THE JOURNAL OF THE ACOUSTICAL SOCIETY OF KOREA Vol.20, No.2E 2001. 6. pp. 46~50

Variation of Radiation Impedance for Piston Source According to Baffle

Soon-Jong Park*, Moo-Joon Kim**, Chun-Duck Kim***

*Dept. of Electronic Eng., Pukyong National University
**Dept. of Physics, Pukyong National University
***Dept. of Electrical Eng., Pukyong National University

(Received 12 April 2001 ; accepted 8 June 2001)

Abstract

The characteristics of radiation impedance are analyzed by algorithms which consists of Finite Element Method (FEM) and Hybrid type Infinite Element Method. The changes of radiation impedance for piston source according to the size and the material properties of baffles are studied. The results of the radiation impedance for rigid finite baffle coincide with other reports. The more the material properties of baffle that comes across the acoustic medium, the more the calculation results of radiation impedance approach the ones without baffle.

Therefore, these results can be applied to the design and the radiation characteristics analysis of acoustic transducers.

Keywords: *Finite element method, Hybrid type infinite element method, Radiation impedance, Piston source, Baffle*

I. Introduction

The calculation of radiation impedance is an important element in analyzing the driving characteristics for the transducers[1,2]. In general, the acoustic transducer has the baffle that reduces backward energy loss. Therefore, the radiation impedance has many different characteristics according to the size and the material property of the baffle[3,4].

The radiation impedance has been calculated under the condition that a source is mounted on the rigid infinite baffle[3-5]. However, the practical transducers have the non-rigid finite baffle. The theoretical calculation has many difficulties and errors for finite baffles[4]. The radiation impedance for the regular square source was investigated in the previous report[2]. Although the radiation impedance have been calculated by the finite element method, it takes a lot of computation time because the electric field, mechanical field, and acoustic field have to be considered. The calculation method is not adequate to analyze changes of radiation impedance by boundary conditions in vibrating surfaces.

In this paper, we calculate the radiation impedance under each condition that the source is mounted on the rigid baffles with various size and non-rigid finite baffles. The calculation algorithm is combined with the two dimensional finite element and the hybrid type infinite element method. The calculation results are compared with the theoretical ones for the rigid infinite baffle. Using the algorithm, the variation of radiation impedance according

---

Corresponding author: Soon-Jong Park (ultrasj@mail1.pknu.ac.kr)
Yong-dang Cam., Pukyong National University, Pusan 608-739, Korea

to both size and material properties of baffles, can be investigated.

II. Finite Element Analysis

The symmetric three dimensional problem can be expressed by the two dimensional problem with the surface including symmetric axis. In order to calculate the radiation impedance of a circular piston source, the algorithm that combines the two dimensional axisymmetric finite element method and hybrid type infinite element method must be applied.

Firstly, the Lagrangian expression for the two dimensional axisymmetric finite element method according to a circular piston source can be derived as follows[6,7]:

$$\mathcal{L} = \sum_{e} \mathcal{L}e = V_e - T_e - W_e \tag{1}$$

$$\begin{aligned} V_e &= \frac{1}{2}\ \frac{2\pi}{\rho_e\, c_e^{\,2}} \int\int_e 2\pi r p^2\, drdz \\ &= \frac{1}{2}\ \frac{1}{c_e^{\,2}}\ \{p\}_e^{\,T}\,[K^{(r)}]_e\{p\}_e \\ T_e &= \frac{1}{2}\ \frac{1}{\rho_e\omega^2} \int\int_e 2\pi r\left\{\left(\frac{\partial p}{\partial r}\right)^2+\left(\frac{\partial p}{\partial z}\right)^2\right\} drdz \\ &= \frac{1}{2}\ \frac{1}{\omega^2}\ \{p\}_e^{\,T}[M^{(r)}]_e\{p\}_e \\ W_e &= \int_{\Gamma_e} 2\pi r\, p u_n\, dl \\ &= U_n\ \{p\}_e^{\,T}\,\{W^{(r)}\}_e \end{aligned}$$

where subscript e refers to an element, and

V : potential energy

T : kinetic energy

W : driving force

$[K^{(r)}]_e$: inertance matrix

$[M^{(r)}]_e$: elastance matrix

$\{W^{(r)}\}_e$: distribution vector

$\{p\}_e$: sound pressure vector

ρ_e : density

c_e : sound speed

ω : angular frequency

U_n : driving displacement.

The discretized equation can be derived by the variational principal($\delta\mathcal{L}=0$).

$$([M^{(r)}] - k^2[K^{(r)}])\{p\} = -\omega^2 U_n\{W^{(r)}\} \tag{2}$$

Secondly, the Lagrangian expression for the hybrid type infinite element corresponding to open region can be written as follows[6],[7]:

$$\mathcal{L}_{\mathrm{Ie}} = \{\beta\}^T[G]_{\Gamma e}\widetilde{\{p\}}_{\Gamma e} - \frac{1}{2}\{\beta\}^T[H]_{\Gamma e}\{\beta\} \tag{3}$$

$$[G]_{\Gamma e} = \int_{\Gamma e}\{A_n\}\{L\}^T dl$$

$$[H]_{\Gamma e} = \int_{\Gamma e}\frac{1}{2}(\{A\}\{A_n\}^T + \{A_n\}\{A\}^T)dl$$

where,

$\{\beta\}$: unknown coefficient vector

$\widetilde{\{p\}}$: unknown vector at boundary

$\{A\}$: function vector

$\{A_n\}$: normal derivation of $\{A\}$

$\{L\}$: interpolation vector of pressure corresponding to boundary.

Stationary condition of Eq. (3) about $\{\beta\}$ yields

$$\{\beta\} = [H]^{-1}[G]_{\Gamma e}\widetilde{\{p\}} \tag{4}$$

Substitution of Eq. (4) into Eq.(3) yields

$$\mathcal{L}_{\mathrm{Ie}} = \frac{1}{2}\ \widetilde{\{p\}}^T[S]_{\Gamma e}\widetilde{\{p\}} \tag{5}$$

$$[S]_{\Gamma e} = [G]_{\Gamma e}^{\,T}\ [H]_{\Gamma e}^{\,-1}[G]_{\Gamma e}$$

where the matrix $[S]_{\Gamma e}$ for an infinite element is admittance matrix at boundary. Using the admittance matrix, which terminate finite elements at boundary, the finite region can be combined with infinite region.

From the algorithm, the radiation impedance Z_{rF} can be calculated as follows:

$$Z_{rF} = \frac{F}{j\omega u_n} = \frac{1}{j\omega u_n}\int\int_{\Gamma_d} p_d dS \tag{6}$$

where u_n is the particle displacement and p_d is the sound pressure on the radiating face.

The initial axisymmetric model for the radiation impedance analysis is shown in Fig. 1. In this figure, R is radius of circular baffle. The area of radiating face and baffle is divided by finite elements. The open boundaries are then

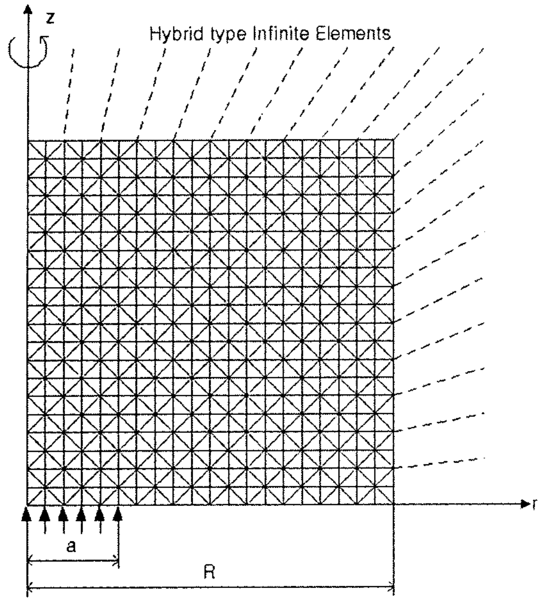


Figure 1. FE model of circular piston source.

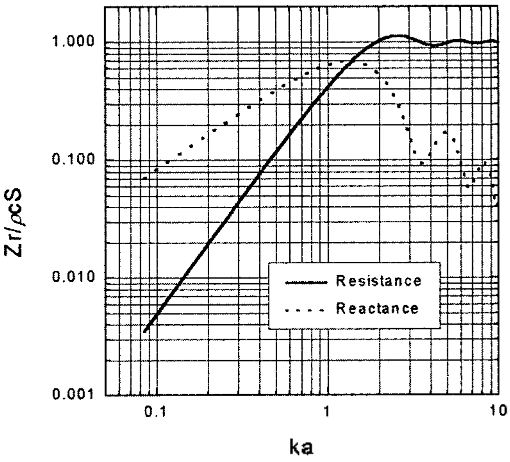


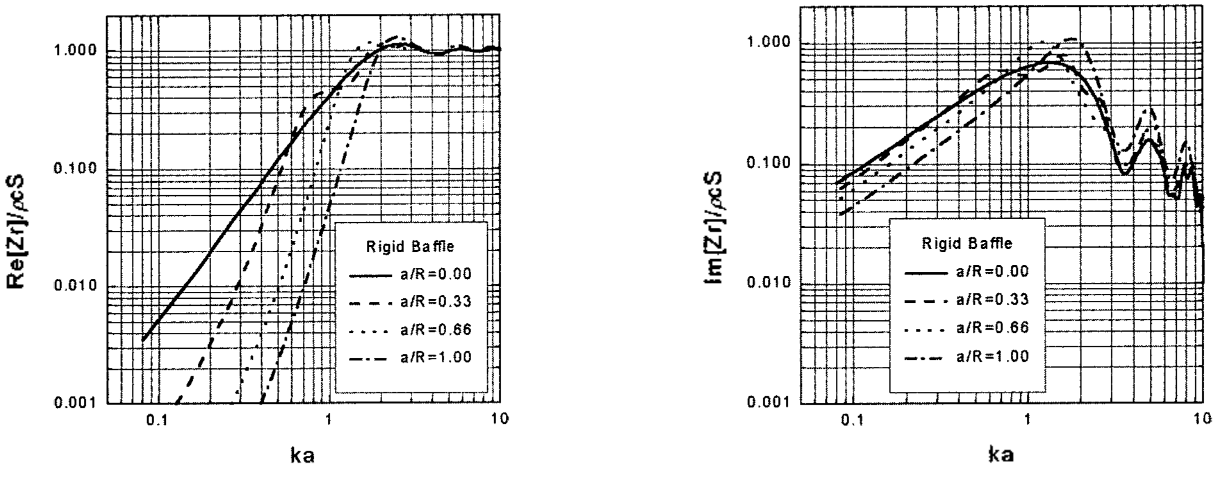
Figure 2. Radiation impedance density for the rigid infinite baffle.

combined with hybrid type infinite elements. The radiating face is assumed to be driven by uniform velocity. Normal derivation of pressure in the rigid baffle are considered to be zero. For the non-rigid finite baffles, material properties of the baffles are substituted to each finite element.

III. Results

Using the FE model as shown in Fig. 1, the radiation impedance density for the rigid infinite baffle is calculated as shown in Fig. 2 when acoustic medium is water. Although the results of radiation reactance have a little difference at large *ka*, the results of radiation impedance obtained by FEM coincide well with the theoretical ones[5]. From the results, it is confirmed that the algorithm can be applied to the calculation of radiation impedance for circular piston source with infinite baffle.

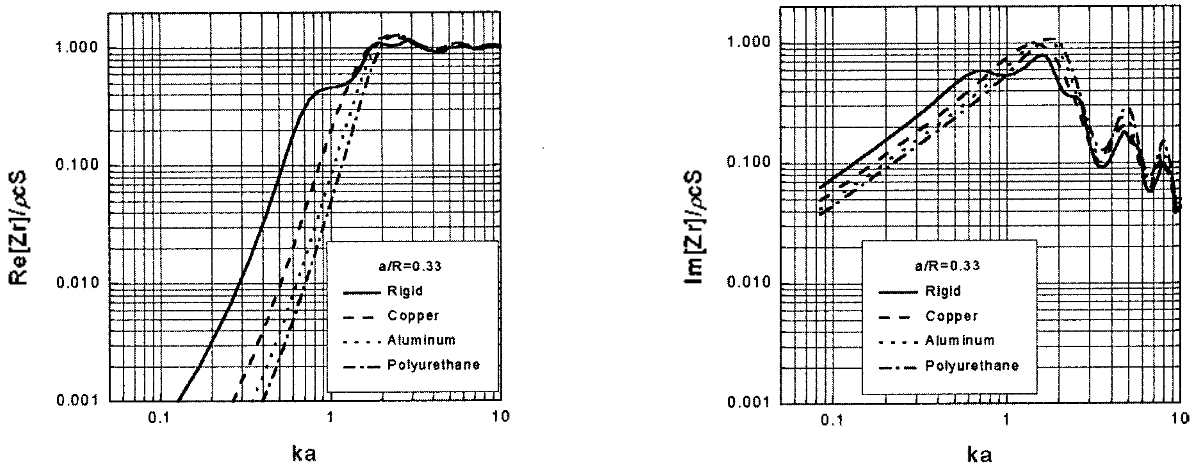
The radiation impedance density is calculated as shown in Fig. 3 when a/R is 0.33, 0.66, and 1.0. In the range of small *ka*, the radiation resistance density without baffle, i.e, a/R=1.0, shows different characteristics from the results



(a) Radiation resistance

(b) Radiation reactance

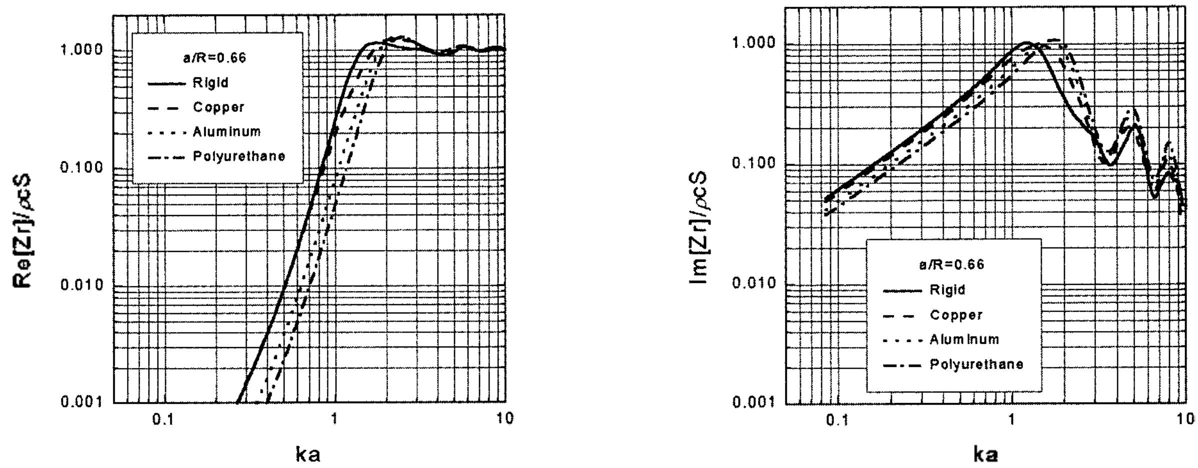
Figure 3. Radiation impedance density for the rigid finite baffle.



(a) Radiation resistance

(b) Radiation reactance

Figure 4. Radiation impedance density for the non-rigid finite baffle (a/R=0.33).



(a) Radiation resistance

(b) Radiation reactance

Figure 5. Radiation impedance density for the non-rigid finite baffle (a/R=0.66).

for infinite baffle, i.e, a/R=0.0. These results coincide well with other reports calculated for rigid finite baffle[4]. The results of radiation reactance, Fig. 3(b), show different peak values by the baffle size. This tendency agrees with the ones of rectangular source calculated by Kim et al.[2].

Since the baffle is practically finite size and non-rigid, the variance of radiation impedance density according to the material properties of finite baffles are calculated. As the non-rigid baffles such as copper, aluminum, and polyurethane are selected with radius ratios of a/R=0.33 and a/R=0.66, respectively. The material properties of non-rigid baffles used in these calculations are shown in Table 1[5].

The radiation impedance density for a/R=0.33 according to each material property is shown in Fig. 4. The radiation impedance density decreased when the acoustic characteristic impedance of the baffle had little value in the range of small *ka*. From the results above, the calculation results for the polyurethane baffle agrees with the ones without baffle. It is considered that the polyurethane baffle can not

Table 1. The material properties of non-rigid baffles.

Material	Density [kg/m^3]	Speed [m/s]
Copper	8900.0	5000.0
Aluminum	2700.0	6300.0
Polyurethane	1020.0	1941.2

play a role of baffle because its characteristic impedance is close to one of water as an acoustic medium.

Fig. 5 shows the results of radiation impedance when the a/R is 0.66. Although the changing tendency of the results are similar to Fig. 4, the variance of change is narrow. It shows that the effect due to the baffle is decreased by reducing the baffle size.

From the results of Fig. 4 and Fig. 5, the size and the material properties of the baffle are effective to the radiation impedance in the range of small *ka*.

IV. Conclusions

To analyze the characteristics of radiation impedance for various baffle conditions, a calculated method which combined the axisymmetric FEM with hybrid type infinite element method is applied.

Using the method, the variance of radiation impedance according to the size and the material properties of the baffles was calculated. From the results of radiation impedance for rigid infinite and without baffle, the effectiveness of this method was confirmed. The dependence of radiation impedance was confirmed for the calculation results of radiation impedance according to the material properties of non-rigid finite baffles.

These results can be usefully applied to the design of practical acoustic transducers.

Acknowledgements

This work was supported by GRANT No. UARC01-31 from the Underwater Acoustic Research Center in Korea.

References

1. C. Kim, H. Seo, D. Kim, J. Yoon, "Input Impedance Analysis of Piezoelectric Cylinder Transducer using Finite Element Method," *J. Acoust. Soc. Kor.* Vol. **11**, No. 6, pp. 32-40, 1992.
2. M. Kim, C. Kim, K. Ha, "A New Calculation Method for the Radiation Impedance of Transducer with Regular Square Vibrating Surface," *J. Acoust. Soc. Kor.* Vol. **18**, No. 1E, pp. 20-25, 1999.
3. Y. Kikuchi, Ultrasonic Transducers, Corona, Tokyo, pp. 347-349, 1969.
4. T. Hayasaka, S. Yosikawa, Theory of Acoustic and Mechanical Vibration, Corona, Tokyo, pp. 603-654(in japanese), 1948.
5. Lawrence E. Kinsler, Austin R. Frey, Fundamentals of acoustics, Top, Seoul, pp. 191-193, 461, 1981.
6. Y. Kagawa, et al., FEM Program Selection-Ⅲ Morikita, Tokyo, pp. 57-66(in japanese), 1998.
7. T. Yamabuchi, et al., "Finite Element Approach to Unbounded Poisson and Helmholtz Problems Using Hybrid-Type Infinite Element," J. IEICE. Jpn. Vol. **68**, No. A3, pp. 239-246, 1985(in japanese).

[Profile]

- **Soon-Jong Park**

He received the B.S. and M.S. degree in Electronic Engineering from Pukyong National University, Busan, in 1992 and 1994, respectively. He is currently a graduate student for Ph. D. degree in Electronic Engineering, Pukyong National University. His current research interests are ultrasonic transducers analysis and its application.

- **Moo-Joon Kim**

The Journal of the Acoustical Society of Korea, Vol 18, No. 1E, 1999.

- **Chun-Duck Kim**

The Journal of the Acoustical Society of Korea, Vol 18, No. 1E, 1999.