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REDISCOVERING THE LEXICOGRAPHIC LINEAR GOAL PROGRAMMING MULTIPLEX MODEL

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Abstract An alternative approach to formulating a special class of linear goal programming (LGP) models is presented. We propose a formulation of the LGP model that can include the decision variables in the objective function. We specifically propose that the position of the decision variables in the objective function be used to eliminate goal constraints whose sole purpose is to indirectly optimize decision variables. For the select group of LGP problems wherever indirect optimization of decision variables are sought, the alternative LGP model formulation is able to reduce the size of these LGP models and in turn the computational effort required for their solution.

1. Introduction

The linear goal programming (LGP) model has been expressed [13, p.670] as:

$$\text{Min}: Z = \sum_{k=0}^{K}\sum_{i=1}^{m} P_k (w_{ik}^{-} d_i^{-} + w_{ik}^{+} d_i^{+})$$

$$\text{subject to}: \sum_{j=1}^{n} a_{ij} x_j + d_i^{-} - d_i^{+} = b_i (\text{for } i = 1,2,\cdots,m)$$

$$\text{and } x_j,\ d_i^{-},\ d_i^{+} \geq 0 \qquad (1)$$

where P_K is the preemptive priority rank($P_1 > P_2 > P_3 >>>> P_K$) assigned to goals **k** to **K** (**k** = 0 is for all system constraints), W_{ik}^{-} and W_{ik}^{+} are numerical or differential weights assigned to the deviation variables of goal **i** at a given priority level **k**, d_i^{-} are negative deviation variables, d_i^{+} are positive deviation variables, a_{ij} are technology coefficients, x_j are decision variables, and b_i are right-hand-side goal targets or numerical objectives.

A variety of simplex based algorithms have been presented in the literature to solve LGP problems [1, 4, 9]. All of these algorithmic methods obtain solution values for the x_j decision variables indirectly by optimizing the prioritized deviation variables, d_i^{-} or d_i^{+}, that are placed in the objective function. Unlike linear programming (LP) simplex methods that require decision variables in the objective function, LGP simplex methods use preemptive priorities to help guide the selection of variables into the solution basis. The arrival of decision variables into an LP solution basis for optimization purposes is based in part on the value of c_j contribution coefficients, rather than the ranking of the preemptive priorities that help drive the LGP simplex algorithms.

Many LGP models are converted LP models with multiple prioritized objectives. It is not uncommon, therefore, in applications of LGP modeling to seek an

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"indirect" optimization of individual decision variables [14, 19, 20, 21]. This is accomplished in any LGP model by structuring a single "budget" [17, p. 73] goal constraint possessing a collection of decision variables or individual goal constraints for any decision variable as follows:

$c_jx_j + d_i^- - d_i^+ = M$ (to seek a maximized x_j) (2)

or

$c_jx_j + d_i^- - d_i^+ = 0$ (to seek a minimized x_j) (3)

where **M** in (2) is an arbitrarily selected large value. The $\mathbf{c_j}$, which is technically an $\mathbf{a_{ij}}$, in either (2) or (3) is optional since the respective $\mathbf{b_i}$ is arbitrary. The deviational variables for these constraints are placed into the LGP model at desired $\mathbf{P_k}$ priority levels in the objective function. The result is an indirect optimization of the $\mathbf{x_j}$ decision variables by a reduction of deviation from the right-hand-side values. These types of goal constraints appear to dominate the existing LGP applied model literature. A sampling of 163 of the LGP applied models listed in [23] were undertaken to determine the predominance of this type of constraint's use in LGP literature. Out of the 163 of the LGP applied models 142(or 87 percent) contained one or more of these indirect optimization constraints.

Unfortunately, this "indirect" approach to the optimization of decision variables in LGP models can increase the number of constraints in a model when a large number of decision variables are selected for direct optimization. For example, in a 1991 information system project selection LGP model over 50 percent of the goal constraints were seeking the indirect optimization of the model's decision variables [21]. In the review of the 163 LGP models from [23], the indirect optimization constraints represented from 2 to 85 percent of the various model's required constraints. If these constraints could be eliminated it might save computational effort. In addition, a reduction of the number of goal constraints in a model might help to avoid serious degeneracy or other computational problems inherent in all simplex based methods [16].

2. An Alternative Formulation procedure

To reduce computational effort and avoid problems that might be caused by the needless addition of goal constraints, we propose an alternative way of formulating LGP models that can universally be solved by any existing LGP simplex algorithm. The alternative LGP model formulation is based in part on the MULTIPLEX model [6].

In 1985 Ignizio [6] presented a general purpose MULTIPLEX model and algorithm designed to address a wide variety of types of multiobjective programming models including LGP models. The "lexicographic minvector" or lexicographic LGP model which is a special case of the MULTIPLEX model permitted the possibility of decision variables to be included in the objective function of the LGP model. The discussion on the MULTIPLEX model did not explain or illustrate how the decision variables, particularly x^{max} variable would or could be used. Nor has any related research on the MULTIPLEX model illustrated the use or possible contribution of the decision variables being included in the LGP model objective function [3, 5].

We now propose a formulation of the LGP model that can include the decision variables in the objective function. We specifically propose that the position of the decision variables in the objective function be used to eliminate goal constraints whose sole purpose is to indirectly optimize decision variables (as previously discussed). This alternative formulation (i.e., alternative from [6]) of the LGP model involves the use of the following objective function:

$$\text{Min:} Z=\sum_{k=0}^{K}P_k\left[\sum_{i=1}^{m}(w_{ik}^-d_i^- + w_{ik}^+d_i^+) + \sum_{j=1}^{n}(c_jx_j^{min} + (-c_j)x_j^{max})\right]$$

(4)

where $\mathbf{x_j^{min}}$ are decision variables that are to be minimized and $\mathbf{x_j^{max}}$ are decision variables that are to be maximized. A model need not include both $\mathbf{x_j^{min}}$ and $\mathbf{x_j^{max}}$ in (4), only what is necessary to model the problem situation. Unlike Ignizio's lexicographic minvector model [6] that only minimizes decision variable values in the objective function, this alternative formulation directly includes the maximization of $\mathbf{x_j^{max}}$ in the objective function.

Using the (4) type of objective function permits the model to avoid the need of adding the individual goal constraints for a budget function for individual decision variables, thus reducing its size and computational requirements. To illustrate this reduction, let's assume we have the following LGP model:

$$
\begin{aligned}
\text{Min: } Z = {} & P_1(d^+_1 + d^-_2 + d^+_2 + d^-_3) \\
& + P_2(d^-_4 + d^+_4 + d^-_5 + d^+_5) \\
& + P_3(d^-_6 + d^+_6 + d^-_7 + d^+_7 + d^-_8 + d^+_8)
\end{aligned}
$$

subject to :

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 + x_5 + d^-_1 - d^+_1 &= 200 \\
x_3 + x_4 + x_5 + d^-_2 - d^+_2 &= 100 \\
x_1 + x_2 + d^-_3 - d^+_3 &= 10 \\
10x_1 + d^-_4 - d^+_4 &= 0 \\
20x_2 + d^-_5 - d^+_5 &= 0 \\
2x_3 + d^-_6 - d^+_6 &= 9999 \\
4x_4 + d^-_7 - d^+_7 &= 9999 \\
10x_5 + d^-_8 - d^+_8 &= 9999 \\
\text{and} \quad x_j,\ d^-_i,\ d^+_i &\geq 0 \qquad (5)
\end{aligned}
$$

In (5) the $\mathbf{x_j^{min}}$ are $\mathbf{x_1}$ and $\mathbf{x_2}$, with $\mathbf{c_j}$'s of 10 and 20, respectively. The $\mathbf{x_j^{max}}$ are $\mathbf{x_3}$, $\mathbf{x_4}$ and $\mathbf{x_5}$, with $\mathbf{c_j}$'s of 2, 4, and 10, respectively. The $\mathbf{x_j^{min}}$ goal constraints are placed at $\mathbf{P_2}$, and the $\mathbf{x_j^{max}}$ goal constraints are placed at $\mathbf{P_3}$. The arbitrary values of 9999 are chosen for the $\mathbf{b_i}$ in the $\mathbf{x_j^{max}}$ goal constraints.

The alternative model formulation equivalent of (5) is:

$$
\begin{aligned}
\text{Min : } Z = {} & P_1(d^+_1 + d^-_2 + d^+_2 + d^-_3) \\
& + P_2(10x_1 + 20x_2) + P_3(-2x_3 - 4x_4 - 10x_5)
\end{aligned}
$$

Subject to :

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 + x_5 + d^-_1 - d^+_1 &= 200 \\
x_3 + x_4 + x_5 + d^-_2 - d^+_2 &= 100 \\
x_1 + x_2 + d^-_3 - d^+_3 &= 10 \\
\text{and} \quad x_j,\ d^-_i,\ d^+_i &\geq 0 \qquad (6)
\end{aligned}
$$

As can be seen, there is a reduction of five goal constraints required for direct optimization of decision variables when using the model in (6). While the $\mathbf{c_j}$ values for the $\mathbf{x_j^{min}}$ are included in the objective function as defined in (5), the $\mathbf{x_j^{max}}$ $\mathbf{c_j}$'s are assigned as negative values to cause the minimization simplex process to maximize the decision variables. No arbitrary values are necessary for this alternative formulation. No other adjustments in the LGP modeling process are required to formulate the model.

The only adjustment to any LGP simplex solution algorithm involves the $\mathbf{P_k}$ preemptive priorities be placed above the appropriate decision variable columns in the simplex tableau. While no one has previously proposed this adjustment in the LGP simplex solution algorithm, our computational experience shows that it will work with any of the LGP algorithms currently in use [1, 4, 9]

3. Computational Results

The alternative formulation of the LGP model objective function substantially reduces goal constraints and their respective deviation variables. This reduction can make a significant reduction in the size of the simplex tableaus (and their resulting computational effort). If we were to formulate the model of (5) using Lee's [9] simplex method as presented in Table 1, the resulting number of simplex elements per tableau required to generate an answer would be 242 elements (i.e., 11 rows x 22 columns). The alternative formulation of (6) again using Lee's method as presented in Table 2 requires only 72 elements (i.e., 6 rows x 12 columns). Clearly the alternative formulation of the LGP model can for some problems substantially reduce the size of the simplex tableau.

In addition to the size reduction advantage there is also the obvious possibility of computation effort reduction in the simplex tableau element computations. To demonstrate the computational efficiency of using the alternative formulation of the LGP model's objective function Lee's simplex based algorithm [9], embedded in the MicroManager 2.0 software[15] was used to solve the models in Tables 1 and 2. The optimal decision variables values presented on the tables for both solutions are exactly the same. There is, though, a unique and beneficial difference in the "Nonachievement" of P_3 observed in the solutions in Tables 1 and 2. The P_3 in Table 1 of 28,997 is a rather meaningless value that represents the sum of deviation from the three arbitrary values of 9,999 selected from the three goal constraints in (5). On the other hand, the P_3 in Table 2 of -1,000 (or \$10 x 100) is the specific amount of maximized profit that will be obtained by using the optimal solution. While the idea of a negative nonachievement is unique to this study, it illustrates a minor feature of the more precise informational efficacy provided by this alternative formulation of the LGP model not addressed in prior research.

The real difference between the two solutions was that the model in Table 1 took 4 iterations or 5 tableaus, while the alternative formulated model in Table 2 took only 2 iterations or 3 tableaus to generate the same answer. Put another way, it took 1,210 (i.e., 5 tableaus x 242 elements) simplex tableau elements to solve for a solution using the model in Table 1 versus only 216 (i.e., 3 tableaus x 72 elements) simplex tableau elements to solve for the same solution using the alternative model formulation in Table 2. While this problem is both small in size and limited in scope from which to draw any real conclusions, the proportions of simplex tableau computational reduction in other sized problems seems to be consistent with those observed in this example.

An additional 25 experimental problems were drawn from the literature or contrived to illustratively present the comparative advantage of the alternative formulation of the LGP model. In Table 3 the 25 problems are summarily described for both LGP and comparable alternative LGP models. Since the alternative LGP model formulation is not limited by type of problem or software, a variety of LGP problems were selected including real, integer and 0-1 solution requirements. For comparison purposes the same software used to process the LGP models were used to process the alternative LGP models. The comparative solution results of the 25 experimental problems are presented in Table 4. Irregardless of solution requirements or type of software for each of the 25 problems, the alternative LGP model required fewer simplex tableaus, fewer elements per tableau, and less CPU time to generate the same decision variable set. As we expected, the reduction in the size of a model by the alternative LGP model formulation results in an almost proportional reduction in the computational effort to obtain a solution.

4. Final Comments

Based on the survey of prior LGP research reported in this note it appears that a majority of LGP models seek to indirectly optimize decision variables with goal constraints. Based on the results of the experimental problems in Table 4 the alternative LGP model formulation procedure described in this note is able to reduce the size of these LGP models and in turn the computational effort required for their solution. We therefore conclude that for the select group of LGP problems wherever indirect optimization of decision variables are sought, the alternative LGP model formulation will save the addition of the goal constraints similar to (2) and (3), and their necessary computational effort.

We feel the implementation of the alternative LGP model requirements on existing software systems can easily be accomplished by software developers or educators who do programming. The only change required in the software is to permit preemptive priorities above the decision variables in the simplex tableau to be treated in the same way as those above the deviation variables. This singular change permits the alternative LGP model to be used with any existing LGP solution procedure.

The use of alternative LGP formulations for LGP models can also have two additional benefits. First, alternative LGP models can have a pedagogical benefit by permitting educators a more direct means of making the transition from LP to LGP. Logically, both LP and LGP are basic content in traditional introductory management science courses. It is also logical to have the simplex method for LP presented before that of LGP. Since the alternative formulation of the LGP model's objective function includes decision variables, educators could use the alternative LGP model as a transition formulation means of moving from the subject of LP to LGP. The transition is aided by the fact that c_j's are used as w_{ik}'s in the alternative LGP model and no confusing arbitrary values, like **M**, are necessary in its formulation.

A second benefit of the alternative LGP model is its unique capability of creating two-dimensional optimization on decision variables. An LP objective function directly optimizes the decision variables and indirectly optimizes slack/surplus variables. An LGP objective function directly optimizes deviation variables (i.e., slack/surplus variables) and indirectly optimizes decision variables. The alternative LGP model permits both decision variables and deviation variables in the objective function to be directly optimized in accordance with a pre-determined ranking. The result is a formulation procedure that offers a new multi-criteria modeling capability different from those in the literature[6, 22]. The exploration of this modeling capability is recommended as a subject for further research.

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<Table 1> Initial Simplex Tableau Formulation of the LGP Model in (5)

Program : Goal Programming
Problem Title : LGP

***** Input Data *****

Min Z = 1P1d+1 + 1P1d+2 + 1P1d-2 + 1P1d-3 + 1P2d+4 + 1P2d-4
+ 1P2d+5 + 1P2d-5 + 1P3d+6 + 1P3d-6 + 1P3d+7 + 1P3d-7

Subject to
C1 1x1 + 1x2 + 1x3 + 1x4 + 1x5 + d-1 - d+1 = 200
C2 1x3 + 1x4 + 1x5 + d-2 - d+2 = 100
C3 1x1 + 1x2 + d-3 - d+3 = 10
C4 10x1 + d-4 - d+4 = 0
C5 20x2 + d-5 - d+5 = 0
C6 2x3 + d-6 - d+6 = 9999
C7 4x4 + d-7 - d+7 = 9999
C8 10x5 + d-8 - d+8 = 9999

***** Program Output *****

Initial Tableau

\Cj Cb\	Basic	Bi	0 X1	0 X2	0 X3	0 X4	0 X5	0 d-1	1P1 d-2	1P1 d-3	1P2 d-4	1P2 d-5
0	d-1	200.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000
1P1	d-2	100.000	0.0000	0.000	1.000	1.000	1.000	0.000	1.000	0.000	0.000	0.000
1P1	d-3	10.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
1P2	d-4	0.000	10.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
1P2	d-5	0.000	0.000	20.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
1P3	d-6	9999.0000	0.000	0.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1P3	d-7	9999.0000	0.000	0.000	0.000	4.000	0.000	0.000	0.000	0.000	0.000	0.000
1P3	d-8	9999.0000	0.000	0.000	0.000	0.000	10.000	0.000	0.000	0.000	0.000	0.000
Zj - Cj	1P3	29997.000	0.000	0.000	2.000	4.000	10.000	0.000	0.000	0.000	0.000	0.000
	1P2	0.0000	10.000	20.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1P1	110.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000

1P3 d-6	1P3 d-7	1P3 d-8	1P1 d+1	1P1 d+2	0 d+3	1P2 d+4	1P2 d+5	1P3 d+6	1P3 d+7	1P3 d+8
0.000	0.000	0.000	-1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	-1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	-1.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	-1.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	0.000	0.000	0.000
1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	0.000	0.000
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	0.000
0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.000	-2.000	-2.000
0.000	0.000	0.000	0.000	0.000	0.000	-2.000	-2.000	0.000	0.000	0.000
0.000	0.000	0.000	-1.000	-2.000	-1.000	0.000	0.000	0.000	0.000	0.000

Final Solution Tableau at Iteration 4

Analysis of decision variables

Variable	Solution Value
X1	10.000
X2	0.000
X3	0.000
X4	0.000
X5	100.000

Analysis of the objective function

Priority	Nonachievement
P1	0.000
P2	100.000
P3	28997.000

<Table 2> Initial Simplex Tableau Alternative Formulation of the LGP Model in (6)

Program : Goal Programming
Problem Title : LGP II

***** Input Data *****
Min Z = 1P1d+1 + 1P1d+2 + 1P1d-2 + 1P1d-3 + 1P2 (10x1 + 20x2)
+ 1P3(-2x3 -4x4 -10x5)
Subject to
C1 1x1 + 1x2 + 1x3 + 1x4 + 1x5 + d-1 - d+1 = 200
C2 1x3 + 1x4 + 1x5 + d-2 - d+2 = 100
C3 1x1 + 1x2 + d-3 - d+3 = 10

***** Program Output *****

Initial Tableau

\Cj Cb\	Basis	Bi	10P2 X1	20P2 X2	-2P3 X3	-4P3 X4	-10P3 X5	0 d-1	1P1 d-2	1P1 d-3	1P1 d+1	1P1 d+2	0 d+3
0	d-1	200.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	-1.000	0.000	0.000
1P1	d-2	100.000	0.000	0.000	1.000	1.000	1.000	0.000	1.000	0.000	0.000	-1.000	0.000
1P1	d-3	10.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	-1.000
Zj-Cj	1P3	0.000	0.000	0.000	2.000	4.000	10.000	0.000	0.000	0.000	0.000	0.000	0.000
	1P2	0.000	-10.000	-20.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1P1	110.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	-1.000	-2.000	-1.000

Final Solution Tableau at Iteration 2

Analysis of decision variables

Variable	Solution Value
X1	10.000
X2	0.000
X3	0.000
X4	0.000
X5	100.000

Analysis of the objective function

Priority	Nonachievement
P1	0.000
P2	100.000
P3	10.000

<Table 3> LGP and Alternative LGP Model Experimental Problem Formulation

Problem No.	X_j	P_k	LGP Model Constraints[1]	Alternative LGP Model Constraints	Constraint Reduction with Alternative Model	Source of LGP Model[2]
1	2	2	3	2	1	[18]
2	2	3	6	4	2	Contrived
3	3	3	6	3	3	Contrived
4	4	2	4	2	2	[8]
5	5	3	8	3	5	Contrived
6	6	4	12	5	7	[21]
7	7	6	17	14	3	[7]
8	8	3	14	6	8	Contrived
9	12	7	16	15	1	[12]
10	14	4	25	11	14	Contrived
11	18	5	35	17	18	Contrived
12	21	5	15	11	4	[17, p. 180]
13	25	6	45	20	25	Contrived
14	29	6	26	17	9	[9, p. 311]
15	35	3	40	20	20	Contrived
16	42	6	12	11	1	[10]
17	45	5	45	37	8	Contrived
18	48	6	35	25	10	[11]
19	65	5	60	40	20	Contrived
20	75	5	70	44	26	Contrived
21	85	5	80	43	37	Contrived
22	95	5	90	55	35	Contrived
23	101	4	153	121	32	[20]
24	170	5	65	46	19	[10]
25	462	5	62	52	10	[14]

1 Only goal constraints (not system constraints) are included in these models.

2 Selection (from literature) or contrivance of models were based on illustration rather than representation purposes. All contrived problems assume goal constraints possess both positive and negative deviation variables.

<Table 4> Experimental Results for Problems From <Table 3>

Problem No	Required No. of Tableaus[1]			Required No. of Tableau Elements[2]		
	LGP Model	Alternative LGP Model	Alternative LGP Reduction	LGP Model	Alternative LGP Model	Alternative LGP Reduction
1	4	3	1	45	28	17
2	6	5	1	135	77	58
3	6	3	3	144	60	84
4	4	2	2	78	36	42
5	8	4	4	242	72	170
6	12	5	7	384	153	231
7	20	13	7	966	720	246
8	16	8	8	629	189	440
9	21	19	2	851	792	59
10	28	12	16	1,885	555	1,330
11	33	14	19	3,560	1,166	2,394
12	23	7	16	940	688	252
13	41	19	22	5,916	1,716	4,200
14	26	18	8	2,108	1,219	889
15	42	20	22	4,988	1,748	3,240
16	19	18	1	1,152	1,071	81
17	48	40	8	6,800	5,040	1,760
18	41	29	12	4,469	3,069	1,400
19	69	59	10	12,090	6,570	5,520
20	73	40	33	16,200	8,036	8,164
21	84	47	37	20,910	8,256	12,654
22	102	63	39	26,220	12,360	13,860
23	153	118	35	63,899	42,875	21,024
24	608	415	193	21,070	13,413	7,657
25	312	222	90	39,262	32,262	7,000

[1] Required number of tableaus includes the original tableau and all required iteration tableaus.

[2] Required number of tableau elements includes only those requiring simplex computation, and not the cj used for column heading

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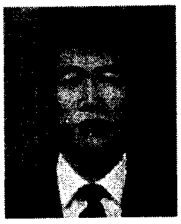

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