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Multi-Criteria Topology Design of Truss Structures

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Abstract

This paper presents a novel design approach that could generate structural design alternatives having different topologies and then, select the optimum structure from them with simultaneously determining its optimum design variables related to geometry and the member size subjected to the multiple objective design environments. For this purpose, a specialized genetic algorithm, called StrGA_DeAl + MOGA, which can handle the design alternatives and multi-criteria problems very effectively, is developed for the optimal structural design. To validate the developed method, plain truss design problems are considered as illustrative examples. To begin with, some possible topologies of the truss structure are suggested based on the stability criterion that should be satisfied under the given loading condition. Then, with the consideration of the given multi-criteria, several different topology forms are selected as design alternatives for the second step of the conceptual design process. Based on the chosen topology of truss structures, the sizing or shaping optimization process starts to determine the optimum design parameters. Ten-bar truss problems are given in the paper to confirm the above concept and methodology.

Keywords: topology and geometry optimization, multi-criteria optimization, StrGA_DeAl, design alternatives

1 Introduction

The major task of engineers is to design and build systems that have a set of desired properties in the design environment. However, it is well known that most design problems, especially in the early stages, would be ill defined, open-ended and unstructured. Even when ignoring the above problems, it is not easy to develop the automatic design system since engineering designs have an important component of creativity. To solve such difficulties, the engineer has determined appropriate design alternatives by his/her intuition or by using some design support systems. Then, more dominant alternatives are selected with the given criteria. This customary approach has been a traditional solving strategy in the general design process for a long time. This approach shows its real ability in situations where the given resource and time are limited, such as the engineers R&D process.

All of the above mentioned may not be an exception in structural optimum designs. Up to date, most research efforts in structural optimization have been concentrated on the problem where structural topology is already fixed; much less effort has been devoted to the optimization of geometry, and relatively little work is related to the optimal design of topology. Note, however, that the optimal design of structural topology can greatly improve the result design(Bendsoe et al 1994). That is, potential improvements affected by topology design are generally more significant than those resulting from fixed-layout optimization. Most recently, some studies have begun to focus on the topology problem with various views. However, the selection of optimal topology under a multi-criteria environment is one of the most difficult problems, and this theme has not actively been studied so far.

Generally, multiple objective problems do not have a unique solution. Multiple optima mean that they expand the width of the decision-makers (DMs) selection, so that the DM can cope with various design situations. In this paper, by employing the conceptual design process of general design into the structural design area, a variation of structural topology is treated as "design alternatives," and the optimal alternatives with optimal geometry and member size are then selected with the consideration of several necessary criteria. Especially for the generation of design alternatives, Han and Lee(1999) generated and provided various design alternatives by utilizing the previous design concepts underlying in the existing 'mechanism' design cases. Even though the application fields are different, the following aspects discriminate between theirs and this approach; the design alternatives of this approach are generated based on the current optimization environment, i.e. no design experience is required for generating design alternatives. Computer automatically guarantees the stable configuration of truss structures, and the unstable ones are eliminated by checking the conceptual stability. Moreover, our approach can cover the preliminary design as well as the conceptual design including the process of applicable evaluation in the multiobjective environments. The process of the design alternatives' generation is developed using the Frontier concept(Sen and Yang 1998) or the Pareto optimum concept(Pareto 1896), which means the strategy to obtain the optimum solutions under the multi-objective environment. To implement this concept, MOGA(Multi-criteria Optimization by Genetic Algorithm developed by Kim(1994)) is adopted. MOGA has a distinguishing feature, in that it can obtain a Pareto optimal set at one run-time by the particular fitness-evaluating system, unlike other approaches, which try to combine all the different objectives into one (ex. weight method) so that a Pareto optimal set can be obtained only one by one. To the best of our knowledge our approach is the first reported on capable of generating design alternatives in the multicriteria environment, and it is successfully applied to truss designs. Meanwhile, the multicrieria researches in the field of ship design has not so activated in Korea. Kim(1994), Lee(1995) and Shin(2000) are the representative ones.

The Genetic Algorithm(GA) is very effective in finding optimal solutions to a variety of structural problems. In addition, the GAs is naturally appropriate for a discrete optimization. This ability is well suited to the uncertain and unstructured nature of the early design stages, especially in representing and evaluating design prototypes.

For effective handling of structural alternatives, this paper uses the Structured Genetic Algorithm (StrGA) developed by Dasgupta(1993) that enables the representation of chromosomes as a hierarchical structure. Moreover, in order to attain the solution more effectively, new operators such as Active Unit-based Crossover and Unit-based Mutation are introduced in the paper. This system would be thus called the Structured Genetic Algorithm for handling Design Alternatives (StrGA_DeAl). This approach have proved its practicability and efficiency in the design field of offshore structures(Ruy and Yang 2001).

As a test-run, optimal truss design problems are considered. Their promising results show that our proposed method can be a very useful tool for design problems encompassing both the

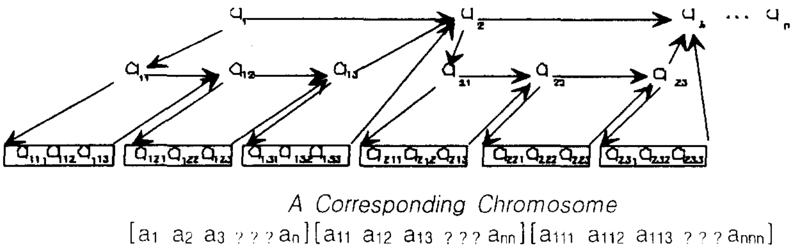

Figure 1: A hierarchical implementation of StrGA

selection of optimal topology and the determination of the optimal sizing of structural members.

This work is organized as follows. In Section 2, the StrGA_DeAl system, developed for effective handling design alternatives, is explained in detail. Then, a novel approach for topology design under a multi-objective environment is presented in Section 3. Section 4 presents the application of our approach for truss structures. Then, the paper concludes with Section 5.

2 Structured genetic algorithm for handling design alternatives

The StrGA_DeAl is derived from the StrGA, whose characteristic lies primarily in its redundant genetic code and a gene activation mechanism. As compared with GA, StrGA has differences in the individual's construction and the encoding-decoding process. In detail, a chromosome is interpreted as the hierarchical structures of the genetic code. For instance, Figure 1 shows a three-level tree structured genome and a flattened linear representation of this structure. In this paper, the chromosome's genes fall into two categories. Generally, high level genes are in charge of the determination of which design alternative is activate, and lower genes governed by the high level genes, are responsible for representing and implementing design variables. From now, the former will be called "unit genes" and the latter will be called "expressible genes". This paper does not pursue the improvement of the GAs performance(Dasgupta 1993), but adopts the StrGA's chromosome structure itself for the purpose of representing and evaluating the design alternatives. In other words, StrGA_DeAl is used in the view of the hierarchical structure of 'the design problem', rather than in respect of the hierarchical structure of 'the chromosome' in StrGA that intends to improve the performance of GA.

To implement this representation scheme for the design alternatives in StrGA_DeAl, it is necessary to use new genetic operators that enable StrGA_DeAl to converge fast to the solution.

2.1 Active unit based crossover

If very long binary strings are used in the process of the chromosomal formation while only small parts of the string participate in the decoding process, the quality of convergence can get worse. In such a case, too much computation time required to obtain a solution whose quality is acceptable for the designer's use. This stems from the fact that unessential information is exchanged between

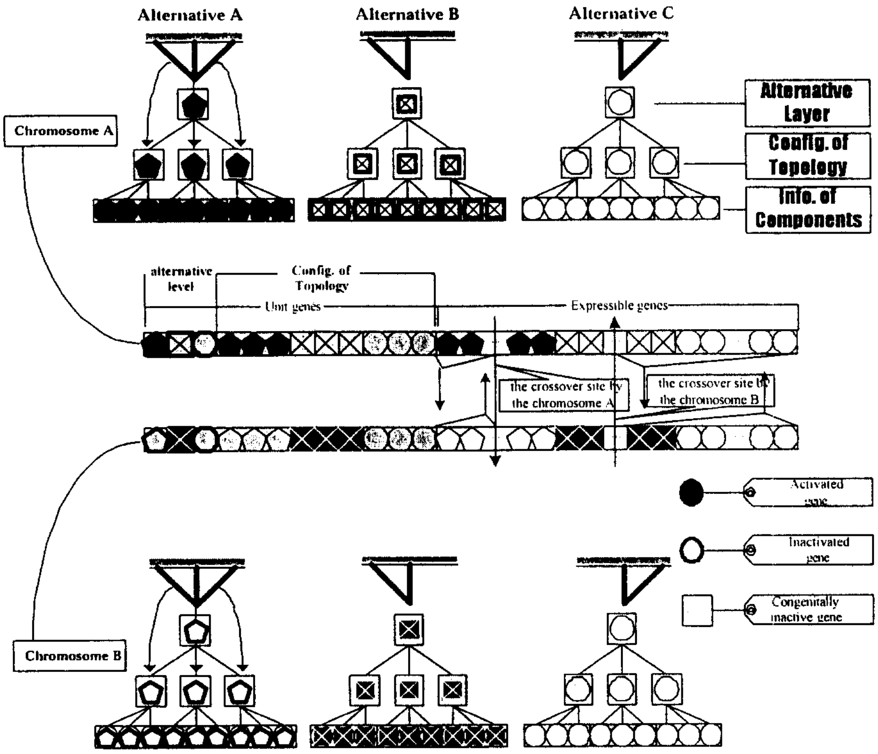

Figure 2: Active unit based crossover: Two chromosomes come across in the mating pool for the crossover. The symbols filled with black mean the active gene regardless of whether it is the unit or expressible one. Note that there are three design alternatives with the relevant tributary genes.

two individuals in the process of crossover. Unfortunately, this occurs in configuring chromosomes for the StrGA_DeAl. In this paper, the crossover site is therefore compulsorily and randomly generated at only expressible genes that belong to the active tributaries, which are a part of the chromosome's binary string. This crossover method is called "Active Unit based Crossover" (AU crossover) which is a new operational strategy of the StrGA_DeAl.

Supposing that the conceptual design generates the design alternatives, and then the chromosomes of the StrGA_ DeAl are constructed with the topological information of the given design alternatives. Generally, the type of genes in the chromosome can be classified according to its role and in a wider sense that it can be one of the followings as depicted in Figure 2: Unit genes, Intermediate genes, and Expressible genes. For reference, the Intermediate genes represent the topological information of each design alter native. Therefore its structure of genes must not corrupt through the process of reproduction.

The AU crossover is designed to maintain the consistency of the chromosomes and to improve the computational efficiency. Supposing that there are two chromosomes as depicted in Figure 2, which are selected to mate for the crossover operation. In case of chromosome A, the activated highest unit gene indicates that the design alternative A is selected. On the other hand, the chromosome B is connected with the design alternative B. Herein, the random site-selection of traditional crossover do not effective in the StrGA_DeAl, because a crossover site on the unit genes can corrupt the topological information of the relevant design alternatives. In addition, a crossover site on the expressible genes under inactive tributaries does not contribute to the improvement of two mated individuals at all. On the contrary, the AU crossover compels the crossover site to locate only on the expressible genes of the activated tributary. As can be shown in Figure 2, AU crossover is a kind of two-point crossover, and is allowed to be used only in activated expressible genes.

2.2 Unit based mutation

Expressible genes evolve through the AU crossover, while unit genes do not have the way to evolve during the generation process. Unit Based Mutation (UB Mutation) enables this process with the following mechanism.

When we handle the problem including design alternatives, the active genes in one set of the unit gene(for example, in Figure 1) have to be restricted to the certain number associated with the characteristics of each layer. Because the general crossover exchanges only the genotypical information between the selected individuals, this crossover is meaningless in obtaining a proper representation of a design alternative. Accordingly, when the generation reaches near the end, a large percentage of chromosomes are fixed to a certain design alternative. Moreover, it is rarely expected to switch to another alternative by the general mutation operator, which switches a selected gene randomly. So, a new mutation operator is devised, which counts the number of unit genes fixed previously, and then randomly generates the active unit genes at a randomly selected site with that number, and sets the other unit genes to zero, which implies a passive gene. This method is called "UB Mutation".

Early on, Yang and Jang(1996) have proposed the method where the chromosome is divided into two parts; one part is the topological variability and the other is the sizing variability. However, they approached to this process in the following limited manner; in the case where only one highest unit gene should be selected and there are four alternatives, they used the constraint in the form of $a_1 + a_2 + a_3 + a_4 = 1$. However, because the constraint-handling method of the evolutionary algorithm has been inapt especially at the equality constraint, they converted this into two inequality constraints. On the other hand, the UB Mutation method automatically prevents individuals from moving to the infeasible region without such a constraint.

3 A new approach for structural design

In the early stages of structural design, there are many cases that demand for the decision of which topological type is suitable for the design objectives. This decision has a big influence on the success of the resultant design. Thus, it is certain that the topology of the structure plays an important role as the design alternatives in the general design process. There are probably several methods that can extract the design alternatives in the design process. Although they can be made only by designer's creative brainstorming in the case of relatively simple structures, Yang et al(1998) used Case Based Reasoning for this situation that supported a way of helping designers to organize their experience and know-how systematically so as to utilize them in current situations. However, they failed to show the formalized process of how to generate design alternatives. On the other hand, unlike their previous work, this paper intends to present a general method that can automatically generate design alternatives of the grid-like type structures.

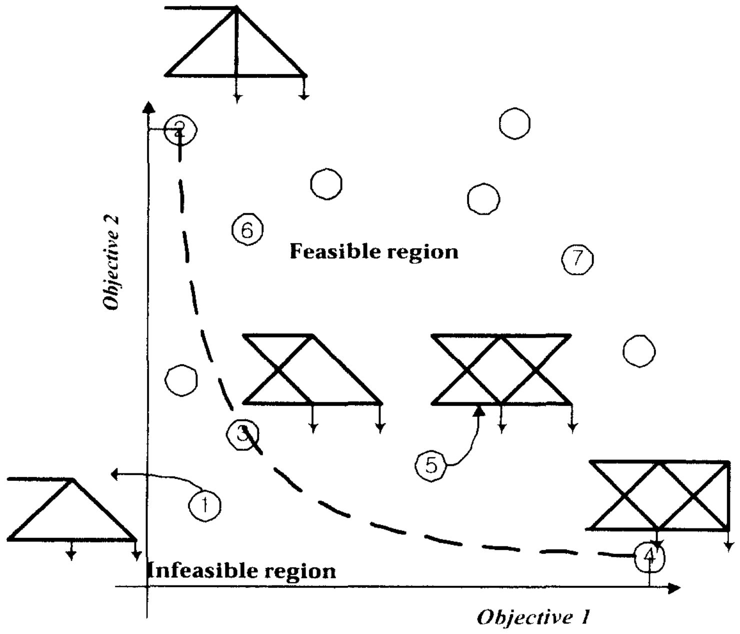

Figure 3: The selection of structural design alternatives

3.1 Step 1: Generation of design alternatives

This part will be explained with possible topologies based on a 10-bar ground structure that should carry two nodal loads. The main process of this step concerns the generation of design alternatives favorable to the design objectives from a purely topological viewpoint. The generation of purely topological alternatives means that design alternatives are extracted only by the topological characteristics of the structures, and the detail design including geometry or sizing is not considered in this step. In order to implement this concept, all members' area sizes of candidates are fixed to a certain resolution, and candidate structures are evaluated on the basis of the given resolution. This process constrains design candidates to be stable under given loads. Structural response constraints are reserved to the next step and the kinematical stability condition is only checked here.

All problems in this paper are made up in the multi-criteria environment. So, topological alternatives favorable to the reference of several criteria must be extracted. For this purpose, a method has been developed, which can generate topologically better design alternatives with the several criteria aspects has been developed, and its summarized scheme is as follows.

Figure 3 shows how this scheme is implemented. One element which connects between the nodes is handled as the design variable in this stage, and then all possible topologies are explored by the MOGA. Viewing Figure 3, all topologies can be designated by circled-numbers, such as (1) or (2), in the figure having axes in terms of objectives. Among them, there are unnecessary topologies that violate the above-mentioned constraint, and meaningful topologies that satisfy the

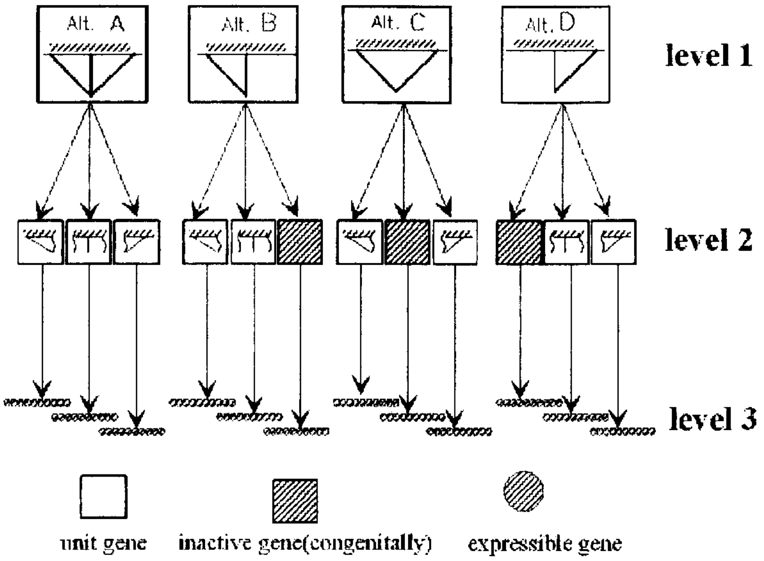

Figure 4: Hierarchical structural chromosome of StrGA_DeAl

stability condition, so that they can be designated in the feasible region. Among the meaningful topologies, there is also the discrimination arisen from the comparison of criteria values. For instance, the topologies numbered 2, 3 and 4 are superior to the ones numbered 5, 6 and 7, and these winners become the design alternatives for the next step. This process is similar to that of obtaining a Pareto set. MOGA is utilized to implement this scheme. Unlike traditional multi-criteria optimization methods where the transformation procedure is used to make single objective function problems, MOGA uses Pareto optimal conditions directly in comparison with each objective function value of multi-design points. This characteristic is at the same time appropriate for the generation of the design alternatives. Meanwhile, all variables in this step are only boolean genes which express the presence or the absence of the members.

There are two assumptions in Step 1. The first is the settling of all of the elements' size with the given resolution-strategy in order to reflect only the topology of a structure on the given criteria. Hajela et al(1993) used this method to extract structures favorable to the weight criterion. The second is that the alternatives selected as design alternatives hold their topological advantages in such way that they have predominance over unselected ones, even though they go through the process of geometric/sizing design. It is very difficult to give the strict proof for such assumptions on the mathematical ground, but rationale behind this assumption is in that in general, the topology can bring out much more significant effects on the final design compared with the preliminary design such as the sizing of structural members through fixed-layout optimization. We will give the partial proof in the case of 10-bar truss through numerical experiments. For the detailed description, see chapter 4.

3.2 Step 2: Representation and evaluation of design alternatives

As compared with the fixed topology problems, the proposed method for the given problems that have several topological design alternatives generated by the foregoing step, should have a obtained at one time, the proposed method must possess more diverse and minute searching ability. So, we have developed a new scheme of StrGA_DeAl + MOGA which can meet the foregoing requirements and effectively handle the design alternatives.

To complete the optimization process, the geometric and sizing problems should be included. Structural response constraints can thus be applied in this step with the consideration of multiple objectives such as minimum weight and minimum displacement. The design variables in this step are the nodal position and the members' area size.

First of all, it should be addressed on how design alternatives are represented and what merit this approach has. To embody design alternatives in StrGA's chromosomes, the roles of the unit and the expressible gene have to be newly assigned. First, according to the highest unit gene's genotype (level 1) condition, the relevant chromosome indicates which alternative is selected, and the switching condition of a lower unit gene at level 2 represents which alternative's module is activated. In the case of Figure 4, even if an intermediate layer directly expresses the element modules used as design variables, according to the problems, it is possible to have the activator controlling lower modules or lower alternatives. In other words, its layer depth can be deeper. On the other hand, the expressible gene is used as a concrete design variable's genotype that is included in a relevant active unit gene's tributary, and it is translated into the phenotype of a relevant alternative's variables in the evaluation step. The number of expressible genes decides the precision of the design variables.

Traditional configuration methods of chromosomes in topology problems have been made up of two parts with the unstructured form. One is a switch gene that determines the constitution of the topology, and the other is a normal gene that expresses design variables of the topologies determined by switch genes(Grierson and Pak 1993, Rajan 1995). So, it can be made out that normal genes are not fixed to the definite variables according to which switch gene is activated. This configuration method may be inappropriate to the structural problem where several different shape structures exist, and the selected topologies can be changed according to the ratio of objectives.

Before the examples are presented, Total Pareto Set(Yang et al 1998) must be defined. Consider multi-criteria problem with the several topological design alternatives. Each Pareto optimal set of topological design alternative can be depicted respectively in coordinate axes of objectives. However, there are also comparative superiority and inferiority among the respective Pareto optimal sets. We define non-inferior set obtained out of the Pareto sets of all topological design alternatives as Total Pareto set. This is the key feature in the problem of topology design in the multi-criteria environment.

4 Example: 10-bar truss

To generate and evaluate the topological alternatives of the relatively complex structure, such as the 10-bar truss in Figure 5, this paper selects the design problem where the optimum structure has to be determined in aspect of topology, geometry and sizing under concentrated loads at nodes 2 and 4. First, the design alternatives, whose topological structure is superior in terms of weight and displacement among the stable structures, are selected through Step 1. The nominal element area of the structures is 1 in2. Meanwhile, Young's modulus and the allowable stress used in the example are 1e7 and 25e3 lb/in2, respectively, and the chromosome length of one variable is set to four so that the design variables can have an integer only from one to sixteen.

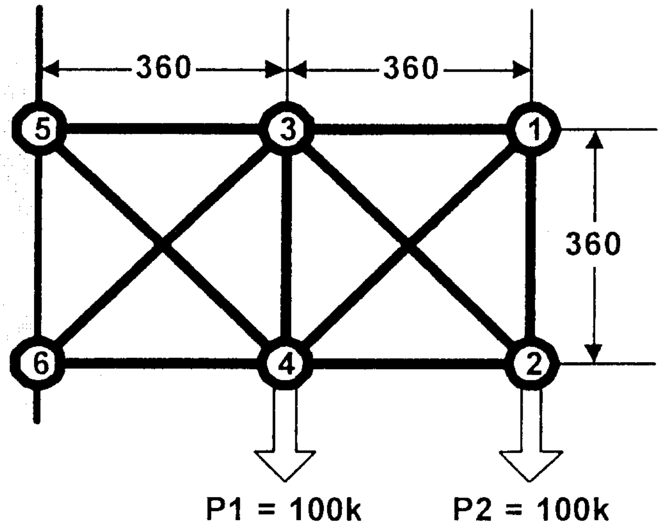

Figure 5: 10-bar truss: Ground structure

example are 1e7 and 25e3 lb/in2, respectively, and the chromosome length of one variable is set to four so that the design variables can have an integer only from one to sixteen.

The alternatives generated according to the different load cases from Step 1 (Generation of Design Alternatives) are depicted in Figure 6. Load Case #1 is loaded with P1 and P2 at the same time, and Load Case #2 is loaded only with P2. Note that the alternative C of Load Case #1 is peculiarly more favorable than a structure whose topology is the same as the alternative D in Load Case #2, in the both terms of weight and displacement.

As the result of numerical experiments, in Load Case #1, Alternatives B and C are the more favorable topological structures in terms of weight and displacement respectively, and in Load Case #2, alternatives A and C are the favorable topological structures.

This paper assumed that the alternative(s) topologically selected as the solution through Step 1 and Step 2 (Representation and Evaluation of Design Alternatives) is a true solution having correct topology among all possible topologies. If so, in order to confirm whether the obtained topologies are changed in the various design environments by changing the material and geometry, the following numerical experiments are conducted, including several additional topologies we can think, which are promising structures. They will be explained on the base of only the problem of Load Case #1.

Figure 7 shows Total Pareto sets according to the variation of the allowable stress. The alteration of the obtained topologies does not occur. However, it can be known that the Pareto set's pattern of alternative B is changed. This phenomenon is due to the fact that in order to reduce weight of alternative B, some elements areas are adjusted in such way that the stress level of these elements nearly reaches to the given allowable stress. On the contrary, the Pareto sets pattern of

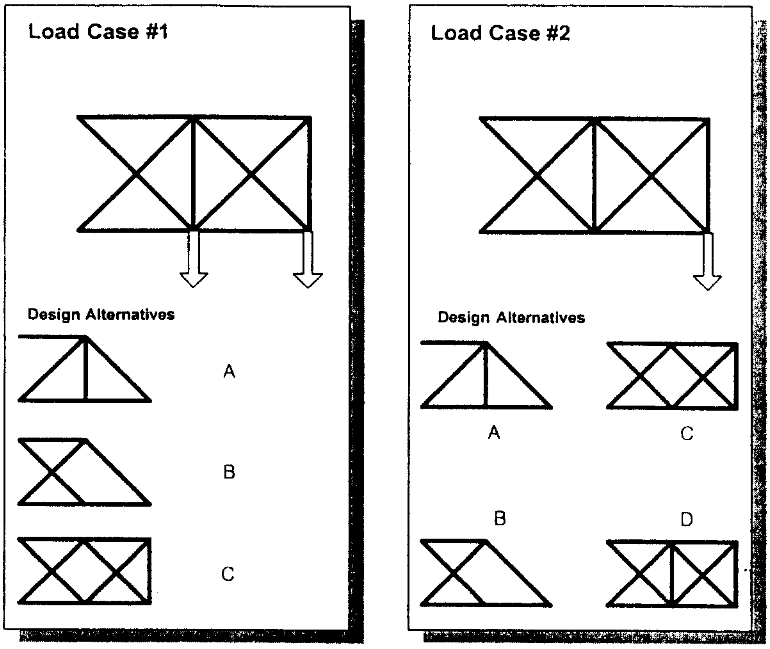

Figure 6: Design alternatives generated by step 1.

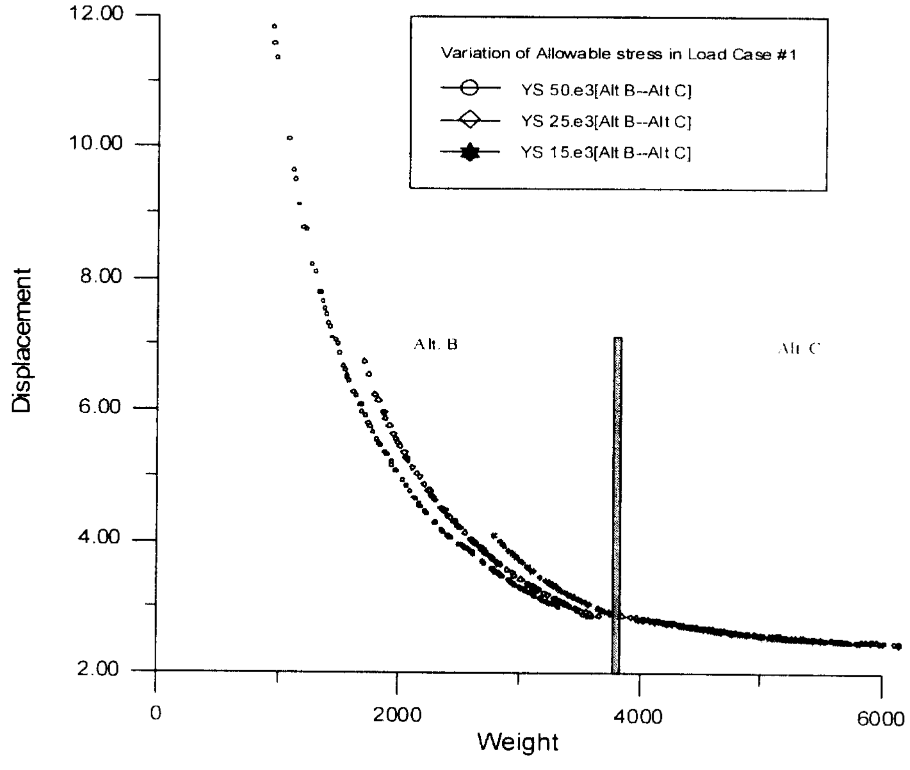

Figure 7: Total Pareto Sets in variation of yield stress

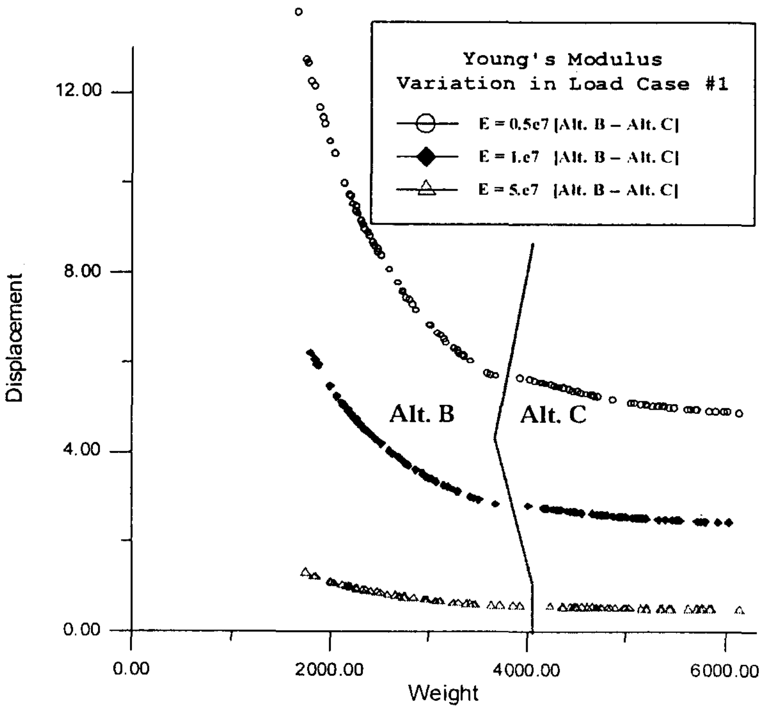

Figure 8: Total pareto sets in variation of young's modulus

alternative C does not change because it is designed to reduce displacement so that it is insensitive to the variation of the allowable stress. In addition, the alteration of obtained topologies also does not occur with the variation of Young's modulus, as shown in Figure 8. Note, however, that the optimal structure of smaller displacement with the same weight can be designed, as the strength of the material increases. In conclusion, these facts state that the material change of overall elements is indifferent to the decision of optimal topology.

Meanwhile, the geometric design is also carried out after the selection of topological alternatives. The geometric variables, which have the same continuous characteristic with sizing variables, are treated in Step 2. The result is given in Figure 9. The variation of geometry is allowed at the y coordinate of only nodes 1, 3 and 5, which are considered to have effects of applied loads, when considering the works done by Rajan(1995). There is also no alteration of obtained topologies. It can be proved that the quality of design is improved without the amelioration of material when considering the variation of geometry, and that topology design might be a higher concept that has a dominant effect over geometric design, judging from the fact that optimal solutions do not alter. Conclusively, it is observed that an optimal structure can be obtained by considering the variation of topology first, before that of material and geometry as well as sizing.

5 Conclusion

The novel paradigm, which can effectively treat topology, geometry and size design of truss structures, is presented in this paper. This structural design support system can generate and handle

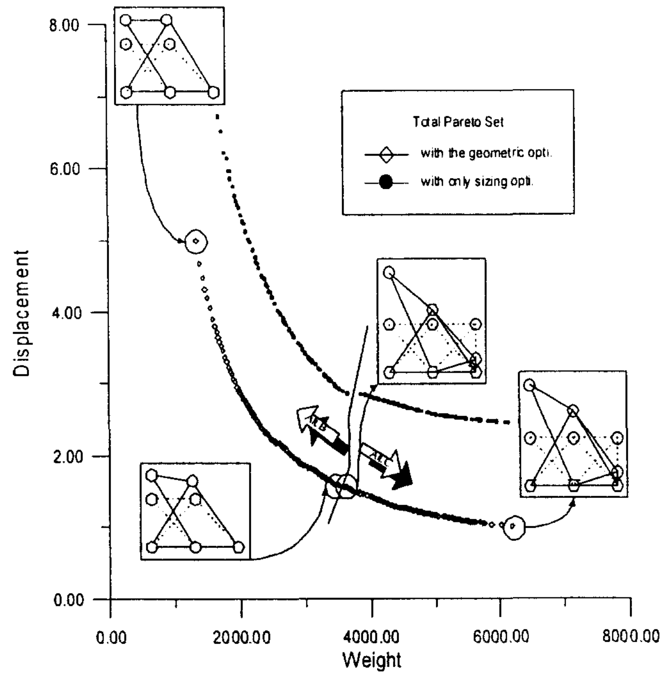

Figure 9: Total Pareto Set with the geometry design in Load Case #1.

various design alternatives on the basis of a multicriteria environment. StrGA_DeAl derived from StrGA takes charge of handling design alternatives, and MOGA is in charge of generating design alternatives in the stage of the conceptual design and in evaluating the several conflicting criteria.

Our approach taken in this paper, which generates design alternatives on the basis of topological aspects and then proceeds with the subsequent design, is motivated by the generic design process, which has usually been implemented with design alternatives as the central bridge between the conceptual and preliminary design. This approach may be more suitable for the situation where the given time for the designer is not plenty. The usefulness and effectiveness are proven by the truss examples. Unlike traditional structural optimization method that covered limited portions of the design process, this system would be expected to cover more extended design concepts including conceptual design as well as preliminary design.

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