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Impedance Imaging of Binary-Mixture Systems with Regularized Newton-Raphson Method

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Abstract — Impedance imaging for binary mixture is a kind of nonlinear inverse problem, which is usually solved iteratively by the Newton-Raphson method. Then, the ill-posedness of Hessian matrix often requires the use of a regularization method to stabilize the solution. In this study, the Levenberg-Marquardt regularization method is introduced for the binary-mixture system with various resistivity contrasts (1 : 2~1 : 1000). Several mixture distributions are tested and the results show that the Newton-Raphson iteration combined with the Levenberg-Marquardt regularization can reconstruct reasonably good images.

1. Introduction

The binary-mixture system can be encountered in many engineering applications like heat exchangers, oil or natural gas pumping system and chemical processing. Because the heterogeneous mixture distribution affects the safety, control, operation and optimization of process, it is important to monitor the mixture process. In this study, the Electrical Impedance Tomography (EIT) technique recently developed for medical purposes is employed to visualize the mixture distribution without disturbing the mixture field[1]. It should be noted that the EIT has good time resolution, which is essential for monitoring the mixture distribution under rapid transient.

In EIT technology, different current patterns are applied to the mixture system through the boundary electrodes and the corresponding voltages are measured. Based on the current-voltage relation, the internal impedance distribution, that is the mixture distribution, is reconstructed. The numerical algorithm used to convert the boundary measurement data to the internal impedance distribution consists of iteratively solving the forward problem and updating the impedance distribution as determined by the procedure of the EIT. In the forward stage of the EIT the boundary voltages is calculated with using assumed impedance distribution, while in the inverse stage the impedance distribution is reconstructed with boundary voltage measurements.

Since the EIT reconstruction problem is a nonlinear ill-posed inverse problem. various regularization methods have been proposed to weaken the ill-posedness and to obtain stable solution. The most often-used regularization methods in EIT for medical imaging are the Tikhonov regularization and the subspace regularization methods[2].

The present study adopts the Levenberg-Marquardt regularization, a variant of the Tikhonov regularization, to moderate the ill-posedness of the impedance imaging problem for the binary-mixture system. Especially, we consider the effect of the resistivity contrast between binary mixtures on the performance of the image reconstruction. In fact, most of previous studies on the regularization methods were carried out for relatively small impedance contrast, mainly up to several tens[3]. This study aims to present the possibility of the EIT for imaging mixture distribution and to investigate the applicability of the Levenberg-Marquardt method, one of most common regularization methods, to various mixtures with various impedance contrasts.

2. Mathematical Model of Impedance Imaging

The schematic diagram of the EIT system is given in Fig. 1. Mathematically, the EIT is composed of the forward problem to obtain the voltage distribution sub-

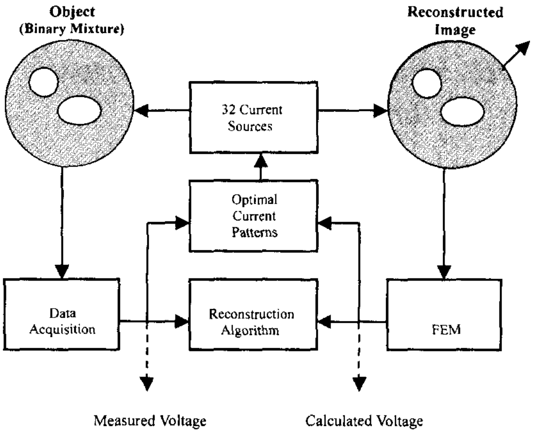


Fig. 1. Schematic diagram of EIT system.

jected to assumed impedance distribution and the inverse problem to reconstruct the impedance distribution under the measured boundary voltages. The forward and inverse problems are discussed as below.

When the impedance distribution $\rho(x, y)$ and boundary current I_l through the l'th electrode e_l are given, the electrical potential distribution u within the problem domain Ω with boundary $\partial\Omega$ is governed by the following Laplace equation and the Neuman type boundary conditions:

$$\nabla\cdot\left(\frac{1}{\rho}\nabla u\right)=0, \qquad \text{in } \Omega \qquad (1)$$

$$\int_{e_l}\frac{1}{\rho}\frac{\partial u}{\partial n}dS = I_l, \qquad l=1,\ 2,\ \ldots,\ L \qquad (2)$$

and

$$\int_{\partial\Omega}\frac{1}{\rho}\frac{\partial u}{\partial n}dS=0, \qquad (3)$$

where n is the outward normal unit vector. In addition, the following two condtions for the injected curent and boundary voltages are needed to ensure the uniquness of the solution:

$$\sum_{l=1}^{L} I_l=0, \qquad (4)$$

$$\sum_{l=1}^{L} U_l=0. \qquad (5)$$

U_l denotes the voltages on e_l.

Since the above equation cant be solved analytically for the arbitrary impedance distribution, the numerical method such as finite element method (FEM) should be employed to obtain the solutions. This work adopts the FEM and the grids shown in Fig. 2 are used. Due to the characteristics of inverse problems, it is recommended that the mesh structure for the image reconstruction process should be different from that for the artificial synthesis of resulting voltages in the numerical experiments. We used finer mesh structures for the artificial boundary voltages.

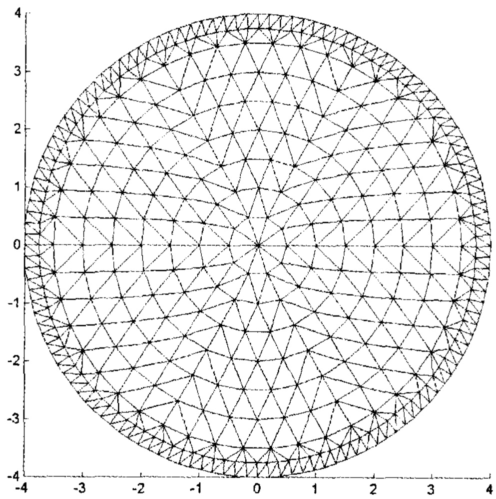


Fig. 2. Finite element meshes.

In most of EIT problems, the impedance within the element is assumed to be constant and the above differential equation is approximated by the system of algebraic equations in the context of the finite element method. The details of the finite element formulation and the solution procedure are given in elsewhere, e.g. Refs.[3][4].

The inverse problem of the EIT maps the boundary voltages from real or artificial experiments to impedance distribution. The objective function may be chosen to minimize the squared error,

$$\Phi(\rho)=\frac{1}{2}[V-U(\rho)]^T[V-U(\rho)] \qquad (6)$$

where V is the vector of measured voltage at the electrodes placed along the boundary and $U(\rho)$ is the calculated boundary voltage vector that must be matched to V.

To find ρ which minimize the above object function, its derivarivative is set to zero as:

$$\Phi'(\rho)=-[U']^T[V-U]=0 \quad (7)$$

where $[U']_{ij}=\partial U_i/\partial \rho_j$ is the Jacobian matrix. For the solution of the above Eq. (7) we use the Newton-Raphson linearization about a resistivity vector ρ^k at the k-th iteration as

$$\Phi'(\rho^{k+1})=\Phi'(\rho^k)+\Phi''(\rho^k)(r^{k+1}-\rho^k)=0. \quad (8)$$

The term Φ'' is called the Hessian matrix, expressed as

$$\Phi''=[U']^TU'-[U'']^T\{I\otimes[V-U]\} \quad (9)$$

where $\otimes$ is the Kronecker matrix product and I stands for the identity matrix. Since U'' is difficult to calculate accurately and relatively small, the second term in the above equation is usually omitted[1]. Therefore, the Hessian matrix is modified as

$$\Phi''=[U']^TU'=J^TJ \quad (10)$$

where J is the Jacobian matrix. Finally, the iterative equation for updating the resistivity vector based on the above object function is derived as

$$\rho^{k+1}=\rho^k+(J^TJ)^{-1}[J^T\{V-U(\rho^k)\}]. \quad (11)$$

The Hessian matrix is known to be ill-conditioned. The ill-conditioning degrades the performance of image reconstruction algorithm. To overcome this ill-posedness, the regularization should be introduced. In standard Tikhonov regularization method[2] the regularization matrix is a diagonal weighting for J^TJ and the equation for the increment $\Delta\rho^k=\rho^{k+1}-\rho^k$ is

$$\Delta\rho^k=(J^TJ+\alpha\ \mathrm{diag}(R^TR))^{-1}[J^T(V-U(\rho^k))]. \quad (12)$$

where α and R are regularization parameter and matrix, respectively. If α=0, of course, the regularization method turns into the conventional Newton-Raphson method. The term including the regularization matrix can be thought to represent an approximation for the second term in the Hessian matrix Eq. (9). In the Levenberg-Marquardt regularization, the regularized term is set to $R^TR=I$[5].

There are many data collecting methods such as neighboring method, cross method, opposite method, multireference method and adaptive method. The characteristics of the methods are summarized in Webster's monograph[2]. Among these, the adaptive method, where desired current distribution can be obtained by injecting current through all the electrode simultaneously, is known to be the best method. In this sutdy, we inject trigonometric current patterns into 32 electrodes simultaneously as follows:

$$I_l^k=\begin{cases}\cos(k\zeta_l) & l=1,2,\cdots,32,k=1,2,\cdots,16\\ \sin(k\zeta_l) & l=1,2,\cdots,32,k=1,2,\cdots,15\end{cases} \quad (13)$$

where $\zeta_l=2\pi_l/32$.

3. Numerical Experiments and Discussions

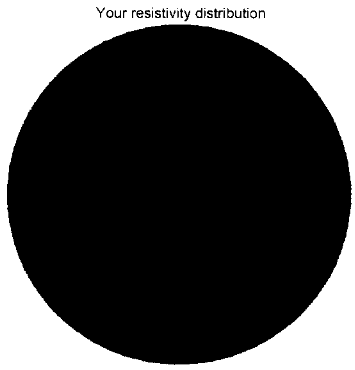
The resolution of the EIT system depends on the various variables, such as impedance contrast, mixture distribution and object size. Therefore, to verify the appropriateness of the proposed EIT algorithm and to illustrate the effect of the impedance contrast on the reconstructed image, a series of simulation is conducted for several numerical examples.

In the present study, the root-mean-squared relative error

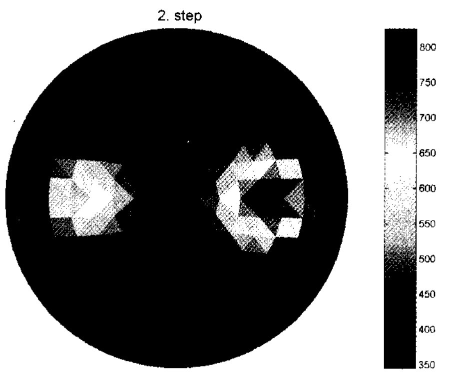
$$\varepsilon=\sqrt{\frac{(U-V)^T(U-V)}{V^TV}} \quad (14)$$

is used as the convergence criterion for the inverse problem. If ε is less than predetermined small value, convergence is assumed and the reconstruction algorithm is stopped. In this study, 10^{-3} is chosen as a stopping criterion. Our experience shows that the quality of reconstructed image does not improve after several iteration steps, so maximum iteration number is set to 10.

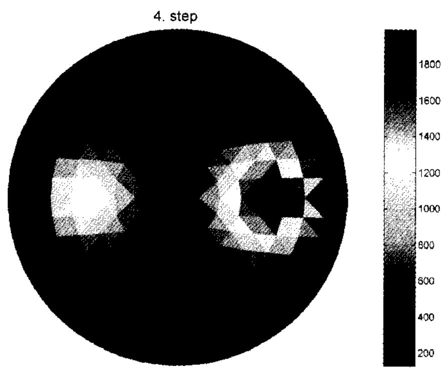
To verify the appropriateness of the EIT to monitor binary mixtures and to investigate the effect of the resistivity contrast, we consider an artificial resistivity distribution with various resistivity contrasts under the synthetic boundary volatages obtained by the forward solver described earlier. Numerical results are given in Fig. 3. Figure 3(a) is the original mixture distribution to be reconstructed. The others, Fig. 3(b)-(e), are the reconstructed images for the above distribution with four resistivity ratios of dispersed phase to continuous phase; 1 : 2, 1 : 10, 1 : 100 and 1 : 1000. It should be noted that in the application of EIT to medical imag-



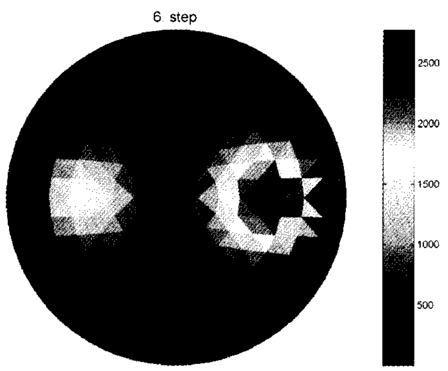
(a) Original mixture distribution



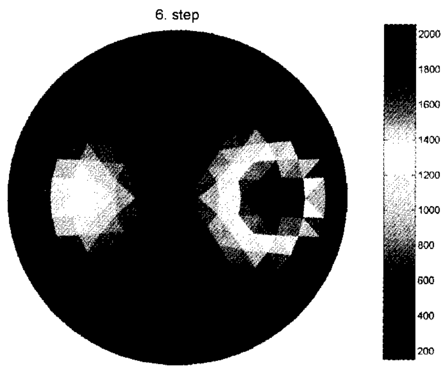
(b) Contrast=2, α=0.00001



(c) Contrast=10, α=0.00001



(d) Contrast=100, α=0.00001



(e) Contrast=1000, α=0.00001

Fig. 3. Reconstructed image of binary mixture for various resistivity contrasts.

ing the resistivity contrast is usually less than 1 : 10[11]. As the contrast increases, the images reconstructed by EIT are expected to become poorer because higher contrast tends to enhance the ill-posedness of the Hessian matrix. As can be seen in Fig. 4 for the binary mixture with resistivity contrast of 2, the EIT algorithm developed in this study can reach the convergence after just 2 iterations and it can predict the location of the dispersed phase. It should be noted that the predicted impedance values are fairly close to

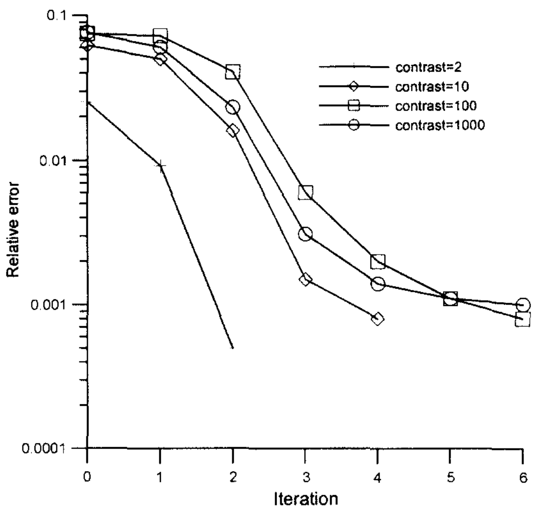


Fig. 4. Relative errors.

the original ones. For contrast of 10, the number of iterations required for the convergence criterion increases to 4. Also, the present algorithm reconstructs a good image and predicts the impedance values quite well. For binary mixtures with resistivity contrasts of 100 and 1000, Fig. 3(d) and 3(e) show that the algorithm still gives good images although more iterations are required. Then, the impedance values are hardly predicted. In view of Fig. 3(c-e), the numerical experiments consistently show that the predicted impedance contrasts are about 10. Nevertheless, the present EIT algorithm employing Levenberg-Marquardt regularization can reproduce the original phase distribution quite reasonably. The trend of relative errors during the iteration shown in Fig. 4 indicates that within 3~4 iterations the relative errors fall down below 0.01 and then decrease gradually.

Now, we consider the effect of the regularization parameter on the image reconstruction. The regularization parameters stated in Fig. 3 are chosen after simulations with $\alpha=10^{-6}$, 10^{-5}, ..., 0.1. For the first 3 cases $\alpha=10^{-5}$ gives the best results, while for the last one with the highest contrast $\alpha=10^{-5}$ fails to give a converged result and $\alpha=10^{-4}$ is selected as the optimized one.

4. Conclusions

The electrical impedance-imaging algorithm is applied to for the visualization of binary mixture system. This EIT technique that has been proposed for the medical application has some obvious advantages like non-intrusive character and rapid response, which are useful for monitoring the behavior of the mixture. However, the inverse problem in the EIT procedure is highly nonlinear and it is required to introduce the regularization method. For the reproduction of the mixture distribution, in the present work, we developed an EIT algorithm employing the Levenberg-Marquardt regularization method. Also, binary mixtures with various impedance contrasts ranging from 1 : 2 to 1 : 1000 are considered in the numerical experiments. As the resistivity contrast increases more iterations are needed to obtain converged results. For the contrast range considered in this study, the numerical results show that the present EIT model with the regularization parameter $\alpha=10^{-5}\sim10^{-4}$ can reconstruct the images of binary mixtures quite reasonably.

Acknowledgement

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