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Rheology and pipeline transportation of dense fly ash-water slurry

Hiromoto Usui*, Lei Li and Hiroshi Suzuki
Department of Chemical Science and Technology, Kobe University
Rokkodai-cho 1-1, Nada, Kobe 657-8501, Japan

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Abstract

Prediction of the maximum packing volume fraction with non-spherical particles has been one of the important problems in powder technology. The sphericity of fly ash particles depending on the particle diameter was measured by means of a CCD image processing instrument. An algorithm to predict the maximum packing volume fraction with non-spherical particles is proposed. The maximum packing volume fraction is used to predict the slurry viscosity under well dispersed conditions. For this purpose, Simha's cell model is applied for concentrated slurry with wide particle size distribution. Also, Usui's model developed for aggregative slurries is applied to predict the non-Newtonian viscosity of dense fly ash - water slurry. It is certified that the maximum packing volume fraction for non-spherical particles can be successfully used to predict slurry viscosity. The pressure drop in a pipe flow is predicted by using the non-Newtonian viscosity of dense fly ash-water slurry obtained by the present model. The predicted relationship between pressure drop and flow rate results in a good agreement with the experimented data obtained for a test rig with 50 mm inner diameter tube. Base on the design procedure proposed in this study, a feasibility study of fly ash hydraulic transportation system from a coal-fired power station to a controlled deposit site is carried out to give a future prospect of inexpensive fly ash transportation technology.

1. Introduction

The deposit of a large amount of fly ash and bottom ash discharged from coal-fired power stations is a serious problem. The amount of fly ash is larger than bottom ash. Considerable amount of recycle use is available, mainly by adding fly ash to cement. However, the addition of fly ash to cement is limited because the production rate of cement is leveled off, and also the concentration of fly ash in cement is limited. Thus, it is expected that large amount of fly ash must be transported from power station to waste material deposit site in the near future. The deposit site is not necessarily located near the power station. Long distance transportation of fly ash with reduced cost must be considered. As a possible technique of fly ash transportation, hydraulic transportation of fly ash-water slurry is investigated in this study. The design of a pipeline transportation system must be accomplished by means of the design procedure based on precise knowledge of rheological behavior of dense fly ash slurries.

Suspensions generally show the complex flow behavior. These complexities may be caused by interactions between solid particles forming the internal structure, particle size distribution, and non-spherical characteristics of suspended particles. Recently, Chryss-Bhattacharya (1999) discussed the prediction method of the maximum packing volume fraction for non-spherical particles. But his discussion was limited to the case of uniform sphericity. Generally speaking, sphericity is the function of particle size, and more sophisticated treatment of sphericity must be carried out in future studies on suspension rheology. The sphericity of fly ash particles was measured for different particle size ranges by means of microscopic CCD camera and automatic particle image processor. Based on these experimental results, a model to predict the maximum packing volume fraction is proposed. On the other hand, one of the present authors (Usui 1995; 1999; 2000) has been discussing the viscosity prediction method of concentrated slurries with wide particle size distribution and with agglomerative nature. The first aim of this paper is to present the viscosity prediction method for non-spherical suspensions which is based on Usui's suspension rheology model. The second aim is to propose a possible way of long distance transportation of large amount of the ash by pipeline.

*Corresponding author: usui@cx.kobe-u.ac.jp

2. Experiments

The dense fly ash-water slurry used in this study was prepared as follows. Sample fly ash was discharged from Matsuura Power Station in Japan where Blair Athol coal was burned. Two kinds of dense fly ash-water slurries; Slurry-A and B were prepared.
Slurry-A; 70 wt% fly ash-water slurry without additive.
Slurry-B; 70 wt% fly ash-water slurry with 0.3 wt%/slurry polystyrene sulfuric acid (PSS) as a dispersing additive. The solid volume fraction corresponding to this concentration was ϕ=0.51. The particle size distribution of pulverized coal was measured by means of the Laser Diffraction Particle Size Analyzer, Horiba-Seisakusho Model 920. Particle size distribution of fly ash was in good agreement with the Rosin-Rammler distribution given by $D(\%)=100\ \exp(-0.061 d_p^{1.24})$, where D(%) is cumulative oversize distribution. Although the particle size distribution is continuous, the particle size distribution must be divided into several parts for the convenience of the following model predictions. Usui *et al.* (2000b) discussed the effect of the subdivided particle size distribution on the accuracy of viscosity prediction. The conclusions obtained from the discussion were as follows. The most important point was the number of minimum sized particles should be much larger than the number of other size particles. The second point was that the number of subdivisions was rather insensitive, and the number of ten to twenty subdivided parts was enough to give an accurate estimation of slurry viscosity. Thus, in this study, the particle size was divided into twelve parts. The detailed method is given as follows. The laser diffraction particle size analyzer gave 52 segments of particle size and the probability of each particle size. Successive four segments data were averaged to give the number mean particle diameter by $d_i = \sum n_k d_k / \sum n_k$. Since the summation of the first four segments did not give enough number of minimum sized particles, first and second segments were combined to give the minimum sized particles. The probability density and the number of particles for each size band are listed in Table 1. The sphericity was measured by means of a flow particle image analyzer, FPIA-2000, Sysmex Co. The well dispersed particle image is taken by a CCD camera, and the sphericity, Ψ, is defined by following equation.

Table 1. Particle size distribution and sphericity of tested fly ash

Particle size (μm)	Probability density (-)	Number based particle size distribution (1/m^3-slurry)	Sphericity (-)
1.02	0.162	1.50×10^{17}	0.93
3.15	0.059	1.84×10^{15}	0.97
4.13	0.067	9.25×10^{14}	0.97
5.43	0.079	4.79×10^{14}	0.97
7.14	0.098	2.62×10^{14}	0.919
9.35	0.124	1.47×10^{14}	0.919
12.24	0.138	7.34×10^{13}	0.919
16.0	0.122	2.90×10^{13}	0.919
20.9	0.088	9.37×10^{12}	0.919
27.2	0.045	2.16×10^{12}	0.919
35.4	0.014	3.03×10^{11}	0.919
45.0	0.002	1.91×10^{10}	0.919

$$\Psi = \frac{(diameter\ based\ on\ area\ of\ particle\ image)}{(diameter\ based\ on\ perphery\ of\ particle\ image)} \tag{1}$$

The image fly ash particles is shown in Fig. 1. The sphericity data were automatically accumulated and the averaged value of sphericity for each particle size band is shown in Table 1. The sphericity of fly ash is a weak function of particle size. The apparent viscosity of fly ash-water slurries was measured by a remoter (Iwamoto Seisakusho Co., Ltd. Model IR-200) with coaxial rotating cylinders. The temperature of the test samples was maintained as 298K.

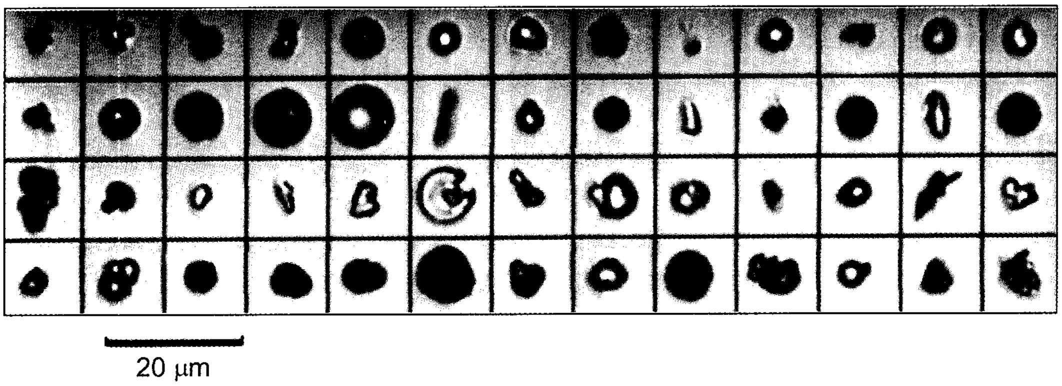
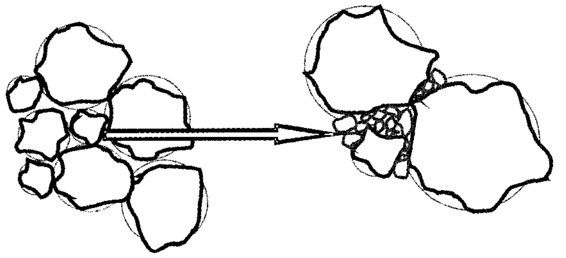


Fig. 1. Photograph of fly ash particles.

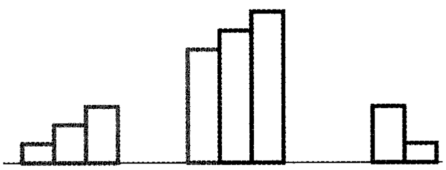
3. Maximum packing volume fraction for non-spherical particle system

It is necessary to predict the maximum packing volume fraction of a non-spherical particle system if we want to predict the slurry viscosity of general suspension systems. Usui's suspension rheology model (Usui, 1999; 2000) needs the value of maximum packing volume fraction estimated for the test sample dispersed in the slurry. As already mentioned above, the fly ash particles have particle size distribution, and the sphericity depends on particle size. Several theories to predict the maximum packing volume fraction have been proposed in the past, e.g. Lee (1970), Yu (1991) and Suzuki-Ohshima (1983). Each theory predict almost the same value of the maximum packing volume fraction, but they are a little different from each other. Since we want to obtain a rough estimation of the maximum packing volume fraction, we would rather select a simple procedure to estimate the maximum packing volume fraction. Thus, Lee's theory has been adopted in this study.

As the first step of the calculation, the continuous particle size distribution is subdivided into two parts, i.e., d_0~d_k and d_{k+1}~d_{max} as shown in Fig. 2. The particle diameter determined by laser diffraction method is interpreted as the volume based diameter. On the other hand, the circularity of the two dimensional trajectory image of a particle is conventionally determined by an image processing instrument (e.g. a flow particle image analyzer, FPIA-2000, Sysmex Co). The well dispersed particle image is taken by a CCD camera, and the circularity, Ψ, is defined by Eq.(1). It is assumed that the circularity, thus obtained, can be used as sphericity for the case of three dimensional calculation. The sphericity data were automatically accumulated and the averaged value of sphericity for each particle size band. The diameter newly defined as $d_{i,nons}=d_i/\Psi_i$ is the diameter which include the effect of non-sphericity. Using the non-spherical diameter and their probability density distribution, we calculate the maximum packing volume fraction when the particles, $d_{k+1,nons}$~$d_{max,nons}$, are packed into the unit volume. This packing condition may be imagined as shown in Fig. 2(a). Since the particle size distribution is very wide, small particles may be packed into the space, which is vacated by the effect of non-sphericity of larger particles. The void space shown in Fig. 2(a) is packed by the smaller particles, $d_{0,nons}$~$d_{k,nons}$. The small area is enlarged in Fig. 2(b) where the non-spherical smaller particles are packed under the maximum packing condition. The maximum packing volume fraction for the occupation of the void in Fig. 2(a) gives the amount of smaller particles, d_0~d_k. This amount of smaller particles must coincide with those calculated by the particle size distribution. If this condition is not satisfied, the border of the two parts, i.e. the number k, is changed, and the above procedure is repeated until good convergence is obtained.



(a) Maximum packing by larger particles; d_k~d_{max} (b) Maximum packing by smaller particles; d_0~d_k



(c) Discrete particle size distribution and the critical particle size dk

Fig. 2. Maximum packing by non-spherical particles.

The calculation procedure is summarized as follows.

1. Give the discrete particle size distribution and the sphericity data.
2. Assume the critical number, k, to separate the particles into two parts as shown in Fig. 2(c), and obtain the ratio of solid volume fractions; ϕ_L/ϕ_S, where ϕ_L and ϕ_S correspond to the solid volume fractions of the smaller particles, d_0~d_k , and the larger particles, d_{k+1}~d_{max}, respectively.
3. Estimate the maximum packing volume fraction, $\phi_{p,max,L}$ for the larger particles, $d_{k+1,nons}$~$d_{max,nons}$, using Lee's algorithm.
4. Calculate the solid volume fraction, $\phi_{p,L,solid}$ which correspond to the value of $\phi_{p,max,L}$ by the following equation.

$$\phi_{p,L,solid} = \frac{\sum_{k+1}^{max} \frac{\pi}{6} d_i^3 N_i}{\sum_{k+1}^{max} \frac{\pi}{6} d_{nons,i}^3 N_i} \phi_{p,max,L} \tag{2}$$

Then, calculate the net solid volume fraction, a which corresponds to the smaller particles, d_0~d_k, by,

$$\alpha = \phi_{p,L,solid}(\phi_S/\phi_L) \tag{3}$$

5. Estimate the maximum packing volume fraction, $\phi_{p,max,S}$, to fill up the volume; 1-$\phi_{p,L,solid}$, by the non-spherical smaller particles; $d_{0,nons}$~$d_{k,nons}$.

6. Calculate the net solid volume fraction, $\beta=\phi_{p,S,solid}$, by the following equation,

$$\beta = \phi_{p,S,solid} = (1-\phi_{p,L,solid})\phi_{p,S,max}\frac{\sum_0^k \frac{\pi}{6} d_i^3 N_i}{\sum_0^k \frac{\pi}{6} d_{nons,i}^3 N_i} \tag{4}$$

7. Compare the values of a and β. If the two values are not the same, new critical number, k, is reassumed, and the procedures 2~7 mentioned above are repeated to give a good convergence.

8. Once the critical number k is determined, the maximum packing volume fraction is calculated by $\phi_{p,max,nons} = \phi_{pLS,solid} + \phi_{p,S,solid}$.

4. Prediction of slurry viscosity for well dispersed condition

Usui (2000) has proved that Simha's model was valid for the viscosity prediction of suspensions, which had both mono-modal and tetra-modal particle size distribution of spherical silica particles. Thus the same procedure is applied for the case of non-spherical fly ash particle suspension with continuous particle size distribution. The calculation procedure of maximum packing volume fraction was explained in the previous section of this paper. The dependence of $a(=\phi_L/\phi_S)$ and $\beta(=\phi_{p,L,solid}/\phi_{p,S,solid})$ on k is indicated in Fig. 3 for the case of fly ash particles. It is evident that a and β are equal at k=3. The value of k, which satisfies the condition mentioned above, may not necessarily be an integer, but we use an integer because of the convenience of calculation. The viscosity predicted by Simha's model using k=3 is shown in Fig. 4 by the broken line. The calculation procedure of Simha's cell model to predict the completely dispersed suspension for given solid volume fraction, particle size distribution and maximum packing volume fraction was reported by Usui *et al.* (2001). The viscosity level shown by the broken line is interpreted as the lower limit of 70 wt% fly ash-water slurry with completely dispersed condition. The experimental data shown in Fig. 4 are much higher than this lower limiting level because of the agglomerative nature of fly ash particles. We will discuss the effect of agglomeration in the next section.

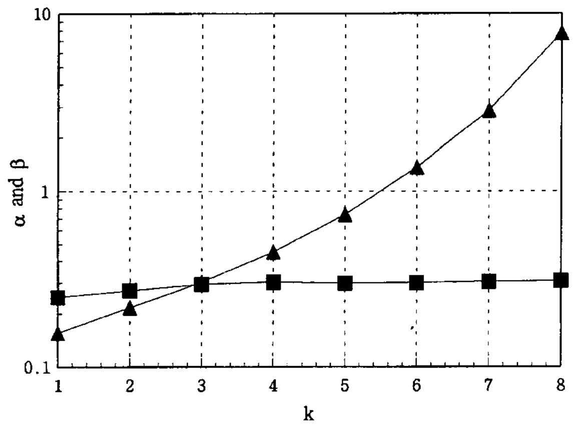


Fig. 3. Solid volume fraction ratio and maximum packing volume fraction ratio depending on subdividing number, k.

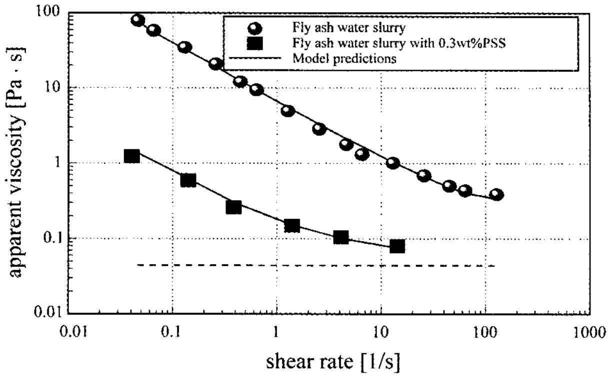


Fig. 4. Apparent viscosity of fly ash-water slurries. Solid line indicates slurry viscosity predicted by Eq.(5) with Simha's cell model. Dotted line indicates viscosity level predicted by Simha's cell model for well dispersed non-spherical particle suspension. Dotted line is viscosity level predicted by Simha's cell model for the well dispersed spherical particle suspension.

5. Prediction of non-newtonian slurry viscosity and discussion

The fly ash particles agglomerate with each other when the shear rate decrease, and the apparent viscosity increase. The particle size distribution in this study is shown in Table 1. The particle size is subdivided into twelve discrete ranges. Usui *et al.* (2000) proposed the viscosity prediction method for slurries with discrete particle size distribution. The basic idea was the combination of Simha's cell model and Usui's non-Newtonian slurry viscosity model. But, Simha's cell model needs the maximum packing volume fraction. We estimate it by the packing model for non-spherical particles proposed in this study. The model proposed by Usui concentrates the effect of the agglomerative nature of suspended particles to the cluster formation by very small primary particles. The number of primary particles contained in a cluster, n, is chosen as a thixotropy parameter. The shear break-up process was determined by taking the force balance on the two spherical clusters in simple shear flow. Usui proposed a rate equation in which the breakup of the cluster and both Brownian coagulation and shear coagulation processes were taken into account. The rate equation is given by

$$\frac{dn}{dt} = \frac{4\alpha_b k_b T N_0}{3\eta_0} + \frac{4\alpha_s \phi n \dot{\gamma}}{\pi} - \frac{3\pi d_0^3 n}{4F_0 N_b}\left(\frac{n}{1-\varepsilon} - 1\right)\eta\dot{\gamma}^2 \tag{5}$$

Since the solid concentration is known, the unknown parameters in Eq. (5) are n, F_0 and ε. The dependence of

ε on the number of primary particles contained in a cluster is assumed to be given by the following equation (Usui, 1999).

$$\varepsilon = \varepsilon_{max}\{1-n^{-0.4}\} \tag{6}$$

where ε_{max} is the void fraction at $n=\infty$. Using the relationship given by Eq. (10), the unknown parameters are reduced to two. When a set of apparent viscosity and shear rate data is given experimentally, the number of primary particles contained in a cluster is calculated as follows.

1. Assume the value of n, and estimate the size and number of cluster by Eq. (10).
2. Calculate the maximum packing volume fraction by $\phi_{p,max}=0.63\Psi^{\beta}$.
3. Use Simhaís cell model (Simha, 1952) to predict the slurry viscosity at a given shear rate.
4. If the predicted viscosity is not the same as the experimentally determined one, the value of n is reassumed until a good convergence is obtained.

The number, n, of the primary particles contained in a cluster is shown in Fig. 5. The data obtained for Slurry-B indicate that the cluster is almost deflocculated at higher shear rate range. The distribution of n obtained for Slurry-A does not reach to unity at higher shear rate range. Moreover, it does not show the tendency to be leveled off. This means that the experimental data obtained for Slurry-A does not show the Newtonian flow characteristics even at higher shear rate range of this experiments. Eq. (5) is used to predict the inter-particle bonding energy of the primary particle. Also the same equation was used to predict the non-Newtonian Slurry viscosity. Fig. 6 shows the dependence of F_0 on the shear rate. The averaged values of inter-particle bonding energy were estimated for 1.02 μm fly ash particle as $F_0=2.38\times10^{-15}$ [J] (without additive) and $F_0=1.06\times10^{-17}$ [J] (with 0.3 wt% PSS), respectively. Then slurry viscosity is predicted for the known particle size distribution. The predicted viscosity is shown in Fig. 4 by the solid line. This diagram indicates that the non-Newtonian slurry viscosity of a non-spherical particle system can be predicted by the slurry viscosity model proposed in this study. The increase in viscosity from the broken line level observed for experimental data in Fig. 4 is attributed to the effect of agglomeration of fly ash particles.

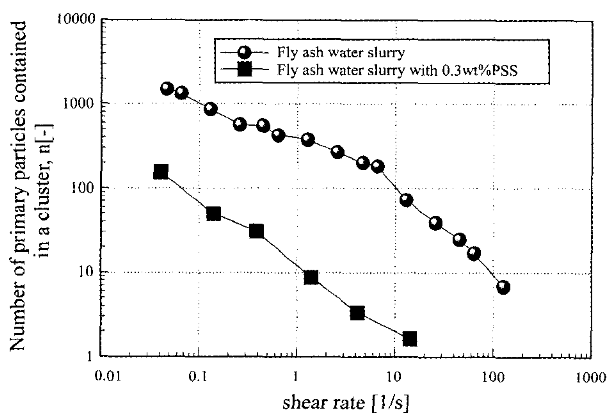


Fig. 5. Number of primary particles contained in cluster as function of shear rate.

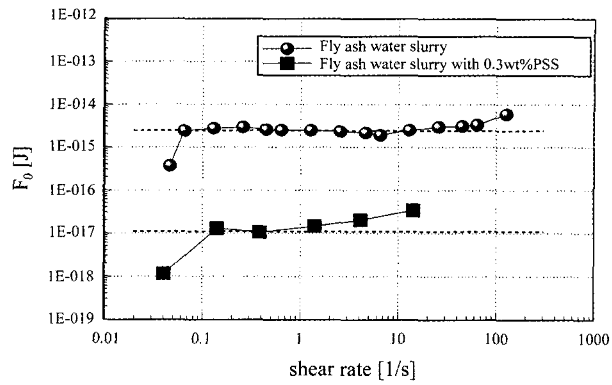


Fig. 6. Inter particle bonding energy determined for each viscosity measured for different shear rate.

6. Design of hydraulic pipeline transportation of fly ash slurry

A model to predict the non-Newtonian viscosity of dense fly ash-water slurries was proposed in the previous section of this paper. Once the value of inter-particle bonding energy, F_0 is determined for slurry at certain solid volume fraction, the relationship between shear rate and shear stress can be predicted even for different solid concentrations with the same fly ash. Using the shear rate-shear stress relationship, the flow rate in a slurry pipeline at a fixed pressure drop can be easily calculated by following relationships.

$$\frac{dP}{dx} = \frac{2\tau_{wall}}{R} \tag{7}$$

$$Q = \frac{8R}{dP/dx}\int_{\tau_{centre}}^{\tau_{wall}} \tau^2 \dot{\gamma}(\tau)\,d\tau \tag{8}$$

Kawasaki Heavy Industries Co. Ltd. is now doing the fly ash slurry transportation experiments using the test rig with 50 mm inner diameter stainless tube. The concentration range of test slurries is between 70 wt% and 75 wt%. Their results are compared with the predicted pressure drop of this study in Fig. 7. Since the reciprocal slurry pump is used in their test rig, the flow is pulsative. Thus, the exact comparison is difficult. However, as shown in Fig. 7, the present model to predict hydraulic pipeline transportation of fly ash slurries gives a reasonable agreement with the experimental results. We can conclude that the rheological

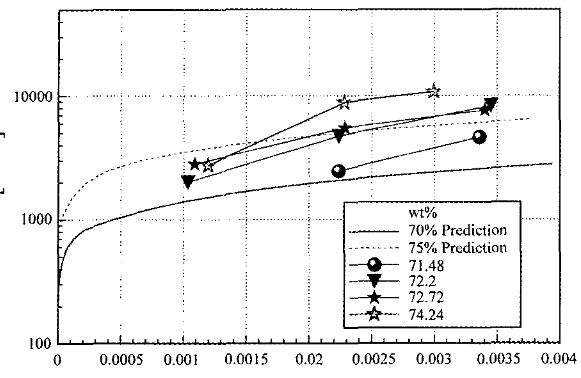


Fig. 7. Comparison of pressure drop estimation with the experimental results.

model proposed in this study for non-spherical particle suspensions with particle size distribution is successfully used for the hydraulic pipeline transportation of dense slurries.

The power consumption of slurry transportation pipeline was estimated for the following design conditions.

1. Discharge rate of fly ash from a power station = 640,000 ton/year, or 88.9 ton/h,
2. Particle size distribution, slurries, density, slurry temperature are the save as the present study,
3. Slurry concentration = 70 wt%,
4. Horizontal pipeline length = 10 km, pipe inner diameter = 125 mm.

The predicted results are;
Total pressure drop = 11.5 MPa,
Power consumption = 500 kW, with pumping efficiency = 0.5.
Flow rate = 0.0220 m^3/s
Average velocity of pipe flow = 1.8 m/s.

Based on the pipe line transportation design mentioned above, the cost estimation of hydraulic transportation and its comparison with the other transportation method, e.g. belt conveying or track conveying, should be done in the future study.

7. Sedimentation stability of dense fly ash-water slurries

Bunn-Chambers (1993) and Ward *et al.* (1999) investigated the hydraulic transportation of dense fly ash-water slurry. The fly ash transportation pipeline from Bayswater power station in NSW, Australia to the deposit site (9.5 km from the power station) has been successfully operated (Ward *et al.*, 1999). The flow rate is 300 ton/hr. Fly ash-water slurry (70 wt%) without additive is transported in this system. Since the fly ash particles sediment very easily, it cause a very high pumping power when the pipeline system is restarted. This is a serious problem because a larger pump with over estimated power consumption must be installed. There are several techniques to avoid the sedimentation of fly ash particles, when the pipeline operation is stopped. Purge of slurries by water from pipeline system is not realistic since the volume of pipeline is quite large if the pipeline length is long, e.g. 123 m^3 for the design example mentioned in the previous section of this study. The use of pig causes the same problem. The use of stabilizing additive to prevent the sedimentation of fly ash particles seems to be appropriate if the cost of additives is reduced. However, the addition of stabilizing additive causes an increase in slurry viscosity. The combined use of dispersing additive to reduce the slurry viscosity may be the solution for this problem.

Several natural or synthetic polysaccharides were selected as the stabilizer for fly ash-water slurries. They are rhamsan gum S-194 (trade name: Alcaligrnes-ATCC 31961, Dainippon Pharmaceutical Co. Ltd.), rhamsan gum S-130 (trade name: Alcaligrnes-ATCC 31555 Dainippon Pharmaceutical Co. Ltd), xanthan gum (trade name Vanzan, R. T. Vanderribdt Co.) and carboxyl methyl cellulose (trade name CMC, Daicel Chemical Co. Ltd.). The concentration of those stabilizer was adjusted so that the viscosity of fly ash slurry with 0.3 wt% naphthalene sulfuric acid formalin condensate (NSF) becomes the same level of fly ash slurry-A without additive. The experimental results are shown in Fig. 8. Using these fly ash-water slurries with different additives, but at the same viscosity level, the gravitational sedimentation tests were carried out. Test slurries were contained in a glass column (38 mm diameter and 200 mm height). After one day or one week since the sedimentation test started, the slurry was sampled from the sampling hole attached on the sidewall of glass column. The solid concentration of each sample was measured, and the results are shown in Fig. 9. It is evident that the additives can act as the stabilizer. Rhamsan gum, S-194, is the best to prevent the sedimentation of fly ash particles. The slurry with 0.02 wt% S-194 showed soft pack slurry even

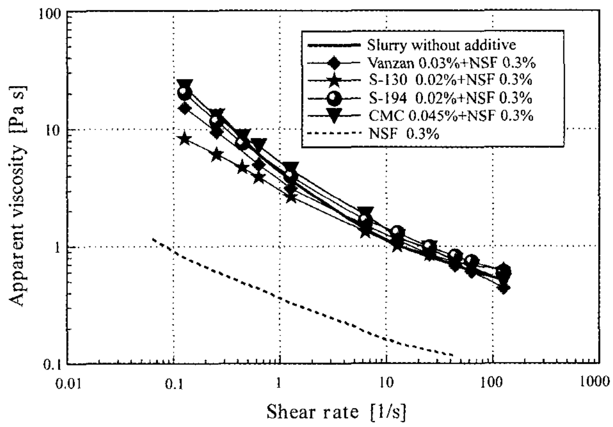


Fig. 8. Apparent viscosity of 70 wt% fly ash-water slurries with various kind of additives.

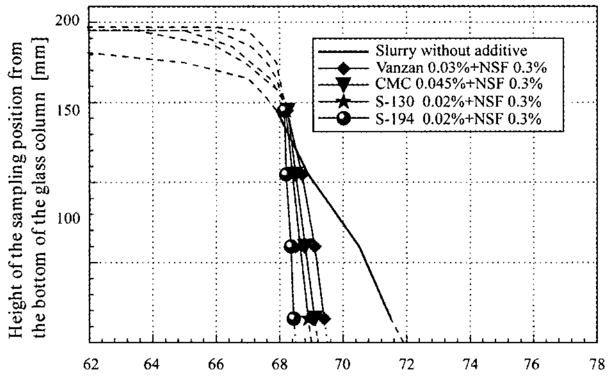


Fig. 9. Sedimentation stability test results obtained for 70 wt% fly ash-water slurries with various kind of additives.

after 7 days. Rhamsan gum, S-130, is the second and CMC follows. Xanthan gum was less effective to prevent the sedimentation. We recommend the use of Rhamsan gum S-194 or S-130 as a stabilizer. CMC may be also recommended.

The results mentioned above indicate that the slurry can be suddenly changed to non-sediment if we can put the additives into pipe flow by means of suitable method. For this purpose we propose the additive injection system shown in Fig. 10. This injection system must be operated periodically just before the pipeline system is to be shut down. Since the average velocity in the pipe is 1.8 m/s as shown in the previous feasibility study of this paper, all the fly ash slurry can be replaced by no-sediment slurry within 31 minutes if we use three additive injection station as shown in Fig. 10. The cost of the additive for one shutdown is roughly estimated as 1,500 US$. The increase of total cost by using the stabilizing additives does not cause a serious problem. Moreover, the decrease of steady pumping cost by adopting the optimum power consumption of pumping system may cause the reduction of total cost. Thus we conclude that the hydraulic fly ash slurry transportation system with periodical addition of stabilizer is worth to be considered in the practical slurry transportation system in the future.

8. Conclusions

A model to predict the maximum packing volume fraction for a non-spherical particle suspension is proposed. Simha's cell model is applied for the suspension with particle size distribution. It is certified that the maximum packing volume fraction for non-spherical suspension is successfully used to predict the slurry viscosity under completely dispersed conditions. Also, a thixotropy model proposed for dense slurries by Usui is combined with the maximum packing volume fraction for non-spherical particle system to give more accurate prediction of non-Newtonian rheological characteristics of slurries. The model results in the estimation of inter-particle bonding force between primary particles in a cluster, and the power consumption and flow rate relationship in hydraulic slurry pipeline transportations system is predicted. A possible way to reduce the total cost of slurry pipeline system by means of a periodical addition of stabilizer is proposed.

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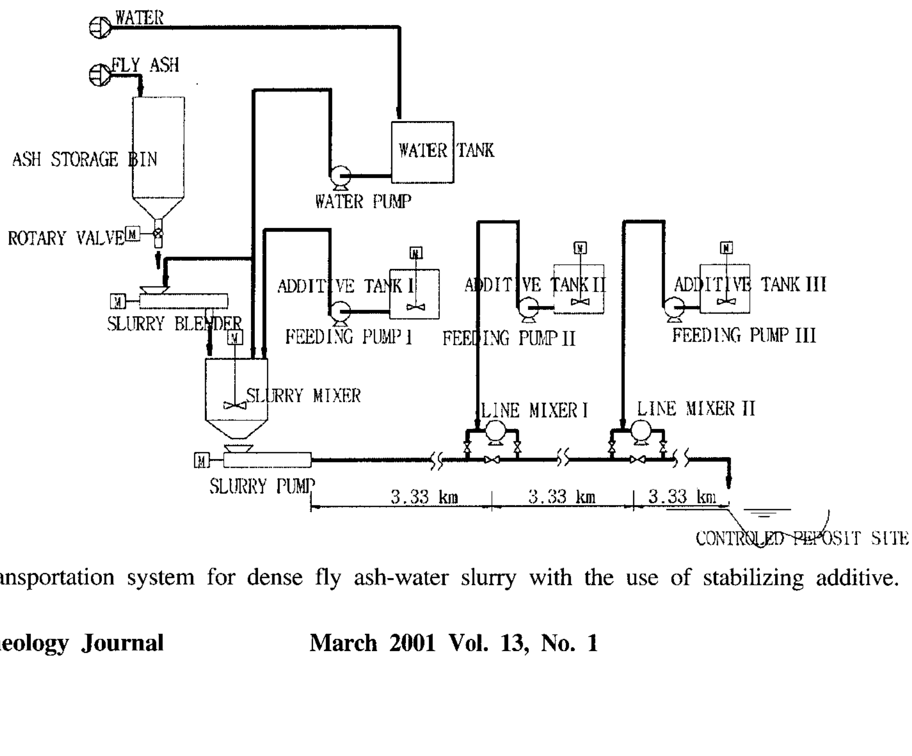

Fig. 10. Hydraulic transportation system for dense fly ash-water slurry with the use of stabilizing additive.

nology, Japan. Also the authors appreciate the valuable discussion provided by Mr. Yutaka Takahashi on the practical slurry transportation system. The experimental assistance by Mr. Yohei Tsukamasa is appreciated.

Nomenclature

a	= radius of spherical particle	[m]
b	= cell radius	[m]
d_0	= diameter primary particle which create the cluster	[m]
F_0	= inter-particle bonding energy	[J]
k_B	= Boltzmann constant	[J/K]
n	= number of the primary particles contained in a cluster	[-]
N_b	= number of chains to be broken-up during the cluster break-up process	[-]
N_0	= total number of minimum sized (primary) particles less than critical	[-]
n	= number of the primary particles contained in cluster	[-]
t	= time	[s]
T	= temperature	[K]
α_b	= Brownian coagulation constant (=0.58)	[-]
α_S	= shear coagulation constant (=0.6)	[-]
$\dot{\gamma}$	= shear rate	[s^{-1}]
ε	= void fraction	[-]
η	= slurry viscosity	[Pa.s]
η_0	= solvent viscosity	[Pa.s]
Ψ	= sphericity	[-]
ρ	= density	[kg/m^3]
τ	= shear stress	[Pa]
ϕ	= volume fraction of solid particles	[-]
$\phi_{p,max}$	= maximum packing volume fraction	[-]

<Subscirpts>

nons.	= value obtained for non-spherical particles
p,max	= maximum packing
L	= value obtained for d_{k+1}~d_{max}
S	= value obtained for d_0~d_k
solid	= solid volume fraction

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