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Dynamic Analysis and Optimal Design of Engine Mount Systems with Consideration of Foundation Flexibility

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Abstract

Equations of motion of an engine mount system including foundation flexibility are derived. Forced vibration analysis is carried out for the given engine mount system excited with the unbalanced force and moment. A new optimal design method for the engine mount system is proposed, in which vibration characteristics of the chassis frame structure are considered as design parameters.

Keywords: *Engine mount systems, Flexible foundation, Optimal design, Vibration analysis*

I. Introduction

One of the important problems encountered in the automotive design is the reduction of engine vibrations and ultimately the dynamic forces transmitted from the engine to the body structure. Rubber mounts or hydraulic mounts have been used to connect the engine to the base structure to isolate the vibrations. In general, rubber mounts are used extensively to control noise and vibration in automotive fields.

Many researches in this area have been made, especially for analytical methods to predict dynamic characteristics and various optimal design techniques for engine mount systems[1-3]. In most of these researches, supporting structures had been assumed as rigid bodies, so that vibration effects of the supporting structures are neglected in dynamic analysis of engine mount systems. However, in real vehicle systems, there exist various kinds of vibration modes in foundation such as those of the suspension system, body structure, and any structural components. If engine and foundation motions are coupled together, coupling effect on dynamic characteristics may be enormous. Recently, researches of vibration analysis of engine mount systems with flexible supporting structure have been made[4,5].

In this paper, a computational method is presented for the accurate dynamic analysis of the engine mount system that is supported by flexible foundation. Equations of motion for the engine mount system including foundation flexibility are derived. Using the equations of motion for the engine mount system, the forced vibration analyses are carried out for the given engine mount system excited with the unbalanced force and moment. Design parameters such as stiffness, location, orientation angle of rubber mounts,

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and natural frequency of the foundation structure are optimally determined for the optimal design of the engine mount system.

II. Equations of Engine Mount System with Flexible Foundation

2.1. Equations of Engine Mount System

Figure 1 shows the configuration of a typical 3-point engine mount system with a supporting structure. The engine-transmission assembly is modeled as a 6-DOF rigid body; the rubber mounts are modeled as linear tranlational springs with a stiffness coefficient in each of the three principal directions; the supporting structure is modeled as a flexible body.

Engine-transmission assembly motion can be expressed with translational and rotational displacement vectors of its mass center, $\{x_{cg}\}$, $\{\theta_{cg}\}$. The translational displacements of the engine-transmission assembly at the mount position $\{x_m\}_i$ can be expressed by the displacement of the mass center and position vector of the mount point $\{r_m\}_i$ with respect to the engine mount system.

$$\begin{aligned}\{x_m\}_i &= \{x_{cg}\} + \{\theta_{cg}\} + \{r_m\}_i \\ &= \{x_{cg}\} + [T]_i\{\theta_{cg}\}\end{aligned} \quad (1)$$

where $[T]_i$ is a transformation matrix for the i-th mount as shown in Eq. (2).

$$[T]_i = \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix} \quad (2)$$

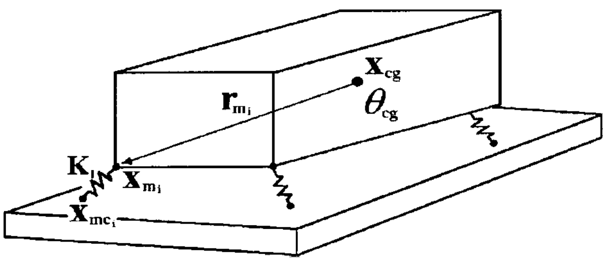

Figure 1. Configuration of an engine mount system with flexible supporting structure.

For a flexible foundation the forces acting on the mounts can be written as

$$\{f\}_i = [K]_i(\{x_m\}_i - \{x_{mc}\}_i) \quad (3)$$

where $[K]_i$ is the stiffness matrix of the i-th elastic mount, and $\{x_{mc}\}_i$ is the foundation displacement vector at the mount locations. Thus the equations of the engine mount system including the displacements of the supporting points can be expressed as

$$[M_e]\{\ddot{x}_e\} + \sum_i [K_e]_i\{x_e\} - \sum_i [K_{mc}]_i\{x_{mc}\}_i = \{f_e\} \quad (4)$$

where

$$\{x_e\}^T = \{x_{cg} \quad \theta_{cg}\}^T \quad (5)$$

$$[K_e]_i = \begin{bmatrix} K & KT \\ T^T K & T^T K T \end{bmatrix}_i \quad (6)$$

$$[K_{mc}]_i = \begin{bmatrix} K \\ T^T K \end{bmatrix}_i \quad (7)$$

In Eq. (4), $\{f_e\}$ is the engine excitation force vector, $[M_e]$ is the mass matrix composed of mass and mass moment of inertia about mass center, $[K_e]_i$ is the stiffness matrix of the engine mount system, and $[K_{mc}]_i$ is the stiffness matrix of the supporting structure, which appears when flexibility of the chassis structure is considered.

2.2. Equations of Supporting Structure

The supporting structure can be modeled using the finite element method. Dynamic flexibilities of the supporting structure are obtained in the form of modal mass, modal stiffness, and mode shape matrices by performing finite element analysis. The equations of the supporting structure can be written as follows.

$$\begin{aligned}&[M_g]\{\ddot{q}_c\} + [K_g]\{q_c\} + [u_C]^T[K_M][u_C]\{q_c\} \\ &- [u_C]^T[K_m]\{x_e\} = [u_C]\{f_c\}\end{aligned} \quad (8)$$

In Eq. (8), $[K_M]$ and $[K_m]$ have nonzero elements only at the mount positions as follows;

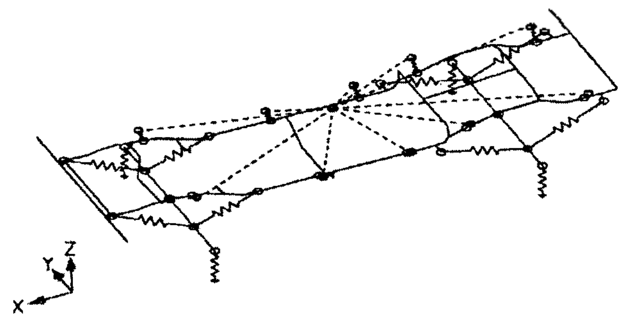

Figure 2. FE model of the supporting structure without engine-transmission assembly.

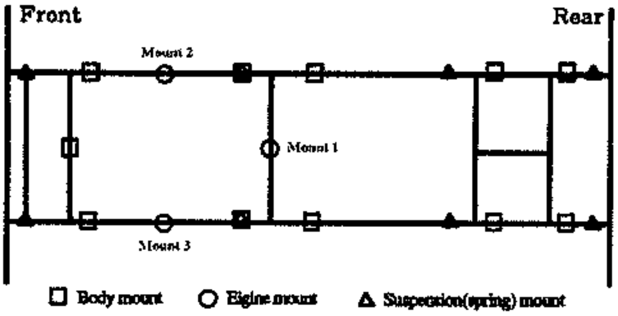

Figure 3. Mounting locations on the frame.

$$[K_M]=\sum_i \begin{bmatrix} 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & K_i & \cdot \\ 0 & \cdot & \cdot & 0 \end{bmatrix} \tag{9}$$

$$[K_m]=\sum_i \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ K^T_{mc_i} \\ \cdot \\ 0 \end{bmatrix} \tag{10}$$

And $[M_g]$ is a modal mass matrix, $[K_g]$ is a modal stiffness matrix, $\{q_c\}$ is a modal coordinate vector, $\{f_c\}$ is an excitation force vector of supporting structure, $[u_C]$ is a modal matrix.

2.3. Equations of Coupled Engine Mount System

By combining Eq. (4) and (8), the equations of the coupled engine mount system can be written as follows.

$$\begin{bmatrix} M_g & \\ & M_e \end{bmatrix}\begin{Bmatrix} \ddot{q}_c \\ \ddot{x}_e \end{Bmatrix} + \begin{bmatrix} K_g+u_c^TK_Mu_C & -u_c^TK_m \\ -K_m^Tu_c & K_e \end{bmatrix}\begin{Bmatrix} q_c \\ x_e \end{Bmatrix} = \begin{Bmatrix} u_c^Tf_c \\ f_e \end{Bmatrix} \tag{11}$$

In Eq. (11), f_e is a engine excitation force vector.

III. Vibration Analysis

3.1. Normal Mode Analysis

The specific engine mount system to be discussed in this paper is a passenger jeep car model with a 4-cylinder engine. Natural frequencies of the engine mount system with the chassis structure are obtained through vibration analysis for the computer simulation model. Figure 2

Table 1. Natural frequencies of the uncoupled system.

Unit: Hz

Mode number	Mode shape	Frequency
1	Sprung mass vertical	1.72
2	Sprung mass roll	2.39
3	Sprung mass roll (Body dominant)	3.39
4	Sprung mass pitch	5.01
5	Sprung mass yaw	8.49
6	Front axle vertical	10.18
7	Rear axle vertical	10.28
8	Body fore-aft	13.97
9	Rear axle roll	14.42
10	Front axle roll	14.79
11	Yaw + torsion	25.34
12	Torsion + bending	34.88
13	Front torsion	34.88
14	Front bending	37.89
15	Rear bending	42.38
16	Rear torsion	47.67
⋮	⋮	⋮
24	3rd bending	194.40

Table 2. Natural frequencies of the coupled system.

Unit: Hz, %

Mode number	MATLAB	NASTRAN	Percent error
1	1.544	1.543	-0.02
2	2.173	2.172	-0.06
3	3.205	3.204	-0.02
4	4.467	4.432	-0.79
5	4.494	4.474	-0.44
6	4.892	4.891	-0.03
7	6.419	6.410	-0.14
8	8.572	8.567	-0.06
9	9.091	8.888	-2.28
10	10.176	10.178	0.02
11	10.276	10.280	0.03
12	11.239	11.079	-1.44
13	12.841	12.524	-2.53
14	13.993	13.991	-0.01
⋮	⋮	⋮	⋮
30	195.952	195.837	-0.06

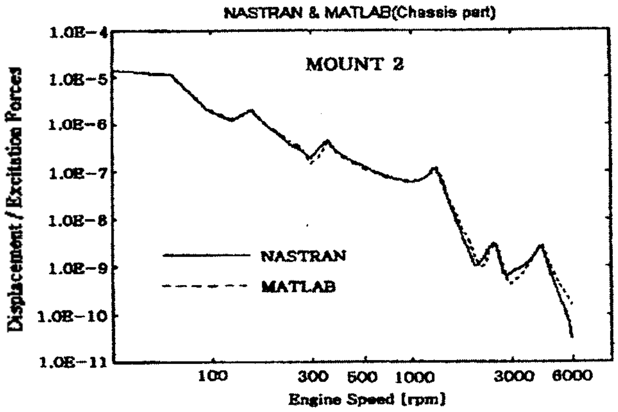

Figure 4. Comparison of force transmissibility through the mount 2.

shows the finite element model of the supporting structure including the body, the frame, and the suspension system. Figure 3 shows the mounting locations on the frame. Table 1 shows the natural frequencies of the model using NASTRAN.

Natural frequencies for the coupled system are computed by solving Eq. (11) using MATLAB, and they are compared with the NASTRAN analysis results in Table 2. The comparison in Table 2 shows that the presented method gives accurate analysis results. Figure 4 shows the comparison of force transmissibility through the mount 2 when vertical excitation force acts on the mass center of engine-transmission assembly.

3.2. Forced Vibration Analysis

In general, unbalanced forces, moments, inertia torques, etc. cause vibrations of the engine frame[6]. In this study, only vertical force and pitch moment are considered for the forced vibration analysis.

In the case of the 4-cylinder 4-cycle engine, the unbalanced forces from the individual pistons are added algebraically, so that the resultant force acting on the engine frame is simply

$$F_z = \frac{4M_p \omega^2 R^2}{L} \cos(2\omega t) \tag{12}$$

Eq. (12) shows that the secondary unbalanced inertia force oscillates with a frequency equal to twice the engine rpm and with amplitude that is proportional to the square of the rpm. In Eq. (12), M_p is the reciprocating mass per cylinder of the engine, ω is the angular velocity of the crankshaft, R is the radius of the crank, and L is the length of the connecting rod.

Although the forces due to acceleration of the pistons in an in-line engine are in the same plane, they do not have the same line of action. In the case of the in-line engine of 4 cylinders, the resultant pitch moment acting on the engine frame exists as shown in Eq. (13) if the midpoint of the 4 cylinders does not coincide with the mass center of the engine.

$$M_y = l_r F_z \tag{13}$$

In Eq. (13), l_r is the distance of the mass center projected on plane of cylinders from the mid-axis of 4 cylinders.

The vibration modes and forced response characteristics of the coupled system can be simulated solving the eigen-value problem of Eq. (11). The displacement of the coupled system can be obtained as follows.

$$\begin{Bmatrix} x_c \\ x_e \end{Bmatrix} = \begin{bmatrix} u_C & 0 \\ 0 & 1 \end{bmatrix} [U] \begin{bmatrix} \ddots & & \\ & \frac{1}{\omega_{n_i}^2 - \omega^2 + j2\xi_i \omega_{n_i} \omega} & \\ & & \ddots \end{bmatrix} [U]^T \begin{bmatrix} u_c^T & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} f_c \\ f_e \end{Bmatrix} \tag{14}$$

In Eq. (14), $[U]$ is a modal matrix of coupled system and ξ_i is the modal damping ratio of the i^{th} vibration mode. A modal damping ratio is assumed to be 0.3%. Substituting Eq. (14) into Eq. (3), we can obtain transmitted forces for the mounts.

3.3. Response Characteristics by Engine Excitation

Figures 5-7 show the transmitted forces through the each rubber mount due to engine excitation. In high speed range of the engine, the transmitted forces are increased, since these engine speeds are near the natural frequencies of bending and torsional modes for the supporting structure.

Figures 8-10 show the comparison of transmitted forces for the engine mount systems with a flexible supporting structure and a rigid supporting structure. In high speed range where structural vibration modes of the flexible

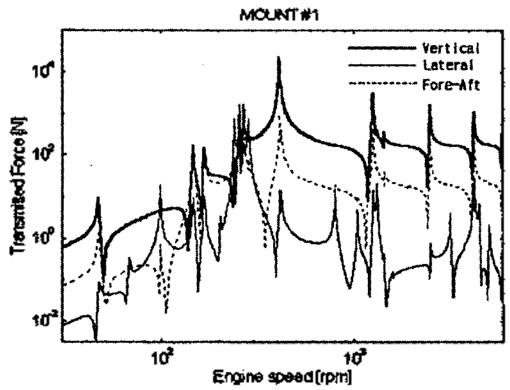

Figure 5. Transmitted forces through the rubber mount 1.

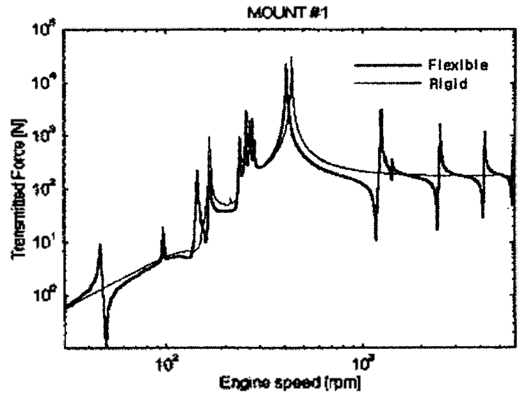

Figure 8. Transmitted forces through the rubber mount 1. (Rigid and flexible foundation model)

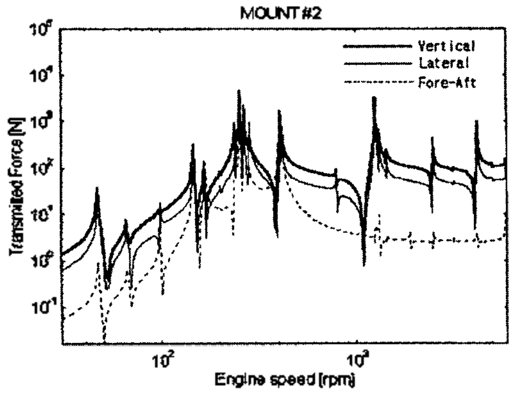

Figure 6. Transmitted forces through the rubber mount 2.

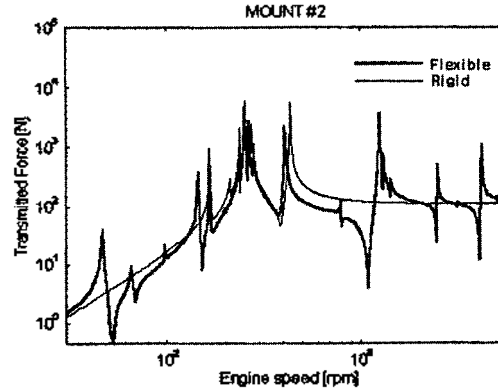

Figure 9. Transmitted forces through the rubber mount 2. (Rigid and flexible foundation model)

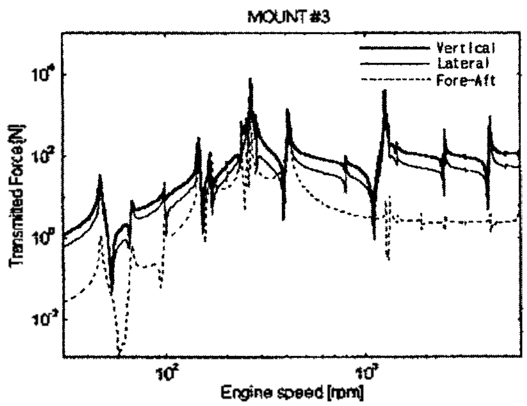

Figure 7. Transmitted forces through the rubber mount 3.

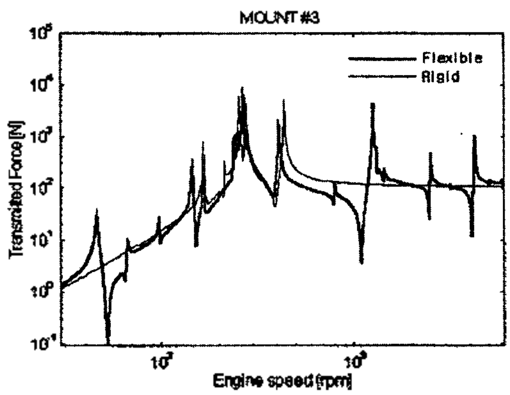

Figure 10. Transmitted forces through the rubber mount 3. (Rigid and flexible foundation model)

foundation exist, the two models show different behaviors for the transmitted force as in Figures 8-10, where vibration picks are not appearing for the rigid foundation model.

IV. Optimal Design of Engine Mount System

Using the computer simulation method discussed in the previous section, design optimization of the engine mount system for vibration isolation is presented, and the effects

Table 3. Design variables and their limits.

Design variable		Unit	Lower limit	Initial value	Upper limit
x_1	Backward distance of the mount 1 from the E/T mass center	m	0.5570	0.6070	0.7070
z_1	Vertical distance of the mount 1 from the E/T mass center	m	0.2290	0.2590	0.2890
x_2	Forward distance of the mount 2 from the E/T mass center	m	0.0500	0.0800	0.1500
y_2	Lateral distance of the mount 2 from the E/T mass center	m	0.1896	0.2096	0.2896
b	Vertical distance of the front mounts	m	0.1050	0.1550	0.1850
y_3	Lateral distance of the mount 3 from the E/T mass center	m	0.2125	0.2325	0.3125
α_1	Inclined angle of the mount 1 from the vertical line	Deg.	0.0000	10.0000	90.0000
α_2	Inclined angle of the mount 2 from the vertical line	Deg.	0.0000	35.0000	90.0000
α_3	Inclined angle of the mount 3 from the vertical line	Deg.	0.0000	35.0000	90.0000
S_1	Shape factor of the mount 1	-	0.3500	0.5000	0.7000
S_2	Shape factor of the mount 2	-	0.3500	0.5000	0.7000
S_3	Shape factor of the mount 3	-	0.3500	0.5000	0.7000
d_1	Diameter of the mount 1	m	0.0150	0.0680	0.0900
d_2	Diameter of the mount 2	m	0.0150	0.0680	0.0900
d_3	Diameter of the mount 3	m	0.0150	0.0680	0.0900

of the foundation flexibility are discussed. In this study, the objective of optimal design is to determine the design parameters of the individual mounts and the natural frequency of supporting structure in order to minimize the response displacements at the mass center of the passenger compartment which is mounted on the supporting structure by elastomers.

The optimization problems are solved using MATLAB optimization function 'fmins', which minimize a nonlinear objective function of several design variables [7]. It is a direct search method that does not require gradients or other derivative information. At each step of the search, a new point in or near the current simplex is generated.

For the optimal design, we focused our attention on the engine idling speed range of 600-1400 rpm, where the structural vibration modes of the supporting structure exist. The objective function for minimizing the displacement of the passenger compartment due to engine excitation can be written as follows.

$$F = Minimize\left\{ |RT|_{\max} + \sum_{i=1}^{NC}[\, W_i(|g_i(x)| + g_i(x))] \right\} \quad (15)$$

where $|RT|_{\max}$ is a maximum displacement of the passenger compartment center, W_i is the weighting factor of the i^{th} constraint equation, $g_i(x)$ is the i^{th} constraint equation, and NC is the total number of constraint equations.

Mounting locations, orientation angles, diameters and shape factors of rubber mounts, and natural frequency of the fore-bending vibration mode of the supporting structure are used for design variables. Table 3 shows the design variables and their limits. The allowable compression and shear strain of the rubber mount are 20% and 30%, respectively.

Deflections at mount positions due to quasi-static forces are constrained to be within 17mm, which is required in the design target.

V. Case Study

To study the flexibility effects of the supporting structure on the optimization of the engine mount system three separate cases are considered.

Case1) The coupling effect is neglected. Transmitted forces through mounts are computed with the rigid foundation. Transmitted mount forces are considered as input data to the finite element modal model of the supporting structure. Mounting locations, orientation angles, diameters and shape factors of rubber mounts are used as design variables.

Case2) The coupling effect is considered with 24

Table 4. Engine mass and inertia properties.

Mass (kg)	Inertia properties (Nm/s²)					
m	I_{xx}	I_{yy}	I_{zz}	I_{xy}	I_{xz}	I_{yz}
225	8.51	24.18	19.09	0.54	5.39	0.27

vibration modes of the flexible foundation. Design variables are same as in case 1.

Case3) 24 vibration modes of flexible foundation are used for the coupling analysis. Mounting locations, orientation angles, diameters and shape factors of rubber mounts, and natural frequency of fore-bending vibration mode for the flexible foundation are used as design variables.

Details of the engine inertia properties are given in Table 4. Optimization results for the 3 cases are compared in Table 5. As shown in Table 5, design values can be improved by considering the coupling effect of the engine mount and supporting structure. The flexible system can reduce the response displacement of the passenger compartment by 26.9 %, but without foundation flexibility response displacement is reduced by 8.6 %. In case 3, the response displacement is reduced by 30.7 %. It is shown that the response displacement of the passenger compartment is significantly influenced particularly by the natural frequency of fore-bending vibration mode of the supporting structure.

VI. Conclusions

In this paper, a computational method is presented for the dynamic analysis of the engine mount system that is supported by flexible foundation. Using the derived equations of motion, the forced vibration analyses are carried out for the given engine mount system excited with the unbalanced force and moment. The forced vibration analysis results show that the flexibility of the supporting structure has significant effects on the vibration modes and the transmitted forces. Especially the flexibility effects become more significant in the range of the natural frequencies of the supporting structure.

Optimal design for the engine mount system is presented, in which mounting locations, orientation angles, shape parameters of rubber mounts, and natural frequency of the supporting structure are selected as design variables to minimize the response characteristics of passenger compart-

Table 5. Comparison of optimization results.

Method	Symbols	Original	Case 1	Case 2	Case 3
Shape parameter of engine mount (m)	S_1	0.5000	0.3507	0.3501	0.3653
	S_2	0.5000	0.3619	0.4369	0.4029
	S_3	0.5000	0.3619	0.4369	0.4029
	d_1	0.0680	0.0556	0.0422	0.0721
	d_2	0.0680	0.0702	0.0689	0.0576
	d_3	0.0680	0.0702	0.0689	0.0576
Location of engine mount (m)	x_1	0.6070	0.5570	0.6433	0.5635
	z_1	0.2590	0.2535	0.2884	0.2886
	x_2	0.0800	0.1008	0.0699	0.0960
	y_2	0.2096	0.2169	0.2525	0.2454
	b	0.1550	0.1850	0.1208	0.1542
	y_3	0.2325	0.2145	0.2419	0.2389
Orientation angle of engine mount (Degree)	α_1	10.0	12.3	9.6	10.1
	α_2	35.0	40.9	42.7	30.0
	α_3	35.0	30.7	50.0	43.7
Natural frequency of foundation (Hz)	37.89	37.89	37.89	37.89	41.1
Objective function	$\lvert RT\rvert_{max}$	2.0354 E-5	1.8612 E-5	1.4867 E-5	1.4095 E-5
	Reduction rate (%)	0	9.6	26.9	30.7

ment. It is shown that improved optimization results can be obtained by considering the flexibility of the supporting structure. The response displacement is significantly reduced particularly when the natural frequencies of fore-bending vibration modes of the supporting structure are considered as design variables in design optimization.

Acknowledgements

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References

1. Bernard, J. E. and Starkey, J. M., "Engine Mount Optimization", SAE Paper No. 830257, 1983.
2. Geck, P. E. and Patton, R. D., "Front Wheel Drive Engine Mount Optimization", SAE Paper No. 840736, 1984.
3. Straw, R. L., "The Development of Isolation Mount", SAE Paper No. 840781, 1984.
4. Ashrafuion, H. and Nataraj, C., "Dynamic Analysis of Engine Mount Systems", *ASME Journal of Vibration and Acoustics*, Vol.114, pp. 79-83, 1992.
5. Kim, J. H., "Dynamic Analysis of Engine Mount Systems with Consideration of Foundation Flexibility", Ph. D. Thesis, The Seoul National University, 1995.
6. Lee, J. C., "Optimal Mounting System of a Variable Displacement Engine", Ph.D. Thesis, Korea Advanced Institute of Science and Technology, 1985.
7. Grace, A., Optimization Toolbox for Use with MATLAB, The Mathworks Inc, 1995.

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The Journal of the Acoustical Society of Korea, Vol. 17, No. 1E, 1998.

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The Journal of the Acoustical Society of Korea, Vol. 17, No. 1E, 1998.