On the Forced Resonant Characteristics of Partially Filled Electrically Small Cavity with Loaded Reactance

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Abstract

This paper presents the forced resonant characteristics of an electrically small cavity partially filled with dielectric material. The method of moments with Galerkins procedure is used to determine the forced resonant characteristics of the small cavity. In order to obtain the equations of the external reactance gives rise to the forced resonance at a given frequency, the cavity with external reactance can be treated as two-port network which has the admittance parameters. Numerical results show that the forced resonance, series or parallel resonance, can be obtained by the controlling the external reactance. To verify the availability of the theoretical analysis, experiments are carried out for the bakelite as the material by measuring the length of external reactance at operating frequencies.

Ⅰ. Introduction

Previously, work has been done on the analysis and application of the forced resonance characteristics for an electrically small cavity with external loaded reactance[1],[2]. The forced resonance of the electrically small cavity is very useful for measuring dielectric properties especially at low frequencies. In fact, small sized cavity is enough to measure dielectric constants, since resonant condition can easily be achieved by controlling the external reactance. In this paper, the analysis of forced resonant characteristics is extended to the electrically small cavity partially filled with dielectric material for easy setting of the material. The method of moments with Galerkin's procedure is used to determine the probe current from which the general characteristics of the small cavity with the externally connected reactance element is calculated. To derive the determining equation for forced resonances, the cavity with external reactance can be treated as two-port network which has the admittance parameters. From this procedure, we can obtain the equations of the external reactance gives rise to the forced resonance at a given frequency.

Numerical results show that the forced resonance can be easily obtained by controlling the externally connected reactance element for the series resonance as well as for the parallel resonance. These resonant characteristics are conformed by the calculation and experiments of the input impedance. In order to verify the availability of the theoretical analysis, the length of external reactance was measured for the bakelite as the dielectric material at given frequencies.

Ⅱ. Formulation of Problem

Fig. 1 shows the geometry of the electrically small cavity with connected external reactance. A cavity with a cross section $a \times b$ and depth u is partially filled with a dielectric material. The excitation post of radius r_1 is located at $x=d$ and $z=s$. The post and cavity walls are assumed to be perfect electric conductors. The external reactance of length l is connected to the excitation post. This reactance is used to enforce the resonance of the electrically small cavity partially filled with the dielectric material.

The cavity dimensions are chosen as the cross section of the cavity $a \times b$ corresponds to the cutoff condition of the waveguide with the cross section $a \times b$ when the cavity is empty. For this reason we named it cutoff cavity.

The resonant condition of the cutoff cavity shown in Fig. 1 is given by

$$Im\ \{Z_{in}(\varepsilon_r, f, X(l))\} = 0 \tag{1}$$

where Im represents the imaginary part of the complex quantity. From this resonant condition, we can determine the forced resonant characteristics of the cutoff cavity partially filled with dielectric by adjusting the length of the external reactance. In (1), the input impedance Z_{in} is the impedance looking to the electrically small cavity partially filled with dielectric material.

The cutoff cavity is excited at $y=0$ with a voltage V by a delta-function generator. We assume a similar delta gap voltage at the loading position $y=b$. Then the integral equation governing the current distribution on the excitation post and the aperture electric field distribution on the dielectric material surface

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($z=c$) can be written as

$$\frac{1}{j\omega\varepsilon_0}\iint_S \overline{K}^I_{ee}(r,r')\cdot J(r')dS' - \frac{1}{j\omega\varepsilon_0}\iint_{S_b} \overline{K}^I_{em}(r,r')\cdot(\hat{z}\times E_b)dS'_b = -V\delta(y)\hat{y} + jXI(b)\delta(y-b)\hat{y} \quad (2)$$

$$\left[\iint_S \overline{K}^I_{he}(r,r')\cdot J(r')dS' + \frac{1}{j\omega\mu_0}\iint_{S_b} \overline{K}^I_{hm}(r,r')\cdot(\hat{z}\times E_b)dS'_b = \frac{1}{j\omega\mu_0}\iint_{S_b} \overline{K}^{II}_{hm}(r,r')\cdot(E_b\times\hat{z})dS'_b\right]_{t.c.} \quad (3)$$

$t.c.$: tangential components

where

$$\overline{K}^I_{ee}(r,r') = (\bar{I}k_0^2 + \nabla\nabla)\cdot\overline{G}^I_e(r,r') \quad (4)$$

$$\overline{K}^I_{em}(r,r') = \nabla\times\overline{G}^I_m(r,r') \quad (5)$$

$$\overline{K}^I_{he}(r,r') = \nabla\times\overline{G}^I_e(r,r') \quad (6)$$

$$\overline{K}^I_{hm}(r,r') = (\bar{I}k_0^2 + \nabla\nabla)\cdot\overline{G}^I_m(r,r') \quad (7)$$

$$\overline{K}^{II}_{hm}(r,r') = (\bar{I}k_0^2\varepsilon_r + \nabla\nabla)\cdot\overline{G}^{II}_m(r,r') \quad (8)$$

$$k = \omega\sqrt{\varepsilon\mu_0} = k_0\sqrt{\varepsilon_r}$$

$$k_0 = \omega\sqrt{\varepsilon_0\mu_0}$$

$$\varepsilon = \varepsilon_r\varepsilon_0$$

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$

and $\bar{I}$ is a unit dyadic. Moreover, $\hat{y}$ indicates a unit vector in the y direction, $\delta(*)$ is the Dirac delta-function k_0, and ω represent the free space wave number and the angular frequency, respectively. And position vectors r and r' are for the observation and source points, respectively. The $J(r')$ represents the surface current density, dS' is an element of area on the surface of the excitation post, and $I(b)$ represents the current at the loading position of the external reactance jX. The time dependence $exp\ (j\omega t)$ is assumed and omitted throughout this paper.

If the radius r_1 of the post is sufficiently small, the current density may be considered to be uniform around the periphery of the post. Thus the integral equation (2) is simply represented as the y direction integral only since $J(r') = \hat{y}I(y')/2\pi r_1$. In (2) the dyadic Green function of the electric type for the cavity is given by [3]

$$\overline{G}^I_e = -\hat{y}\hat{y}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\varepsilon_{0m}}{ab\Gamma^I_{nm}}\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b} \cdot \sin\frac{n\pi x'}{a}\cos\frac{m\pi y'}{b}L^I_{nm}(z,z') \quad (9)$$

$$L^I_{nm} = \begin{cases} \dfrac{\sinh\Gamma^I_{nm}z'}{\sinh\Gamma^I_{nm}c}\sinh\Gamma^I_{nm}(z-c), & z\ge z' \\ \dfrac{\sinh\Gamma^I_{nm}(z'-c)}{\sinh\Gamma^I_{nm}c}\sinh\Gamma^I_{nm}z, & z\le z' \end{cases} \quad (10)$$

$$\overline{G}^I_m = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\varepsilon_{0n}\varepsilon_{0m}}{ab\Gamma^I_{nm}}(\hat{x}\hat{x}N_x + \hat{y}\hat{y}N_y)M^I_{nm}(z,z') \quad (11)$$

$$M^I_{nm} = \begin{cases} \dfrac{\cosh\Gamma^I_{nm}z'}{\sinh 2\Gamma^I_{nm}c}\cosh\Gamma^I_{nm}(z-2c), & z\ge z' \\ \dfrac{\cosh\Gamma^I_{nm}(z'-2c)}{\sinh 2\Gamma^I_{nm}c}\cosh\Gamma^I_{nm}z, & z\le z' \end{cases} \quad (12)$$

$$\overline{G}^{II}_m = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\varepsilon_{0n}\varepsilon_{0m}}{ab\Gamma^{II}_{nm}}(\hat{x}\hat{x}N_x + \hat{y}\hat{y}N_y)M^{II}_{nm}(z,z') \quad (13)$$

$$M^{II}_{nm} = \begin{cases} \dfrac{\cosh\Gamma^{II}_{nm}(z'-z_1)}{\sinh 2\Gamma^I_{nm}u}\cosh\Gamma^{II}_{nm}(z-z_2), & z\ge z' \\ \dfrac{\cosh\Gamma^{II}_{nm}(z'-z_2)}{\sinh 2\Gamma^I_{nm}u}\cosh\Gamma^{II}_{nm}(z-z_1), & z\le z' \end{cases} \quad (14)$$

$$\varepsilon_{0n} = 1\ (for\ n=0),\ 2\ (for\ n\ne 0)$$

$$z_1 = c-u, \quad z_2 = c+u$$

$$\Gamma^I_{nm} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 - k_0^2} \quad (15)$$

$$\Gamma^{II}_{nm} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 - k_0^2\varepsilon_r} \quad (16)$$

$$N_x = \sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}\sin\frac{n\pi x'}{a}\cos\frac{m\pi y'}{b} \quad (17)$$

$$N_y = \cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b}\cos\frac{n\pi x'}{a}\sin\frac{m\pi y'}{b} \quad (18)$$

The integral equation (2) for the unknown current $J(y')$ and the aperture electric field E_b can be solved numerically by the method of moments with Galerkin's procedure. Let $J(y')$ and E_b are expanded in a series of cosine functions as

$$\mathbf{J}(y') = \hat{y}\sum_{l=0}^{L}I_lF_l(y') \quad (19)$$

$$E_b(x',y') = \hat{x}\sum_{p=0}^{P}\sum_{q=1}^{Q}E_{xpq}C_p(x')S_q(y') + \hat{y}\sum_{p=1}^{P}\sum_{q=0}^{Q}E_{ypq}S_p(x')C_q(y') \quad (20)$$

$$F_l(y') = \cos\frac{l\pi y'}{b} \tag{21}$$

$$C_p(x') = \cos\frac{p\pi x'}{a}, \quad S_q(y') = \sin\frac{q\pi y'}{b}, \tag{22}$$

$$S_p(x') = \sin\frac{p\pi x'}{a'}, \quad C_q(y') = \cos\frac{q\pi y'}{b}, \tag{23}$$

where the I_l, E_{xpq} and E_{ypq} are complex expansion coefficients. Substituting (19) and (20) into the integral equation (2) and (3) we obtain

$$\begin{bmatrix} [Z_{l'l}] & [B_{xl'pq}] & [B_{yl'pq}] \\ [C^x_{p'q'l}] & [Y^x_{xp'q'pq}] & [Y^x_{yp'q'pq}] \\ [C^y_{p'q'l}] & [Y^y_{xp'q'pq}] & [Y^y_{yp'q'pq}] \end{bmatrix} \begin{Bmatrix} \{I_l\} \\ \{E_{xpq}\} \\ \{E_{ypq}\} \end{Bmatrix} = \begin{Bmatrix} \{V_{l'}\} \\ \{0\} \\ \{0\} \end{Bmatrix}. \tag{24}$$

The elements of the matrix are represented at appendix.

Ⅲ. Resonant Lengths of External Reactance

The cutoff cavity shown in Fig. 1 can be treated as a two-port network which has the following admittance representation

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11}(\varepsilon_r, f) & -y_{12}(\varepsilon_r, f) \\ -y_{21}(\varepsilon_r, f) & y_{22}(\varepsilon_r, f) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{25}$$

We choose port 1 and 2 as the driving point and the externally loaded reactance point, respectively. Loading the reactance jX imposes the following constraint on port 2:

$$i_2 = -\frac{v_2}{jX} \tag{26}$$

Combining (25) and (26) yields the following input impedance

$$Z_{in} = \frac{y_{22} + (jX)^{-1}}{y_{11}\{y_{22} + (jX)^{-1}\} - y_{12}^2} \tag{27}$$

The input impedance on the port 1 is the impedance looking to the cavity partially filled with dielectric material.

To derive the determining equation for forced resonances, we split the components y_{ij} in (27) into the real and imaginary parts. Then substituting (27) into (1), we obtain the following equation for the external reactance:

$$X^{-2} y^I_{11} - X^{-1} E - F = 0 \tag{28}$$

where

$$E = 2 y^I_{11} y^I_{22} + (y^R_{12})^2 - (y^I_{12})^2 \tag{29}$$

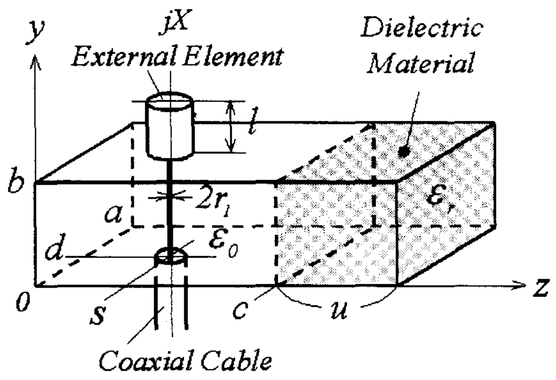


Fig. 1. Configuration of an electrically small cavity partially filled with dielectric material.

$$\begin{aligned} F = y^I_{22}(y^I_{12})^2 - y^I_{22}(y^R_{12})^2 - y^I_{11}(y^I_{22})^2 \\ - y^I_{11}(y^R_{22})^2 + 2 y^R_{22} y^R_{12} y^I_{12} \end{aligned} \tag{30}$$

and $y_{ij}{}^R$ and $y_{ij}{}^I$ represent the real part and imaginary part of y_{ij}, respectively. For calculation of these admittance parameters, we can use the method of moments as described previously.

The solution of (28) is simply obtained as follows:

$$X^{-1} = \frac{E \pm \sqrt{E^2 + 4 y^I_{11} F}}{2 y^I_{11}} \tag{31}$$

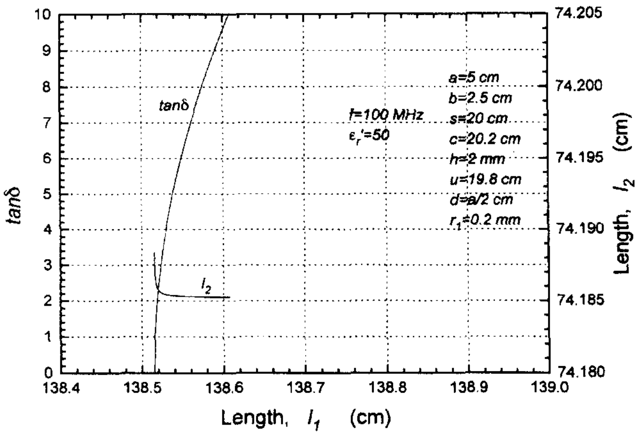
Now we use the short-circuited transmission line of length l and characteristic impedance Z_c as a external reactance component. In this case the reactance can be expressed as $X = Z_c \tan(k_0 l)$. Substituting this relation into (31), then (31) can be expressed as

$$\tan(k_0 l) = \frac{2 y^I_{11}}{Z_c(E \pm \sqrt{E^2 + 4 y^I_{11} F})}. \tag{32}$$

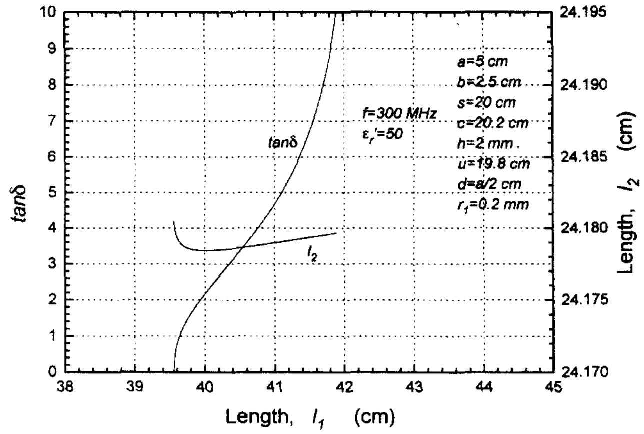
From this equation, we can obtain the equations of the lengths of the external reactance gives rise to the forced resonance at a given frequency as follows:

$$l_{1,2} = k_0^{-1} \tan^{-1}\left(\frac{2 y^I_{11}}{Z_c[E \pm \sqrt{E^2 + 4 y^I_{11} F}]} \right). \tag{33}$$

As is evident from (33), two independent lengths of the external reactance satisfy the resonant condition for the dielectric constant of the material at the same operating frequency. The plus sign in (33) corresponds to the series resonance and the



(a)



(b)

Fig. 2. Forced resonant characteristics as a parameter of frequency. (a) f=100 MHz, (b) f=300 MHz

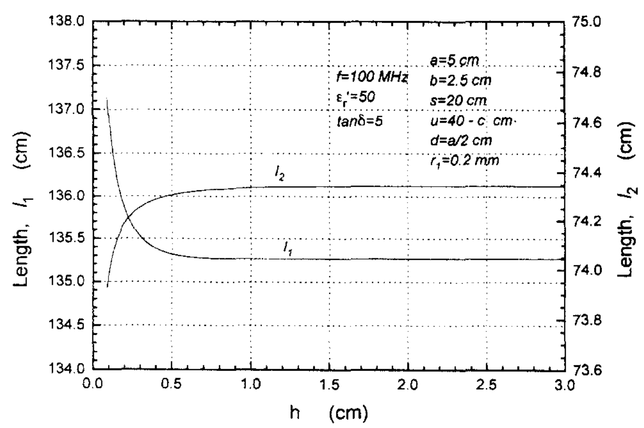


Fig. 3. The length of external reactance vs. the distance *h*.

minus sign to the parallel resonant. These resonant characteristics are conformed by the calculation and experiments of the input impedance.

Ⅳ. Numerical and Experimental Results

Fig. 2 shows the relation between the dielectric constant (ε_r', $tan\ \delta$) and the length of external reactance l giving the resonance calculated by using (33) in the case of 100 MHz and 300 MHz. In Fig. 2, as mentioned above, we represent l_1 for the series resonance and l_2 for the parallel resonance, respectively. As shown in Fig. 2 the cavity can be resonated at the given frequencies at two independent fixed lengths. For example, in the Fig. 2(b), the length of $l_1 = 41.099\ cm$ is resonated at the frequency of 300 MHz as a series resonance when $tan\ \delta = 5$. And the length of $l_2 = 24.179\ cm$ is also resonated at 300 MHz as a parallel resonance when $tan\ \delta = 5$. It is found from these results that the dielectric constant can be determined easily by controlling the length of external reactance.

Fig. 3 shows the effect of the distance h $(= c - s)$ between the post and the dielectric material in the case of 100 MHz. The result shows that the cavity resonant is independent on the dielectric material above $h \approx 1cm (\approx 0.003\lambda)$. And the effect of the distance s between the post and the cavity wall is also independent on the input impedance above $s \approx 0.37\lambda$. These are very important things for the forced resonant characteristics with a reduced structure parameter.

Fig. 4 represents an example of the current distribution on the driving post for the series resonance. The phase of the current is a constant along the post, but the amplitude of the current is a approximately constant.

Fig. 5 shows an example of the electric field distribution on the $z = c$ for the series resonance at the same structure parameters as shown in Fig. 4. The electric field intensity is a strong at the loading point. From the electric field distributions, the effect of the dielectric material for the forced resonance characteristics is mainly affected by the electric field around the loading position.

In order to verify the availability of the theoretical analysis, the length of external reactance was measured for the two cases of the air and bakelite as the dielectric material at given frequencies. The air is very good dielectric material for experiments because it has a unit value (1.0006 at atmospheric pressure) of the dielectric constant.

A measurement setup comprised of an HP-8753D vector network analyzer and an electrically small cavity partially filled with dielectric material is shown in Fig. 6, and the variable reactance made by Nihonkosyuha Co. (model No. SS-SP-91, Z_c

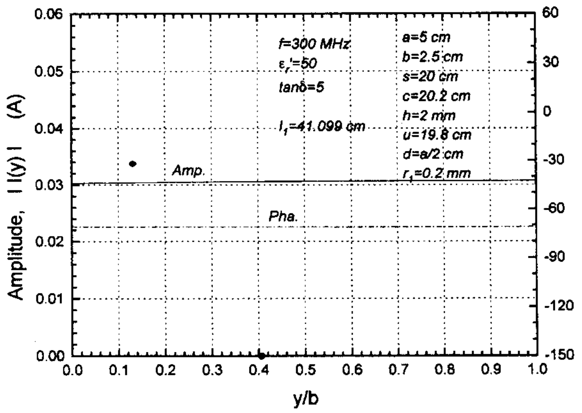


Fig. 4. Current distribution of the post.

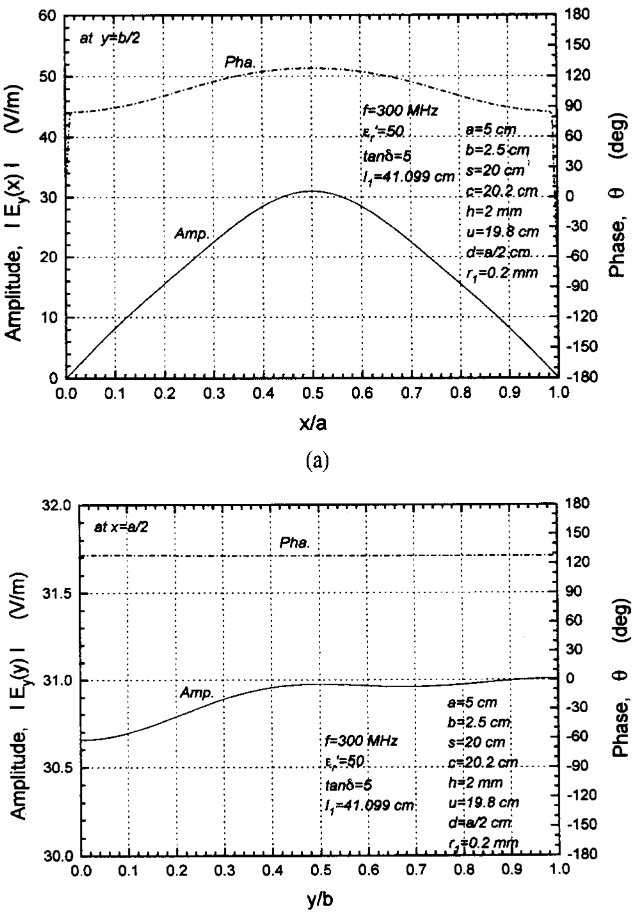


Fig. 5. Electric field distributions at $z=c$. (a) $E_y(x)$, (b) $E_y(y)$.

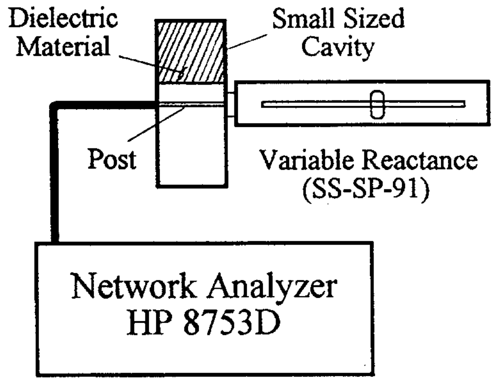


Fig. 6. Experimental setup.

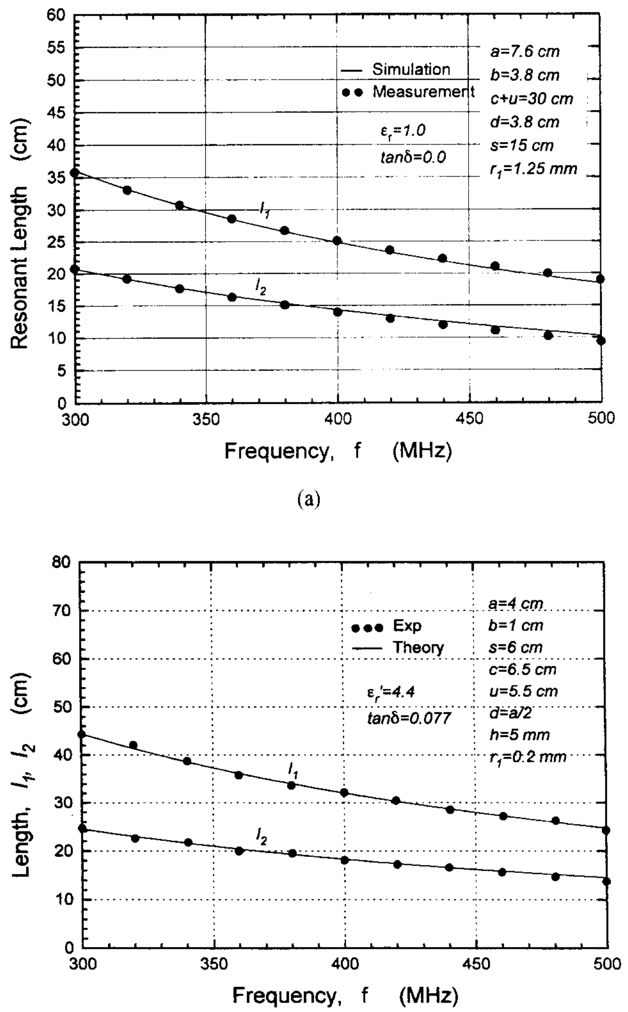


Fig. 7. Theoretical and experimental resonant lengths for the air (a) and bakelite (b).

$=50\,\Omega$) was used as the external reactance. For searching the resonant length, we adjust the variable reactance to enforce cavity resonance($\mathrm{Im}\{Z_{in}\}=0$) at given frequency. One difficulty in resonant length measurement is the errors caused by the measurement setup. These errors are known as systematic errors which can be removed by an external calibration procedure.

Two types of cavity were used for experiments. For the specimen of air, we have used the cavity size of $7.6 \times 3.8 \times 40$ cm which is a commercial *Al* size. And for the bakelite specimen, we have used the other type of cavity made of *Cu* plate of which dimension is $4 \times 1 \times 12$ cm, and operating frequencies varies from 300 to 500 MHz.

Figs. 7(a), (b) show the measured and calculated resonant lengths of the air and bakelite materials over the frequency range 300 - 500 MHz, respectively. It is shown that the measured lengths of the external reactance at given frequencies are almost equal to calculated ones based on the theory presented in this paper.

V. Conclusion

We have analyzed the forced resonance characteristics of the electrically small cavity partially filled with dielectric material. To derive the determining equation for forced resonances, the cavity with external reactance can be treated as two-port network which has the admittance parameters. It is demonstrated that the forced resonance characteristics of the cavity by the externally loaded reactance element. This resonant characteristics can be applied especially for measuring the complex dielectric constant at low frequencies, since it has the resonant condition on small sized cavity by the externally controlled element.

References

[1] K. C. Kim and S. Tokumaru: "On the Method for measuring dielectric constants of low-loss materials by forced resonance", *Trans. of IEICE*, vol. J79-B-II, no.4, pp.268-271, 1996.

[2] K. C. Kim, K. Uchida, and S. Tokumaru: "Analysis of a Forced Resonance Characteristics and Its Application to Measuring Dielectric Constants of Low-Loss Materials", *J. of EEIS*, vol. 1, no.3, pp.65-74, 1996.

[3] R. E. Collin: "*Field Theory of Guided Waves*", IEEE Press, 2nd edition, Chap.5, 1990.

Appendix

The expressions for the elements of the matrix appered in Eq. (24) are as follows.

$$Z_{l'l} = \frac{1}{j\omega\varepsilon_0}\iint_S \iint_{S'} F_{l'}(y)\hat{y}\cdot\overline{\boldsymbol{K}}^{I}_{ee}(\boldsymbol{r},\boldsymbol{r}')\cdot\hat{y}F_l(y')dS'dS \quad \text{(A-1)}$$

$$B_{xl'pq} = -\iint_S \iint_{S'_b} F_{l'}(y)\hat{y}\cdot\overline{\boldsymbol{K}}^{I}_{em}(\boldsymbol{r},\boldsymbol{r}')\cdot\hat{y}C_p(x')S_q(y')dS'_b dS \quad \text{(A-2)}$$

$$B_{yl'pq} = \iint_S \iint_{S'_b} F_{l'}(y)\hat{y}\cdot\overline{\boldsymbol{K}}^{I}_{em}(\boldsymbol{r},\boldsymbol{r}')\cdot\hat{x}S_p(x')C_q(y')dS'_b dS \quad \text{(A-3)}$$

$$C^{x,y}_{p'q'l} = \iint_{S_b} \iint_{S'} (\hat{y}C_{p'}(x)S_{q'}(y) - \hat{x}S_{p'}(x)C_{q'}(y)) \cdot\overline{\boldsymbol{K}}^{I}_{he}(\boldsymbol{r},\boldsymbol{r}')\cdot\hat{y}F_l(y')dS'dS_b\Big|_{x,y} \quad \text{(A-4)}$$

$$Y^{x,y}_{xp'q'pq} = \frac{1}{j\omega\mu_0}\iint_{S_b} \iint_{S'_b} (\hat{y}C_{p'}(x)S_{q'}(y) - \hat{x}S_{p'}(x)C_{q'}(y)) \cdot[\overline{\boldsymbol{K}}^{I}_{hm}(\boldsymbol{r},\boldsymbol{r}') + \overline{\boldsymbol{K}}^{II}_{hm}(\boldsymbol{r},\boldsymbol{r}')]\cdot\hat{y}C_p(x')S_q(y')dS'_b dS_b\Big|_{x,y} \quad \text{(A-5)}$$

$$Y^{x,y}_{yp'q'pq} = \frac{-1}{j\omega\mu_0}\iint_{S_b} \iint_{S'_b} (\hat{y}C_{p'}(x)S_{q'}(y) - \hat{x}S_{p'}(x)C_{q'}(y)) \cdot[\overline{\boldsymbol{K}}^{I}_{hm}(\boldsymbol{r},\boldsymbol{r}') + \overline{\boldsymbol{K}}^{II}_{hm}(\boldsymbol{r},\boldsymbol{r}')]\cdot\hat{x}S_p(x')C_q(y')dS'_b dS_b\Big|_{x,y} \quad \text{(A-6)}$$

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