = \sigma_0\sqrt{\pi a}\int_0^\infty \bar{q}(\bar{x}_0)\,h_c(\bar{x}_0,\bar{y}_0)d\bar{x}_0 \tag{12}$$

Introducing $\bar{r}_0 = r_0/a$,

$$h(\bar{x}_0,\bar{y}_0) = \frac{2}{\pi}\left\{\frac{\sin\theta_0}{\bar{r}_0} - \frac{2\bar{y}_0}{(x+1)\bar{r}_0^{3}}(\bar{x}_0\cos 3\theta_0 + \bar{y}_0\sin 3\theta_0)\right\} \tag{13}$$

In conclusion, the stress intensity factor of the crack is obtained by equations (9) and (12) as follow,

$$\frac{K_I}{\sigma_0\sqrt{\pi a}} = 1 - \bar{s}_0 - \bar{\lambda} \tag{14}$$

where, $\bar{\lambda} = \lambda/\sigma_0\sqrt{\pi a}$ is the normalized stress intensity factor.

2.2 The method of solving the singular integral equation

Substituting from $\bar{x} = (1+\tau)/(1-\tau)$ and $\bar{x}_0 = (1+\xi)/(1-\xi)$ into equation (11), we obtain the following equation,

$$(1-\bar{s}_0)\,\mathbf{M}_c(\tau,\bar{y}_0,1) = \frac{2}{\pi}\int_{-1}^{1}\frac{\bar{q}(\xi)}{(1-\xi)^2}\left[\mathbf{H}_c(\tau,\xi,\bar{y}_0,1) + \frac{1}{\gamma}\mathbf{I}_r(\tau,\xi,\bar{y}_0)\right]d\xi \tag{15}$$

Since $\bar{q}(\xi)$ is nonsingular in the case of $\xi = \pm 1$, we introduce the nonsingular function, $\phi(\xi)$ as follow,

$$\bar{q}(\xi) = \phi(\xi)\sqrt{1-\xi^2} \tag{16}$$

Substituting from equation (16) into equation (15) and introducing Erdogan and Gupta's method[13], equation (15) is reduced to the following equation,

$$(1-\bar{s}_0)\,\mathbf{M}_c(\tau_k,\bar{y}_0,1) = \frac{2}{N+1}\sum_{n=1}^{N}\left\{\left(\frac{1+\xi_n}{1-\xi_n}\right)\phi(\xi_n)\beta(\tau_k,\xi_n,\bar{y}_0)\right\},\; k=1,2,\ldots,(N+1) \tag{17}$$

where,

$$\xi n = \cos\left(\frac{n\pi}{N+1}\right), \quad \tau\text{k} = \cos\left[\frac{(2\text{k}-1)\pi}{2(\text{N}+1)}\right]$$

$$\beta(\tau_k,\xi_n,\bar{y}_0) = \mathbf{H}_c(\tau_k,\xi_n,\bar{y}_0,1) + \frac{1}{\gamma}\mathbf{I}_r(\tau_k,\xi_n,\bar{y}_0)$$

The unknown function, $\phi(\xi)$ in equation (17) has been obtained with N = 90. Substituting from the result into equation (16) and equation (12), the stress intensity factor is calculated by equation (14). In the case of N = 100, the change of the stress intensity factor is less than 0.1%.

3. Examination

$v = 0.3$ is used in the following procedure.

Finite Element Method with ANSYS 5.3 is used for verifying the theory and eight noded rectangular element is used for numerical analysis.

The Table 1 is the comparison in dimensionless stress intensity factors between the present solution and the FEM analysis. From the table 1, the value from the FEM analysis is greater about 1~10% than that from the present theory, which seems to be due to the reinforcement force in the x-direction.

Table 1. Comparison in dimensionless stress intensity factors between the present solution and the FEM analysis

y_0/a	λ=0.1		λ=0.5		λ=1.0		λ=2.0	
	Present	FEM	present	FEM	present	FEM	present	FEM
0.5	0.753	0.787	0.384	0.411	0.234	0.254	0.123	0.137
1.0	0.833	0.845	0.491	0.512	0.316	0.335	0.171	0.188
2.0	0.876	0.885	0.576	0.592	0.392	0.412	0.223	0.243
5.0	0.896	0.903	0.622	0.637	0.439	0.453	0.258	0.280
10.0	0.899	0.907	0.631	0.642	0.447	0.462	0.265	0.285

Fig. 3 shows the stress intensity factors as a function of seam welding location and the stiffness ratio. In Fig. 3, the solid line indicates the value by present theory and the point, • that by FEM analysis and we see that the reinforcement effect gets better as welding line is closer to the crack and stiffness ratio of the cracked plate and the reinforcement sheet becomes larger and that the stress intensity factor approaches zero as y_0 is closer to zero.

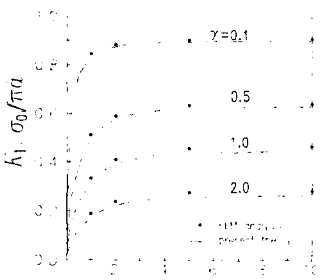


Fig.3 Dimensionless stress intensity factors as a function of seam welding location and stiffness ratio.

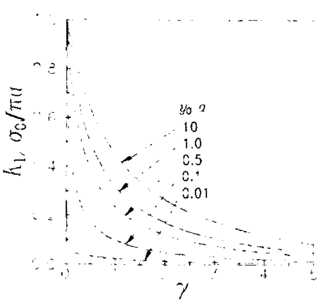


Fig.4 Dimensionless stress intensity factors as a function of stiffness ratio.

Fig. 4 shows the stress intensity factor as a function of the stiffness ratio, γ. The stress intensity factor deceases as the stiffness ratio increases and the normalized stress intensity factor approaches 1 as the stiffness ratio leads to zero. Besides from the equation (8), we can see that the influences to affect reinforcement effect between shear modulus ratio and thickness ratio are the same.

4. Conclusion

Through the theoretical analysis to calculate the stress intensity factor, the cracked plate reinforced with a thin plate by symmetric seam welding, the following conclusions are obtained

1. The stress intensity factor of the cracked plate reinforced with a thin plate by seam welding has been solved with the singular integral equation method.
2. The theory has been verified by comparing the result of this theory with that of numerical analysis.
3. The reinforcement effect gets better as welding line is closer to the crack and stiffness ratio of the cracked plate and the reinforcement plate becomes larger.

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