Random Vibration Analysis of Nonlinear Stochastic System under Earthquake Using Statistical Method

지진하중을 받는 비선형 추계적 시스템의 불규칙진동해석

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국문요약

본 논문에서는 비선형 추계적 구조시스템의 지진에 대한 동적응답 해석방법을 제안하였다. 부분구조합성법에 기초한 섭동법을 응용하여 지진외력에 의한 불규칙진동의 시간응답과 주파수응답 해석과정을 정식화하였다. 이 방법에서는 대형·동적 시스템의 지배방정식인 비선형 미분방정식을 몇 개의 비선형 모달방정식으로 근사 변환한다. 각 분계는 비선형 복원력항을 모드좌표로 근사변환함으로써 선형화하여 합성되어진다. 모드좌표에서 섭동법을 이용하여 비선형 운동방정식의 불규칙 진동에 대한 해를 구함으로 해석과정이 축소되어진다. 제안된 방법의 적합성과 유효성을 평가하기 위하여 비선형성을 가진 기계구조 시스템을 해석하였다. 이 해석결과는 불규칙 진동 응답을 해석하는데 유효한 접근방법으로 판단되며 내진 설계에 기여할 것으로 예상된다.

주요어 : 비선형구조시스템, 지진, 추계적 문제, 내진설계, 확률론적 해석

ABSTRACT

Industrial machines are sometimes exposed to the danger of earthquake. In the design of a mechanical system, this factor should be accounted for from the viewpoint of reliability. A method to analyze a complex nonlinear structure system under random excitation is proposed. First, the actual random excitation, such as earthquake, is approximated to the corresponding Gaussian process for the statistical analysis. The modal equations of overall system are expanded sequentially. Then, the perturbed equations are synthesized into the overall system and solved in probabilistic way. Several statistical properties of a random process that are of interest in random vibration applications are reviewed in accordance with nonlinear stochastic problem. The obtained statistical properties of the nonlinear random vibration are evaluated in each substructure. Comparing with the results of the numerical simulation proved the efficiency of the proposed method.

Key words : nonlinear structure system, earthquake excitation, stochastic problem, seismic design, statistical method

1. Introduction

In recent years, the trend in mechanical systems has been toward high-speed and lightweight ones in many industrial machines. These conditions can cause trouble of a nonlinear vibration in mechanical systems. Hence, it has become important to consider the nonlinear characteristics in vibration analysis, design of the structure system. For a nonlinear system, exact solutions are generally not possible and approximate solutions can be obtained numerically. This method can be extremely time-consuming and is not practical, especially for large DOF system. Therefore, Iwatsubo et al.[(1),(2)] have proposed a new method to analyze the vibration of a nonlinear mechanical system. Moon et al.[(3)] have reported study on the vibration of mechanical system to analyze the dynamic problems of nonlinear MDOF systems. They developed the SSM(substructure synthesis method) technique to reduce the overall size of the problem for the nonlinear structure, and obtained approximate solutions of the nonlinear system using a perturbation method. On the other hand, it is necessary that a high-speed rotating system used for the jet engine of an aircraft, power plant turbine, etc. promptly pass a critical speed. Accordingly, the casing is often modeled elastically to decrease the critical speed. When random excitation excites such a mechanical system, the casing is excited to contact with the rotor and there is a danger that the bearing will be damaged. Therefore, the investigation of the random response of rotating machinery is very important from the viewpoint of disaster protection. Soni and Srinivasan[(4)] have reported the earthquake analysis of rotor system using the response spectrum method and time response method in deterministic system. Matsushita et al.[(5)] have reported seismic analysis of a rotor system. They used the real earthquake data to analyze the linear response. However, the vibration of nonlinear rotor systems under a random excitation force has been investigated few so far. Moreover, the vibration analysis of a nonlinear rotor-bearing-casing system utilizing a statistical approach to a seismic wave is not found in past research. Therefore,

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this paper proposes an analytical method for nonlinear vibration of mechanical system against a random excitation by applying the statistical method.

2. Method of analysis

2.1 Nonlinear system excited by earthquake

For the simple explanation, a single DOF vibratory system with the nonlinear restoring force of the system, which is excited by earthquake, is considered. The nonlinear equation of motion can be expressed in higher order terms of the displacement $\varepsilon\omega_0^2 x^3$ as

$$\ddot{x} + 2\varsigma\omega_0\dot{x} + \omega_0^2 x + \varepsilon\omega_0^2 x^3 = \ddot{X}_0 \tag{1}$$

where ς, ω_0 are damping ratio and natural frequency, respectively. ε is a small parameter. $\ddot{X}_0$ is earthquake excitation, which has spectrum density $S_N(\Omega)$. The excitation $\ddot{X}_0$ is regarded as a random process; hence, it is extremely difficult to obtain an exact solution $x(t)$. Thus, solutions can be obtained approximately. It should be noted that the solution itself for random inputs is not the ultimate goal in stochastic analysis of a nonlinear system. Meaning of the stochastic analysis such as Eq. (1) is to decide the statistic information of displacement x. Generally, the statistic characteristic of random process is decided from the PDF (probability density function) and PSD(power spectrum density) function of the system. Accordingly, for the probabilistic analysis of nonlinear random response, PSD and PDF of excitation forces should be obtained. To elaborate, a nonlinear system with the inputs, which are assumed to be a Gaussian random process, is considered. Because of the nonlinear characteristic, the output is no longer a Gaussian random process. Hence, the statistic characteristic of its vibration cannot be evaluated easily. Therefore, an adequate method to evaluate the statistical properties of the response of a nonlinear structure system should be developed. For this reason, the earthquake excitation needs to be approximated to Gaussian stationary process by reasonable procedures. Thus, an adequate approach to handle the earthquake as random process is introduced hereafter. A typical record of earthquake-induced ground acceleration is shown in Fig. 1. Treating this graph as a sample function of the underlying stochastic process, it is clear that ground acceleration is inherently non-stationary.[6] However, if the principal shock duration T_s, as indicated in Fig. 1, is limited to the period corresponding to the strong-motion portion over which the peak structural response occurs, the process can be regarded as a stationary random process from the viewpoint of the amplitude envelope of earthquake graph. A simple stationary representation of earthquake-induced acceleration can be expressed based on the study of frequency content of a number of strong ground-motion records.[6] Input PSD of the absolute ground acceleration can be expressed as

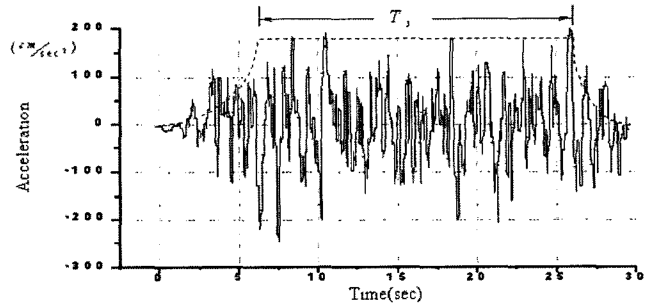


Fig. 1 Earthquake excitation

$$S_N(\Omega) = \frac{\omega_g^4 + 4\varsigma_g^2\omega_g^2\Omega^2}{2\left(\omega_g^2 - \Omega^2\right)^2 + 4\varsigma_g^2\omega_g^2\Omega^2} S_0 \tag{2}$$

where ω_g, ς_g and S_0 are a dominant frequency, damping ratio of filter and spectrum intensity of random process, respectively. ω_g, ς_g and S_0 are parameters to be determined from the earthquake records. To observe the effect of earthquake and the response of the system, a formulation of PSD function is introduced as

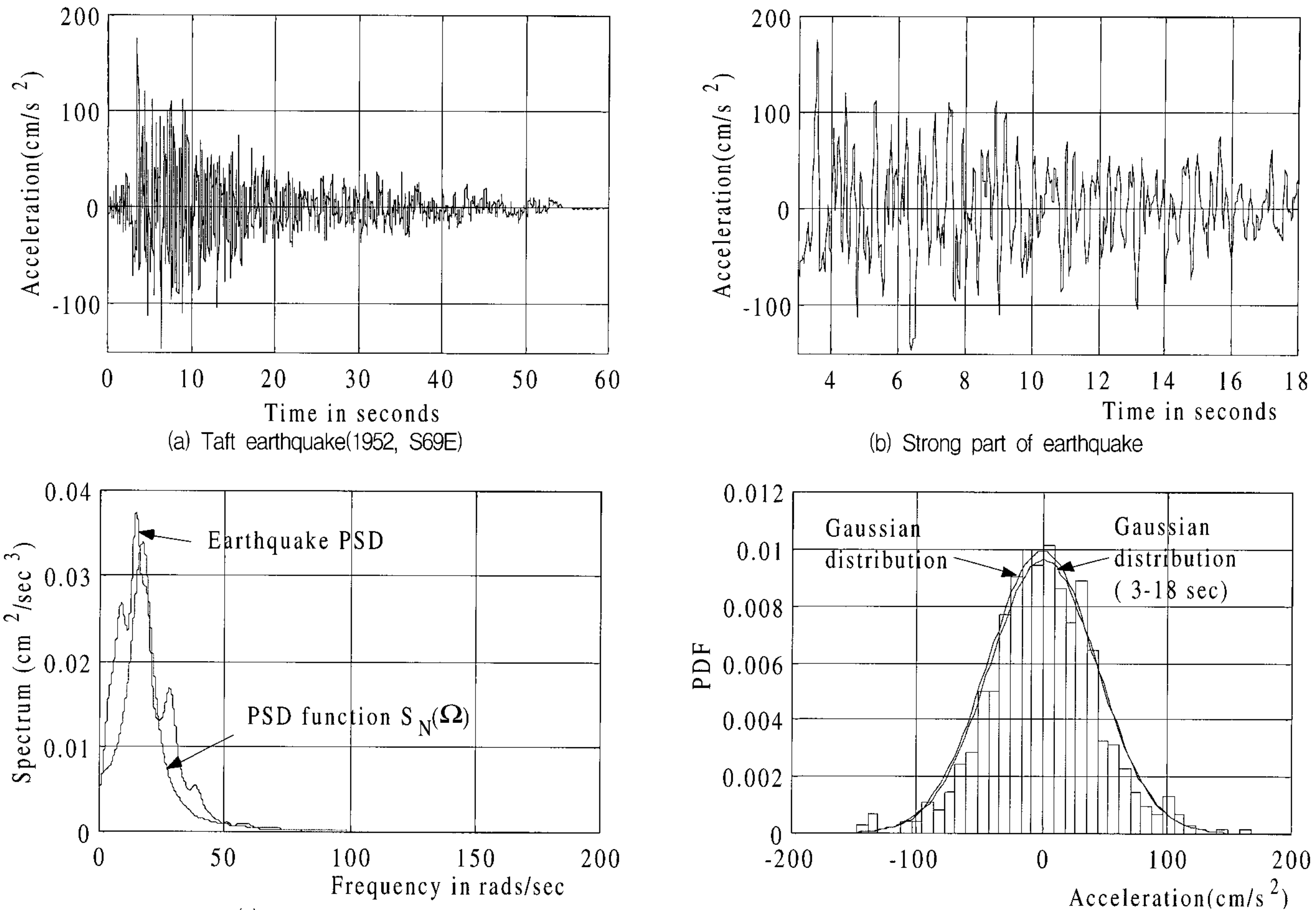
$$S_{N\alpha}(\Omega) = S_N(\Omega)\alpha \tag{3}$$

where α is a maximum value of earthquake input.

For instance, Fig. 2 shows Taft earthquake(1952) and its PSD and PDF, which is regarded as stationary narrow band process(ς_g = 0.41, $\omega_g = 18.75\,\mathrm{rad/sec}$, $\alpha = 1.75\,\mathrm{m/sec^2}$, S_0= 0.0132$\mathrm{m^2/sec^4/Hz}$). Fig. 2(b) shows the corresponding erratic earthquake PSD and the PSD function $S_{N\alpha}(\Omega)$, which shows good agreement. The erratic earthquake PSD is obtained from the part of earthquake data during 3~18 seconds, which can be regarded as stationary process from the viewpoint of the amplitude envelope of earthquake graph. From the Fig. 2(b), it is estimated that $S_{N\alpha}(\Omega)$ can be applicable to solve the response statistically. PDF of earthquake input is obtained from the part of earthquake during 3~18 seconds, which shows a Gaussian distribution, as shown in Fig. 2(d). The part of the earthquake during 3~18 seconds has the statistic properties as mean(= -0.0062). Thereby, the part of earthquake excitation process can be regarded as Gaussian, stationary random process with mean zero.

2.2 Response analysis by perturbation method

According to the procedure described above, the random



(a) Taft earthquake(1952, S69E)

(b) Strong part of earthquake

(c) PSD function

(d) PSD function

Fig. 2 Taft earthquake(1952)

excitation is expressed by Gaussian stationary process. Then, the nonlinear equation of motion reinstate with approximated Gaussian process.

$$\ddot{x}+2\varsigma\omega_0\dot{x}+\omega_0^2 x+\varepsilon\omega_0^2 x^3=W(t) \qquad (4)$$

The excitation $W(t)$ is a Gaussian stationary narrow band random process with zero-mean, which has the spectral density function $S_N(\Omega)$. $x(t)$ is the stationary response of a linearly damped Duffing system subjected to a Gaussian process excitation. Here, $x(t)$ is not Gaussian process, generally. However, if the system is lightly damped and if the nonlinearity is small(that is, $\varepsilon \leq 1$), then $x(t)$ is still expected to be a narrow band random process, as shown in Fig. 3($\varepsilon=0.3$, $\varsigma=0.1$, $\omega=5.23$).

The response has the mean(= -0.0044). PDF of nonlinear response shows a Gaussian distribution with mean zero. Thus, the response of Eq. (4) can be analyzed approximately by applying the perturbation theory. In Eq. (4), the small parameter ε can be perturbed as $x=x_0+\varepsilon x_1$.

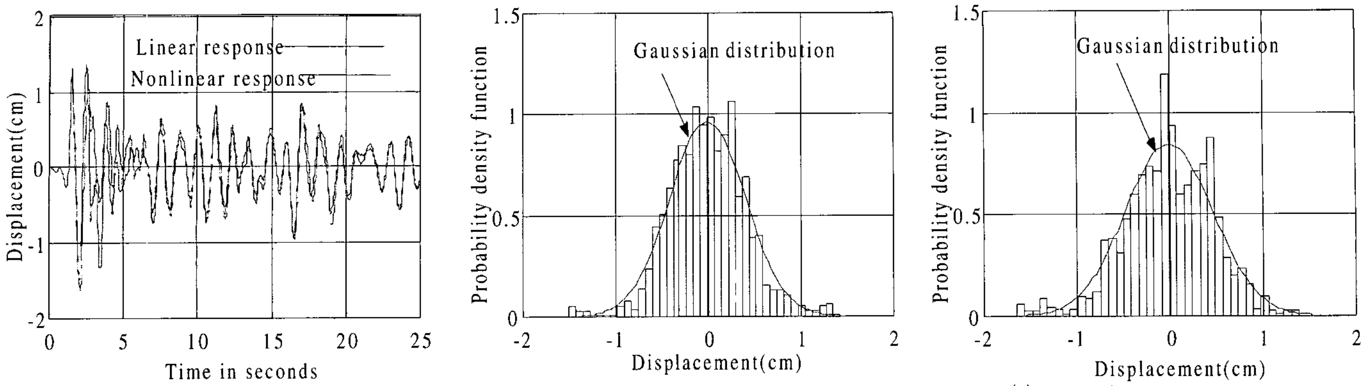
$$\ddot{x}_0+2\varsigma\omega_0\dot{x}_0+\omega_0^2 x_0=W(t) \qquad (5)$$

$$\ddot{x}_1+2\varsigma\omega_0\dot{x}_1+\omega_0^2 x_1=f_p(x_0) \qquad (6)$$

where $f_p(x_0)=-\varepsilon\omega_0^2 x_0^3$. From Eq. (5), (6), the approximating functions x_0, x_1 can be obtained sequentially. The zeroth order approximation x_0 is Gaussian process. Its probabilistic characteristics can be obtained by classical methods of linear random vibration. However, the characterizaticn of x_1 is less simple because the input is now a non-Gaussian process whose mean and covariance function are not generally available in a closed form. Moreover, the determination of the second moment characterization of the approximate solution $x=x_0+\varepsilon x_1$ also requires the correlation function of x_0, x_1, which is not readily available. In handling this problem, the objective is the determination of the stationary mean and covariance function of the first order approximation, as given in Eq. (6). Covariance function becomes due to its stationary characteristic,

$$E\{x(t+\tau)x(t)\}=R(\tau)=E\{x_0(t+\tau)x_0(t)\}$$
$$+\varepsilon E\{x_0(t+\tau)x_1(t)\}+\varepsilon E\{x_1(t+\tau)x_0(t)\} \qquad (7)$$

where each term can be obtained from the random vibration



(a) Time response (b) PDF of nonlinear response (c) PDF of linear response

Fig. 3 Random response

of linear system as

$$E\{x_0(t+\tau)x_0(t)\} = \int_0^\infty \int_0^\infty E\{W(t+\tau-\theta_1)W(t-\theta_2)\}h(\theta_1)h(\theta_2)d\theta_1 d\theta_2 \quad (8)$$

$$E\{x_0(t+\tau)x_1(t)\} = -\int_0^\infty \int_0^\infty E\{W(t+\tau-\theta_1)f_p(t-\theta_2)\}h(\theta_1)h(\theta_2)d\theta_1 d\theta_2 \quad (9)$$

$$E\{x_1(t+\tau)x_0(t)\} = -\int_0^\infty \int_0^\infty E\{W(t-\theta_1)f_p(t+\tau-\theta_2)\}h(\theta_1)h(\theta_2)d\theta_1 d\theta_2 \quad (10)$$

where $h(\theta)$ is the impulse response function corresponding to $\varepsilon = 0$. The determination of the expectations in Eq. (8), (9), (10) involves lengthy but straightforward calculations of expectations of polynomials in Gaussian variables. Then, the spectral density of the nonlinear response is obtained by taking the Fourier transform of Eq. (7).

$$S_{xx}(\Omega) = S_N(\Omega)|H(\Omega)|^2 \left[1 - 6\varepsilon\omega_0^2\sigma_{x_0}^2 \operatorname{Re}\{H(\Omega)\}\right] \quad (11)$$

where $\sigma_{x_0}^2$ is the stationary variance of the linear response $x_0(t)$. $H(\Omega)$ is frequency response function of linear equation between the excitation and the displacement of response. The corresponding variance can be obtained from the covariance $R(\tau)$ of the system by letting $\tau = 0$, which is the same value of the mean square response of the nonlinear vibration. Since the mean response, $E[x(t)]$, is zero, the variance is equal to the second moment $E[x(t)]$

$$E[x^2(t)] = R(0) = \sigma_{x_0}^2 \left[1 - 6\varepsilon\omega_0^2 \int_0^\infty \{R(\tau)h(\tau)\}d\tau\right] \quad (12)$$

The mean square value of the linear response in terms of the system response function and the spectral density of the input random process can be obtained as

$$\sigma_{x_0}^2 = \int_0^\infty S_N(\Omega)|H(\Omega)|^2 d\Omega \quad (13)$$

This procedure is applied to analyze the complex multi-DOF nonlinear system.

2.3 Modeling of nonlinear rotor system

For the analysis, the system shown in Fig. 4 is considered as a mechanical system. The rotor has nonlinearity with respect to its material property. For the dynamic analysis of complex systems, the SSM can be applied. The overall system is divided into three components, i.e., the rotor is the nonlinear component, the casing is the linear component, and the bearing is the assembling component. The coordinate system of the rotor-bearing-casing system is shown in Fig. 4. The $O_r - X_r Y_r Z_r$ coordinate system is fixed in rotor, such that the origin coincides with the center of the shaft where the X_r −axis is vertically upwards, the Y_r − axis is horizontal and perpendicular to the shaft, and the Z_r − axis is along shaft. The $O_c - X_c Y_c Z_c$ coordinate system is fixed in casing. The $O_0 - X_0 Y_0 Z_0$ is an absolute coordinate system, which is fixed in basement. The $U_r(= X_r - X_c, Y_r - Y_c, Z_r - Z_c)$ is a relative displacement between rotor and casing. The $U_c(= X_c - X_0, Y_c - Y_0, Z_c - Z_0)$ is a relative displacement between casing and basement. $\ddot{X}_0$ is an acceleration of the earthquake input.

The shaft and casing components are modeled using

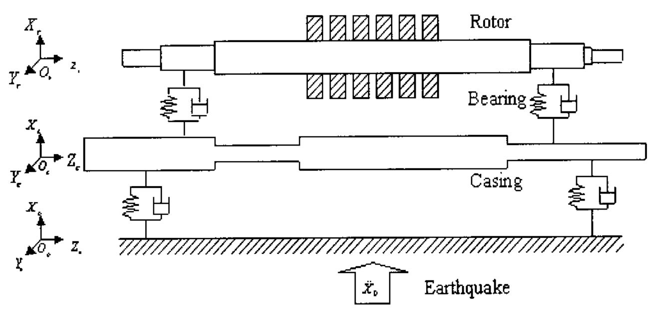


Fig. 4 Mechanical system for analysis

FEM. When the rotor is modeled by FEM, the nonlinear stiffness term of the rotor is determined in approximately. Then, the nonlinear restoring force is formulated in a complex form including nonlinear coupling terms of displacement. However, for the first stage of the analysis, it is considered that the type of nonlinear restoring force $\{R_e\}$, which is represented as[(1),(2)]

$$\{R_e\}=[^1K_e]\{^1X_e\}+\varepsilon[K_{Ne}]\{^1X_e^3\},$$
$$\{^1X_e\}=\{x_r,\theta_{rx},y_r,\theta_{ry}\}^T,\ \{^1X_e^3\}=\{x_r^3,\theta_{rx}^3,y_r^3,\theta_{ry}^3\}^T \qquad (14)$$

where $[^1K_e]$, and $[K_{Ne}]$ are the stiffness matrix and nonlinear stiffness term of the element, respectively. ε is a small parameter. The superscript 1 denotes the nonlinear component. $(\bullet)^T$ denotes the transpose of $(\bullet)$. The external force, which acts on the rotor, is considered as the unbalance force and the earthquake acceleration. Internal force is taken into consideration because the nonlinear component can be synthesized through the internal force with the other components. By considering the boundary conditions, the equation of motion for the nonlinear component can be written as[(3),(4)]

$$[^1M]\{^1\ddot{U}_r\}+[^1K]\{^1U_r\}+\varepsilon[K_N]\{^1U_r^3\}$$
$$=\{^1F_u(t)\}+\{^1F_E\}+\{^1F_b\} \qquad (15)$$

where $[^1M]$ is a mass matrix. $\{^1F_u(t)\}$ is an external force vector by unbalance of rotor. $\{^1F_b\}$ is an internal force vector. $\{^1F_E\}$ is an external force against the earthquake acceleration

$$\{^1F_E\}=-[^1M]\{I\}\{\ddot{X}_0\} \qquad (16)$$

where $\{I\}$ is a vector which shows the direction of the earthquake. In order to apply modal analysis, modal coordinate system is introduced by using the modal matrix $[^1\Phi]$ of the only linear system. Then, the displacement $\{^1U_r\}$ in physical coordinate system can be transformed into the modal displacement approximately as[(3),(4)]

$$\{^1\ddot{\xi}\}+[^1\omega^2]\{^1\xi\}+\varepsilon[^1k_N]\{^1\xi^3\}$$
$$=\{^1f_u(t)\}+\{^1f_E\}+\varepsilon\{^1f_b\} \qquad (17)$$

where $\{^1f_u(t)\}(=[^1\Phi]^T\{^1F_u(t)\})$ and $\varepsilon\{^1f_b(t)\}(=[^1\Phi]^T\{^1F_b\})$ are the external and internal forces in modal coordinates. $\{^1f_E\}(=[^1S]\{\ddot{X}_0\}$, $[^1S]$ is a mode-participation factor of its mode) is the external force against earthquake in modal coordinates. $\varepsilon[^1k_N]$ is the nonlinear term in modal coordinates. According to the previous section, Eq. (17) can be expressed with approximated Gaussian stationary random process $W(t)$ as

$$\{^1\ddot{\xi}\}+[^1\omega^2]\{^1\xi\}+\varepsilon[^1k_N]\{^1\xi^3\}=\{^1W(t)\}+\varepsilon\{^1f_b\} \qquad (18)$$

Here, the perturbation method is introduced to solve the nonlinear equation. In Eq. (18), the small variant $\varepsilon[^1k_N]$ can be regarded as the perturbation parameter term, because the variant $\varepsilon[^1k_N]$ is small relative to $[^1\omega^2]$. The dynamic responses $\{^1\xi\}$ can be expanded in terms of a series of the small parameter ε expressed as

$$\{^1\xi\}=\{^1\xi^{(0)}\}+\varepsilon\{^1\xi^{(1)}\}+\varepsilon^2\{^1\xi^{(2)}\}+\cdots, \qquad (19)$$

where superscripts denote the perturbation order. Then, the perturbed equations are evaluated as

$$\{^1\ddot{\xi}^{(0)}\}+[^1\omega^2]\{^1\xi^{(0)}\}=\{^1W^{(0)}\}+\{^1f_b^{(0)}\},$$
$$\{^1\ddot{\xi}^{(1)}\}+[^1\omega^2]\{^1\xi^{(1)}\}=\{^1f_p(^1\xi^{(0)})\}+\{^1f_b^{(1)}\} \qquad (20)$$

where $\{^1f_p\}$ includes the nonlinear stiffness term. $\{^1W^{(0)}\}$ is exciting Gaussian narrow band stationary random process. $\{^1f_b^{(0)}\}$ and $\{^1f_b^{(1)}\}$ are perturbed internal forces expressed by modal displacements. $\{^1f_p(^1\xi^{(0)})\}=-[^1k_N]\{^1\xi^{(0)3}\}$.

2.4 Equations of motion of an assembled system

The casing is modeled as linear component and the equation of motion is obtained readily

$$[^2M]\{^2\ddot{U}_c\}+[^2K]\{^2U_c\}=\{^2F_E\}+\{^2F_b\} \qquad (21)$$

where $[^2M]$ and $[^2K]$ are the mass and stiffness matrix, respectively. $\{^2F_b\}$ is the internal force vector. $\{^2F_E\}(=-[^2M]\{I\}\{\ddot{X}_0\})$ is an earthquake external force term. After the eigenvalue analysis, the equation of motion is changed into modal coordinate($\{^2U_c\}\equiv[^2\Phi]\{^2\xi\}$). The internal force is introduced in the equation because the linear component can be synthesized through the internal force with the other components. Even the casing component is linear system, this component is perturbed as same as the nonlinear component, because the higher order harmonic oscillation which is occurred in the nonlinear component is translated through the higher order perturbed equation as[(4)]

$$\{^2\ddot{\xi}^{(0)}\}+[^2\omega^2]\{^2\xi^{(0)}\}=\{^2W^{(0)}\}+\{^2f_b^{(0)}\},$$
$$\{^2\ddot{\xi}^{(1)}\}+[^2\omega^2]\{^2\xi^{(1)}\}=\{^2f_b^{(0)}\} \qquad (22)$$

where $\{^2f_b^{(0)}\}$ and $\{^2f_b^{(1)}\}$ are the perturbed internal forces. $\{^2W^{(2)}\}$ is the external force term in modal coordinate system, which is expressed with approximated Gaussian stationary random process $W(t)$. To apply the SSM, as an assembling component, simple linear ball bearings are considered. Generally, there is a damping term in the bearing, but it is ignored in this study. The restoring force of the bearing is modeled as the linear term, where the force and displacement are expressed in matrix form as

$$[{}^1k_{b1}]\{{}^1U_{rb}\}=\{{}^1f_b\},\ [{}^2k_{b2}]\{{}^2U_{rb}\}=\{{}^2f_b\} \tag{23}$$

where $[{}^jk_{bj}]$ $(j=1,2)$ are the terms of bearing coefficient. $\{{}^1f_b\}$, $\{{}^2f_b\}$ are the internal force vectors of the nonlinear component and linear component, respectively. $\{{}^jU_{rb}\}$ is a relative displacements between the rotor and casing corresponding to the bearings. In order to obtain the overall equation, the perturbation parameter ε of the nonlinear component is introduced. Then, the displacement is expressed as

$$\{{}^jU_{rb}\}=\{{}^jU_{rb}^{(0)}\}+\varepsilon\{{}^jU_{rb}^{(1)}\}\quad (j=1,2) \tag{24}$$

Accordingly, by using Eq. (24), the internal force vectors can be perturbed as

$$\{{}^jf_b\}=\{{}^jf_b^{(0)}\}+\varepsilon\cdot\{{}^jf_b^{(1)}\} \tag{25}$$

In order to synthesize the components, Eqs. (20), (22) and (25) are combined and rewritten according to the perturbation order. The equation of order $\varepsilon^{(l)}$ $(l=1,2)$ can be expressed as

$$\{\ddot{\xi}^{(l)}\}+[\bar{K}^{(l)}]\{\xi^{(l)}\}=\{F^{(l)}(t)\} \tag{26}$$

$\{\xi^{(l)}\}=\left\{\{{}^1\xi^{(l)}\}^T,\{{}^1U_{rb}^{(l)}\}^T,\{{}^2U_{rb}^{(l)}\}^T,\{{}^2\xi^{(l)}\}^T\right\}^T$ $\{F(t)^{(l)}\}=\left\{\{{}^1W^{(l)}\}^T,\{-{}^1f_b^{(l)}\}^T,\{{}^2f_b^{(l)}\}^T,\{{}^2W^{(l)}\}^T\right\}^T$, and $[\bar{K}^{(l)}]$ is the stiffness matrix of the overall system which is composed of all of the components. The overall system is obtained by assembling those component equations. The equation of order $\varepsilon^{(l)}$ is obtained as

$$\begin{Bmatrix} {}^1\ddot{\xi}_i^{(l)} \\ {}^2\ddot{\xi}_i^{(l)} \end{Bmatrix}+\begin{bmatrix} [{}^1\omega_i^2]+[a_1] & [a_2] \\ [a_3] & [{}^2\omega_i^2]+[a_4] \end{bmatrix}\begin{Bmatrix} {}^1\xi_i^{(l)} \\ {}^2\xi_i^{(l)} \end{Bmatrix}=\begin{Bmatrix} {}^1f_\eta^{(l)} \\ {}^2f_\eta^{(l)} \end{Bmatrix} \tag{27}$$

where $[a_1]=[\phi_{b1}]^T[{}^1k_{b1}][\phi_{b1}]$, $[a_2]=[\phi_{b1}]^T[{}^2k_{b1}][\phi_{b2}]$, $[a_3]=[\phi_{b2}]^T[{}^1k_{b2}][\phi_{b1}]$, $[a_4]=[\phi_{b2}]^T[{}^2k_{b2}][\phi_{b2}]$ The external force term is $\begin{Bmatrix} \{{}^1f_\eta^{(l)}\} \\ \{{}^2f_\eta^{(l)}\} \end{Bmatrix}=\begin{Bmatrix} [\phi_{a1}]^T\cdot\{{}^1W^{(0)}\}^T \\ [\phi_{a2}]^T\cdot\{{}^2W^{(0)}\}^T \end{Bmatrix}[\phi_{bi}]$ $(i=1,2)$, the eigenvector matrix of the assembling region, which is derived from the eigenvector of each substructure corresponding to its bearing. The external force term of order $\varepsilon^{(1)}$ is obtained as

$$\begin{Bmatrix} \{{}^1f_\eta^{(1)}\} \\ \{{}^2f_\eta^{(1)}\} \end{Bmatrix}=\begin{Bmatrix} [\phi_{a1}]^T\cdot\{-[{}^{\backslash 1}k_{N\backslash}]\{{}^1\xi^{(0)3}\}\}+[\phi_{b1}]^T\cdot\{{}^1f_b^{(1)}\} \\ [\phi_{a2}]^T\cdot\{0\}+[\phi_{b2}]^T\cdot\{{}^2f_b^{(1)}\} \end{Bmatrix}.$$

2.5 Statistical properties of nonlinear response

The earthquake is used as the excitation wave, which is regarded as the Gaussian stationary random process, by considering the strong motion duration. After the eigenvalue analysis of the overall system with Eq. (27), the order $\varepsilon^{(l)}$ coordinate $\{\eta^{(l)}\}$ of the overall system is introduced for modal analysis as

$$\{\eta^{(0)}\}=[\Phi_t]\{\eta^{(0)}\},\ \{\eta^{(1)}\}=[\Phi_t]\{\eta^{(1)}\} \tag{28}$$

$[\Phi_t]$ is the eigenvector matrix of the overall system. The equation of the motion of order $\varepsilon^{(0)}$ is

$$\ddot{\eta}_i^{(0)}+2\zeta\omega_{ti}\dot{\eta}_i^{(0)}+\omega_{ti}^2\eta_i^{(0)}=W_{\eta i}^{(0)},(i=1,2,3,\cdots,n) \tag{29}$$

ω_{ti}^2 is the eigenvalue of the overall system. $W_{\eta i}^{(0)}$ is the external force term in modal coordinates. The damping of the system is assumed to be the proportional damping of the eigenvalue. According to the linear random vibration theory, the solution $\eta_i^{(0)}(t)$ of the linear differential equation may be readily obtained. Then, the equation of the motion of order $\varepsilon^{(1)}$ can be described as

$$\ddot{\eta}_i^{(1)}+2\zeta\omega_{ti}\dot{\eta}_i^{(1)}+\omega_{ti}^2\eta_i^{(1)}=f_{\eta i}^{(1)}(\eta_i^{(0)}),\ (i=1,2,\cdots,n) \tag{30}$$

where $f_{\eta i}^{(1)}(\eta_i^{(0)})(=-\beta_i^2\eta_i^{(0)3})$ is the external force term. β_i^2 is the nonlinear coefficient. The response is

$$\eta_i^{(1)}(t)=-\beta_i^2\int_0^\infty \eta_i^{(0)3}(t-\tau)h_i(\tau)d\tau \tag{31}$$

$h_i(t)$ is the impulse response function of the linear system. Accordingly, the response of a nonlinear system can be evaluated by perturbation theory as

$$\eta_i=\eta_i^{(0)}+\varepsilon\eta_i^{(1)} \tag{32}$$

The equations of $\eta_i^{(0)}$, $\eta_i^{(1)}$ can be used to compute various statistical properties of the response. The covariance of the nonlinear response, computed to the first order of ε, can be obtained as

$$R_{\eta i}(\tau)=\int_0^\infty\Big\{\tfrac{1}{2}S_{Ni}^{(0)}(\Omega)\left|H_i(\Omega)\right|^2 -\tfrac{3}{2}\varepsilon\sigma_{\eta i^{(0)}}^2S_{Ni}^{(0)}(\Omega)\left|H_i(\Omega)\right|^2H_i^*(\Omega)\cos\Omega\tau\Big\}d\Omega \tag{33}$$

where $H_i^*(\Omega)$ is conjugate function of $H_i(\Omega)$. Then, the spectral density $S_{\eta i}(\Omega)$ of the nonlinear response is obtained by taking the Fourier transform of the covariance function as

$$S_{\eta i}(\Omega) = S_{Ni}^{(0)}(\Omega)\left|H_i(\Omega)\right|^2\left[1-6\varepsilon\beta_i^2\sigma^2_{\eta i^{(0)}}\,\mathrm{Re}\left[H_i(\Omega)\right]\right] \quad (34)$$

where $\mathrm{Re}[H_i(\Omega)]$ is the real part of $H_i(\Omega)$. The corresponding variance can be obtained from the covariance $R_{\eta i\,\eta i}(\tau)$ of the system by letting $\tau=0$, which is the same value of the mean square response of the nonlinear vibration

$$\sigma^2_{\eta i} = \sigma^2_{\eta i^{(0)}}\left[1-6\varepsilon\beta_i^2\int_0^\infty\{R_{\eta i}(\tau)h_i(\tau)\}d\tau\right] \quad (35)$$

The stationary variance $\sigma^2_{\eta i^{(0)}}$ is the mean-square value of the linear response. Examining $\sigma^2_{\eta i}$, it appears that if the system is nonlinear with light damping, weak nonlinearity and the excitation random process is Gaussian stationary, then the response spectral density, covariance function, and mean square value, can all be calculated from the knowledge of the spectral density $S_N(\Omega)$ of the excitation random process and the magnitude $|H(\Omega)|$ of the frequency response.

3. Numerical results

3.1 Model for Analysis

A nonlinear rotor system, which is shown in Fig. 4, is considered. The rotor is considered as a uniform beam and the casing is also considered as a uniform beam approximately for the simplicity of calculation. The casing is constrained to a foundation at both ends of the casing. As a support, ball bearing is considered for aircraft engine turbine or power plant turbine. The properties of the rotor system are tabulated in Table 1.

The rotor is modeled by the twenty beam elements and the casing is also modeled by the twenty beam elements. The modal damping ratios of the system is given by ζ = 0.05. The exciting force of the rotor by unbalance is assumed as $F_u(t) = F_0\Omega^2\cos(\Omega t+\phi)$ in x-direction, where F_0, ϕ are unbalance and phase angle, respectively. Rotating speed (Ω=540rad/sec) is assumed to be near first critical speed (ω_{n}=583rad/sec) of the rotor system. The unbalance of the rotor system is located at the middle of the shaft with a value of $0.044\,Kg\cdot m/(\frac{rad}{sec})^2$.

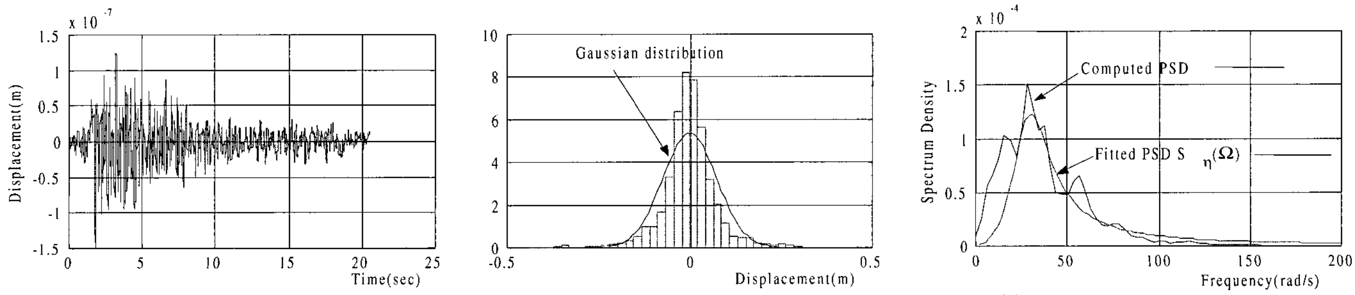
Table 1 Properties of the rotor system

Length of shaft L(mm)	800
Length of casing L(mm)	800
Diameter of shaft D_R(mm)	16
Diameter of casing D_P(mm)	50
Young's modulus of shaft, casing(N/m^2)	2.1×10^{11}
Density of shaft & casing ρ (kg/m^3)	7.81×10^3
Bearing coefficients k_b(N/m)	1.0×10^6
Nonlinear coefficient Y	0.1

3.2 Response of the nonlinear system

Linear response of the system against the random excitation is examined. The responses are computed by SSM with modal analysis procedure, which are shown in Fig. 5.

As can be noticed from Fig. 5(a), the response includes the earthquake response of the system. Probability density of the system is showed in Fig. 5(b) in accordance with relative amplitude of the system. Average value of response is 0.000186. These properties prove that the response is Gaussian stationary random process. In Fig. 5(c), the power spectrum of the response is shown. It can be observed that a typical earthquake response power spectrum is obtained, which has a low frequency component. The simulated PSD shows well fitted with the analytical response of PSD $S_\eta(\Omega)$, which is obtained with ε=0.0. The spectral density is related to the stationary variance $\sigma^2_\eta(0)$ = 0.0059, which is the same as the mean-square value of the linear response.



(a) Response of rotor (b) Gaussian distribution (c) Power spectrum density

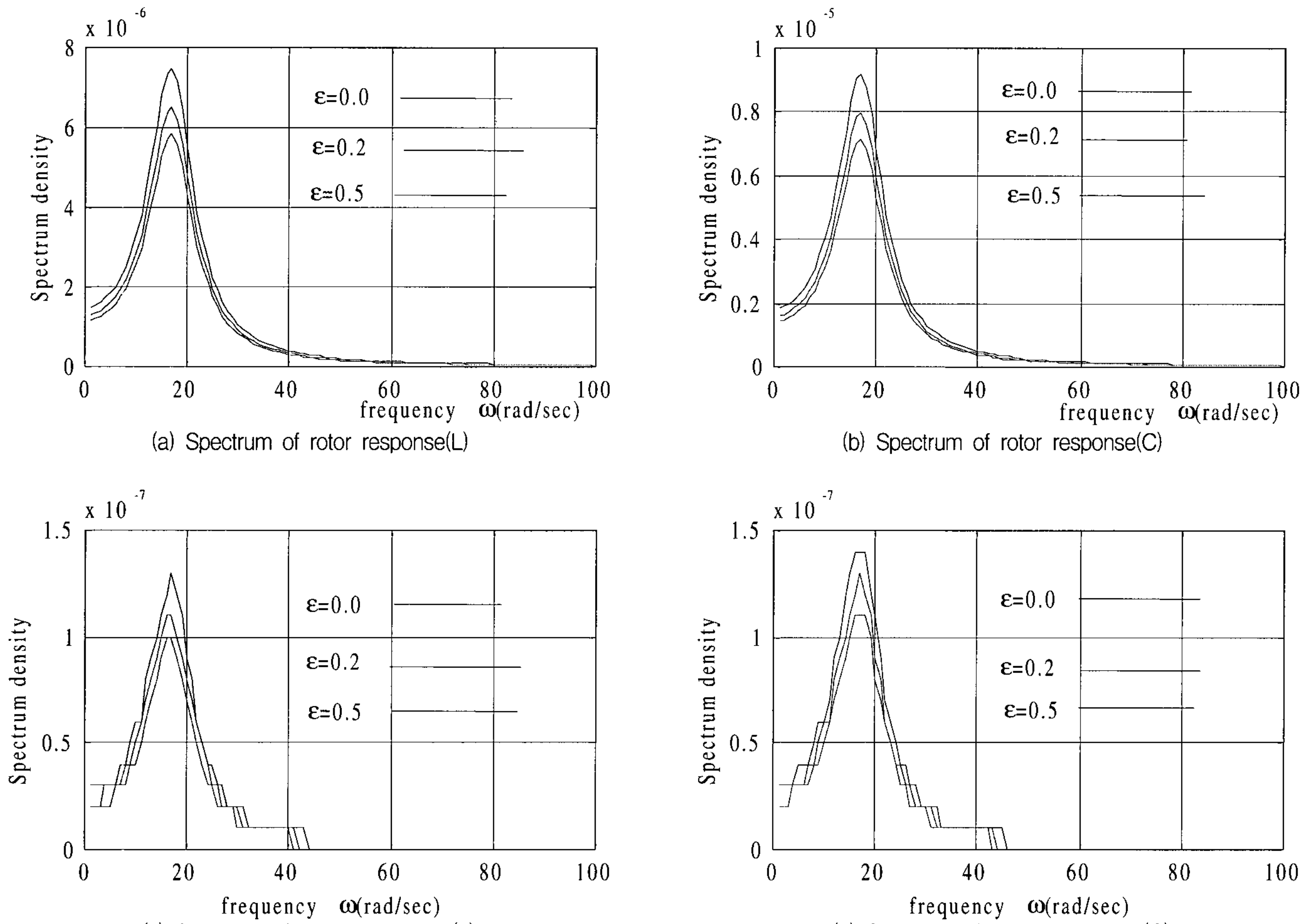
Fig. 5 Response and its probabilistic properties

Here, the statistical properties of nonlinear random vibration of the stochastic system verses the strong motion duration of excitation are investigated. Correlation, spectrum density and its variances, which are important to reliability analysis, are considered.

Those properties of nonlinear responses are calculated according to the procedure of nonlinear random vibration analysis, which is computed to the perturbation first order. Spectrum of harmonic excitation and earthquake excitation are obtained and applied to the system. It is regarded that the nonlinear response depends on the size of the perturbation parameter(ε), which shows nonlinear characteristics. To this end, two kinds of analysis are carried out, i.e., the perturbation parameters are set to ε=0.2, ε=0.5. In Fig. 6, the PSD of the nonlinear responses of the system at the middle of shaft and casing is showed with various perturbation parameters.

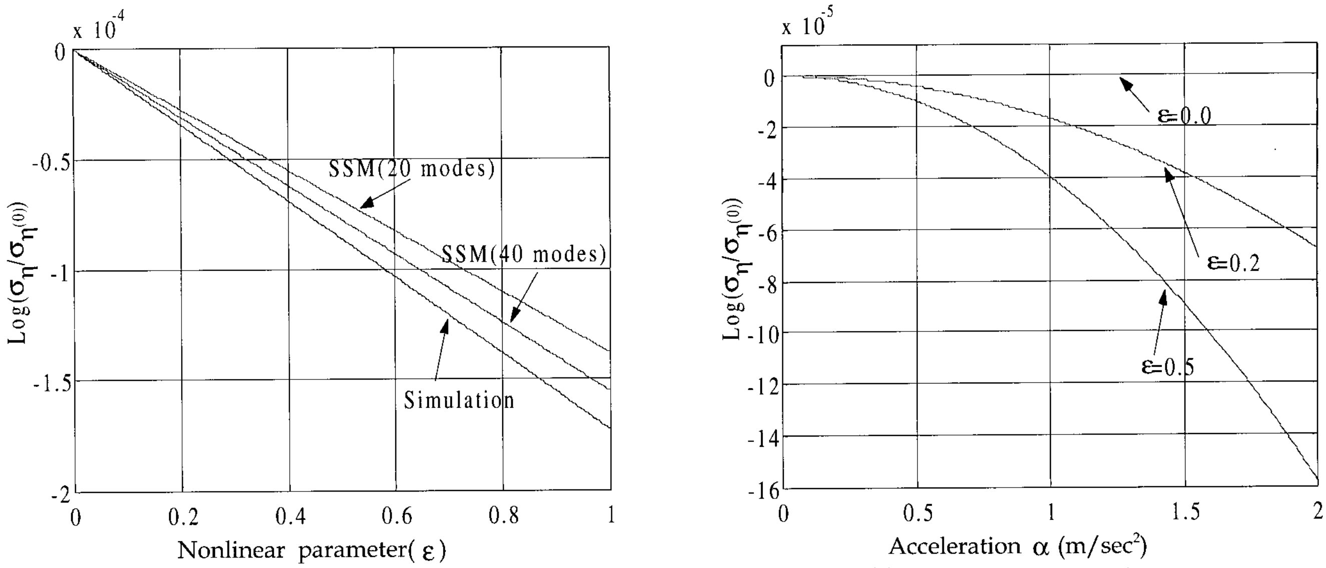
The response of SSM is calculated by taking 20 modes. When the nonlinear parameter is ε=0.0, the PSD shows a linear response. The each PSD of the system, such as rotor and casing, shows a characteristic of earthquake random process. Investigation of the PSD reveals that the PSD is smaller when the perturbation parameter becomes bigger, which is the nonlinear response in terms of the nonlinear characteristic. Variance of the nonlinear response is evaluated from the nonlinear response.

Fig. 7(a) shows the variance of the nonlinear response at the middle of rotor by changing its numbers of adopting mode 20, 40. The calculated variances are investigated for various values of nonlinear parameter. To prove the computing efficiency, those values are compared with the results of simulation. The response of simulation is calculated by direct integration the equation of motion against the excitation where the overall equation of motion of the system has 168 DOF. The rotor has 84 DOF because it has 21 nodes and there are 4 variables per mode. The casing also has 84 DOF so that the total DOF is 168. Investigation of the variance reveals that the value shows a decreases with ε in the spread of displacement about equilibrium point when ε=0. This is consistent with our intuition which suggests that stiffer systems exhibit smaller displacements, and with the observation that the system stiffness increases with nonlinear parameter. This result also reveals that the variance displacement of a hardening spring nonlinear



(a) Spectrum of rotor response(L)
(b) Spectrum of rotor response(C)
(c) Spectrum of casing response(L)
(d) Spectrum of casing response(C)

Fig. 6 Comparing the PSD with nonlinear parameters



(a) Variance with nonlinear parameters ε

(b) Variance with acceleration α

Fig. 7 Displacement variance of the system

system is always less than that of the corresponding linear system. Fig. 7(b) shows the variance of the nonlinear response at the middle of rotor by using its numbers of adopting mode 20.

The calculated variances are investigated for various values of nonlinear parameter against maximum values of exciting acceleration. To prove the nonlinear effect, those values are compared with the results of linear one, which are obtained by same calculation condition. It is observed that according to the input level, variance of the responses become large. When the nonlinear parameters become large, then the response changes a lot.

Next, the computing time to analyze the nonlinear random vibration verses the strong motion when the duration (=3～18 seconds) of excitation is compared. The variances for the values of the result to be compared are used, as shown in Fig. 7(a). The computer used in this analysis is a *LOGIX, IBM* personal computer. In the case of the numerical simulation, it takes 20 minutes 45 seconds. But for the proposed method; it takes 5 minutes 28 seconds, 7 minutes 30 seconds to obtain the result, when the number of adopting modes are 20,40, respectively. As a result, it can be shown that a drastic reduction in calculation time can be achieved, keeping its computing accuracy. This is an important factor in the analysis of structural dynamics against random excitation with a large number of degrees of freedom.

4. Conclusions

In this paper, the random vibration analysis method of a nonlinear stochastic system was theoretically formulated when the actual random excitation is regarded as a Gaussian stationary random process. The formulation is concerned with reducing the number of degrees of freedom for each component by modal substitution. All of the components are then assembled together and the random response of the overall system is analyzed statistically against earthquake excitation. It is shown that nonlinear random responses could be efficiently calculated according to the selected number of vibration modes. Several statistical properties of the random responses that are of interest in nonlinear random vibration applications are reviewed. The results reported herein provide a better understanding of the nonlinear random vibration. Moreover, it is believed that those properties of the results can be utilized in the dynamic design of the nonlinear stochastic system.

References

1. Iwatsubo, T., Kawamura, S., and Moon, B. Y., "Non-linear vibration analysis of rotor system using sub-structure synthesis method," *JSME International Journal Series C*, Vol. 41, No. 4, 1998, pp. 727-733.
2. Iwatsubo, T., Shimbo, K., Kawamura, S., and Moon, B. Y., "Nonlinear vibration analysis of rotor system using component mode synthesis method(Analysis using the perturbation method)," *Trans. Jpn. Soc. Mech. Eng.,(in Japanese) Series C*, Vol. 65, No. 637, 1999, pp. 3499-3506.
3. Moon, B. Y., Kim, J. W., and Yang, B. S., "Nonlinear vibration analysis of mechanical structure system using substructure synthesis method," *KSME International Journal*, Vol. 13, No. 9, 1999, pp. 620-629.
4. Soni, A. H. and Srinivasan, V., "Seismic analysis of a

gyroscopic mechanical system," *Trans. ASME, Journal of Vibration, Acoustics, Stress and Reliability in Design*, Vol. 105, 1983, pp. 489-455.

5. Matsushita, O., Takagi, M., and Kikuchi, K., "Seismic analysis of a rotor system," *Trans. Jpn. Soc. Mech. Eng. (in Japanese), Series C*, Vol. 49, No. 442, 1983, pp. 971-981.
6. Soong, T. T., *Random Vibration of Mechanical and Structural Systems*, MIRCEA GRIGORIU, pp. 64-65.