Computational Structural Engineering
An International Journal
Vol. 1, No. 2, pp. 97~106 (2001)

Computational Structural Engineering Institute

Probabilistic Reliability Assessment of Steel Frame with Leaning Columns

Leo Václavek† and Pavel Marek[1]

†Ing. Leo Václavek,CSc., VŠB Technical University of Ostrava Faculty of Mechanical Engineering, 17.listopadu 15, 708 33 Ostrava-Poruba, Czech Republic, E-mail: leo.vaclavek@vsb.cz
[1]Prof.Ing. Pavel Marek,DrSc., (a) Institute of Theoretical and Applied Mechanics Academy of Sciences of Czech Republic, Prosecká 76 , 190 00 Prague 9, Czech Republic, E-mail: marekp@itam.cas.cz,
(b) VŠB Technical University of Ostrava Faculty of Civil Engineering , 17.listopadu 15, 708 33 Ostrava-Poruba

Received May 2000; Accepted September 2001

ABSTRACT

Safety and serviceability of a planar steel frame are assessed. Attention is turned to the individual main steps in the assessment procedure, i.e., to the definition of loads, selection of transformation model, determination of the response of the structure to the loading, and to the definition of the limiting values (considering safety and serviceability of the structure). The potential of the method using direct Monte Carlo technique as a powerful tool is emphasized.

Keywords: reliability, safety, serviceability, frame, Monte Carlo simulation

1. Introduction

The transition from primitive computational tools to the application of dramatically growing potential of the computer technology affects, among others, the development of structural reliability assessment concepts. In the current specifications there are applied methods, such as Allowable Stress Design and Partial Safety Factors, which had been introduced in the specifications for structural design in the "slide rule" era. Such deterministic and semi-probabilistic methods will not able to compete with qualitatively new concepts reflecting the advent of the computer and information technology era. It can be expected that simplistic reliability assessment tools will gradually diminish and much more sophisticated concepts will be developed. In order to cross the divide between the old and new qualitatively completely different concepts, a reengineering of the entire safety, durability and serviceability assessment procedures would be inevitable. The new era will require, first of all, a transition from a deterministic to a probabilistic "way of thinking" of designers and all others involved in structural design.

Several different approaches can be considered in the development of structural reliability assessment concepts applicable in designers work in the near future. Some researchers prefer analytical approach, other experts are advocating applicability of numerical methods, while computer oriented specialists claim that simulation technique using powerful computers is the optimum solution in this dispute. One of the possible alternatives of simulation-based reliability assessment concepts, SBRA, is applied in this paper.

A recently developed concept SBRA is documented in (Marek, Guštar and Anagnos, 1995). This transparent and user-friendly method is based on expressing all input variables by bounded (non-parametric) histograms, on transformation models allowing calculation of the response of the structure to the loading, on the analysis of variables and their interaction using direct Monte Carlo method as a tool, on expressing the reliability by comparing the calculated probability of failure P_f and the target probability P_d contained in specifications, and on clear "rules of the game" corresponding to the Limit States method. Following pilot example explains and illustrates the application of the SBRA concept.

†Corresponding author
Tel.: +420-69-699-4555 ; Fax: +420-69-691-6490
E-mail address: leo.vaclavek@vsb.cz

2. Transformation Models

2.1 Model of the Frame with Leaning Columns

Transformation model serves for determining of stress and strain response of the frame to external loading. Rough drawing of given steel frame and its loading are shown in Fig. 1. Examination of reliability of the structure supposes loading in the plane of the frame which is secured against deviation from its plane, and the assumption of elastic behaviour of columns and crossbars.

The solution supposes quasistatic response to imposed loading and nécessity to respect the influence of deformation of the structure to resulting response in correspondence with the second order theory. The derived transformation model and its alternatives, which are the subject of the brief evaluation and comparison with the model based on FEM, is used in chapter 3 to the probabilistic assessment of structure by SBRA method. For deriving of the transformation model was used original method based on the theoretical background which provides e.g. (Gere and Timoshenko, 1990; Timoshenko, 1971). First the subject of study is model of one leaning column with one fixed end, then the result is applied in formation of the model of frame with leaning socketed columns.

2.2 Column with Fixed End Loaded at Free End in Vertical and Horizontal Direction

In Fig. 2 is marked out by dashed line the central line of straight unloaded column. Unloaded column is deviated from vertical by angle α, imperfection a represents distance between free end of unloaded column and vertical going through fixed end. The central line of deformed column, which is loaded on free end by forces F and H, is delineate in Fig. 2 by fat full line. Flexural rigidity of column is EI, where E is Young's modulus of elasticity and I is second moment of area about the neutral axes. Length of the column is l. The subject of calculation is horizontal

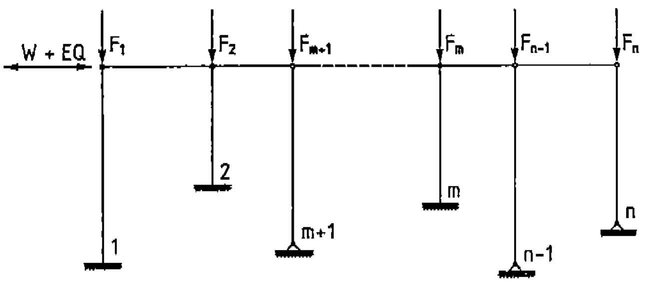

Fig. 1. Loading and scheme of the frame with leaning socketed columns. Total number of columns is n, one fixed end have m columns.

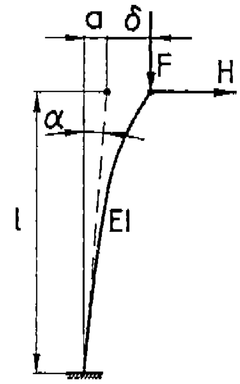

Fig. 2. Column with fixed end loaded by forces at its free end.

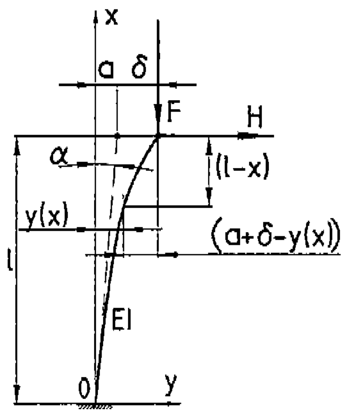

Fig. 3. Column with fixed end loaded by vertical and horizontal-forces at its free end. Origin of the coordinate system is in the place of fixed end, line coordinates are parallel to loading forces.

displacement δ of upper (free) end of column. It is obvious, that the calculation must be carried out with application of the second order theory, because it is necessary to respect the influence of displacement on magnitude of bending moment. As small elastic strain is supposed, it is possible to use simple form of the flexure equation. Calculation is in the next carried out on the assumption (a) elastic deformation of structure in the plane of imposed forces F, H, (b) constant second moment of area along the whole length of column, and perfectly fixed end of column. It is possible to perform appurtenant calculation with use of different coordinate systems. Two different choices of coordinate system are shown in Fig. 3 and Fig. 4. The results obtained are discussed in the end of this chapter.

With respect to coordinate system in Fig. 3 there is flexure equation

$$y''(x) = +\frac{M(x)}{EI} \tag{1}$$

where $M(x)$ is bending moment. For loading shown in

Fig. 3 we find

$$M(x) = H(l-x)+F(a+\delta-y) \quad (2)$$

Setting

$$\omega^2 = \frac{F}{EI} \quad (3)$$

and applying the boundary conditions

$$y(0) = 0 \quad (4)$$

$$y'(0) = tg\alpha = \frac{a}{l} \quad (5)$$

we obtain after integration (1) the equation for deflection of column

$$y(x) = \left(\frac{H}{F}\cdot l+a+\delta\right)cos\,\omega x+\left(\frac{a}{l}+\frac{H}{F}\right)\frac{sin\ \omega x}{\omega}$$
$$+\frac{H}{F}(l-x)+(a+\delta) \quad (6)$$

Displacement δ of upper end of column in the direction of force H (see Fig. 3) is $\delta = y(l)-a$. Eq. (6) gives

$$\delta = \left(\frac{a}{l}+\frac{H}{F}\right)\left(\frac{tg\,\omega l}{\omega l}-1\right)\cdot l \quad (7)$$

With respect to coordinate system in Fig. 4 there is flexure equation

$$y''(x) = \frac{M(x)}{EI} \quad (8)$$

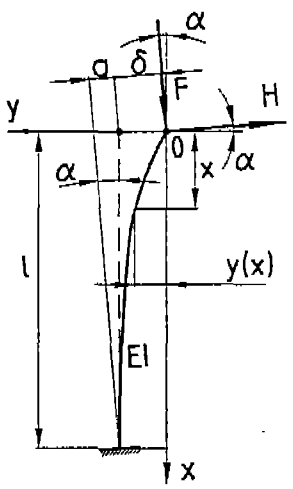

Fig. 4. Column with fixed end loaded by vertical and horizontal forces at its free end. Line coordinates are rotated through angle which is given by deviation of unloaded column from vertical. Origin of the coordinate system is situated at the free end of column which is deflected due to loading by forces F, H.

Bending moment $M(x)$ is

$$M(x) = (F\cdot\cos\alpha-H\sin\alpha)\cdot y(x)+(F\sin\alpha-H\cos\alpha)\cdot x \quad (9)$$

Setting

$$\omega^2 = \frac{F\cdot\cos\alpha-H\sin\alpha}{EI} \quad (10)$$

and applying the boundary conditions

$$y(0) = 0\,, \quad (11)$$

$$y(l) = 0\,, \quad (12)$$

we obtain after integration Eq. (8) the equation for deflection of column

$$y(x) = \frac{F\sin\alpha+H\cos\alpha}{F\cos\alpha-H\sin\alpha}\left(\frac{\sin\omega x}{\omega\cdot\cos\omega l}-x\right) \quad (13)$$

Displacement δ of upper end of column in direction of force H is $\delta = y(l)\cdot\cos\,\alpha$. By using Eq. (13) and $tg\alpha = a/l$ (see Fig. 4), we get

$$\delta = \frac{\frac{a}{l}+\frac{H}{F}}{1-\frac{aH}{lF}}\left(\frac{tg\,\omega l}{\omega l}-1\right)\cdot l\cos\alpha \quad (14)$$

Comments to the model

(a) The small difference between computational schemes in compliance with Fig. 3 and Fig. 4 is based on the fact, that the axis of coordinates in Fig. 4 are slightly rotated with respect to the preceding by angle α and that the length of column l come in the calculation as the length of inclined column, not as its projection onto vertical in Fig. 3.

(b) Computational scheme does not comprise displacement of free end of column in direction of y-axis (see Fig. 3, Fig. 4)-it is small in comparison with displacement in x-axis direction.

(c) As angle α is small, it means $a<<l$, then $\cos\alpha$ tends to unit and $\sin\alpha$ to zero. If there is boath $(a/l)\cdot(H/F)<<1$ and $a<<l$, Eq. (14) gives the same result as Eq. (7). If there are not aforementioned conditions satisfied, it is suitable to use Eq. (10) and Eq. (14) because it provides more accurate result.

(d) In concrete terms, if $a/l=0.1$, $H/F=1$, $F=5\cdot10^5\ N$, $E=2.1\cdot10^5$ MPa, $I=1.89\cdot10^{-4}\ m^4$, $l=6\ m$, we get displacement δ of free end of column
$\delta=1.2194\ m$–according to Eq. (3), Eq. (7)
$\delta=1.1858\ m$–according to Eq. (10), Eq. (14)
$\delta=1.1377\ m$–FEM, software Ansys (100 beam elements).

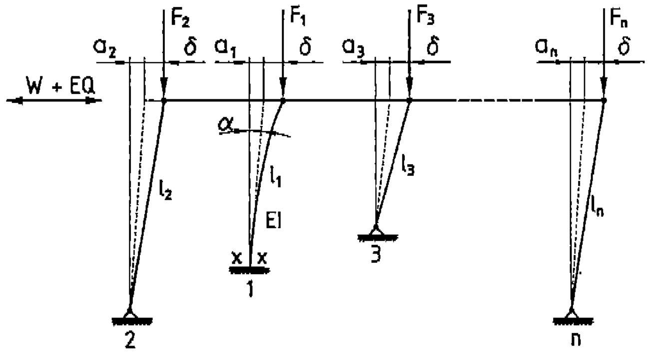

Fig. 5. Scheme of plane structure before loading (dashed line) and after loading (fat full line). Column marked with number 1 has fixed end, remaining columns are socketed.

2.3 Leaning Socketed Columns Rested Against one Fixed Column

2.3.1 Transformation Model of the Frame

The plane structure, scheme of its is shown in Fig. 5, contains n columns, connected on the top by joints with $(n-1)$ crossbars. As transformation model serves to find stress and strain response of constant flexural rigidity fixed column to external loading of structure, it is possible to consider crossbars and leaning socketed columns as undeformable. Fat full line in Fig. 5 shows deformed shape of fixed column (marked with number 1) and positions of loaded socketed columns and crossbars after loading by vertical forces $F_1, F_2, \ldots, F_n$ and horizontal forces W, EQ. Lengths of columns are $l_1, l_2, \ldots, l_n$, lengths of crossbars are not marked out in Fig. 5 since they do not enter the transformation model (as for changes of lengths as a consequence of temperature changes are not considered). Central lines of unloaded columns are marked out by dashed lines. Geometrical imperfections, such as vertical deflection, lack of initial straightness, support deviations and necessary small eccentricities at joints are supposed to be included in equivalent geometrical imperfections $a_1, a_2, \ldots, a_n$, marked in Fig. 5. Horizontal displacement of upper end of fixed column 1 (and consequently displacements of upper ends of remaining columns too) is in Fig. 5 marked as δ.

From the arrangement shown in Fig. 5 results, that column with fixed end have to catch resulting horizontal force H, which is given by

$$H = W + EQ + \sum_{i=2}^{n} \frac{a_i}{l_i} F_i + \left(\sum_{i=2}^{n} \frac{F_i}{l_i} \right) + \delta \tag{15}$$

When $a_1 << l_1$ (see Fig. 5), we can take with sufficient accuracy $\cos \alpha = 1$, $\sin \alpha = a_1/l_1$. Using (14) and (10) we obtain displacement of the upper end of fixed column

$$\delta = \frac{\frac{a_1}{l_1} + \frac{H}{F_1}}{1 - \frac{a_1}{l_1}\frac{H}{F_1}} \left(\frac{tg\,\omega l}{\omega l} - 1 \right) \cdot l_1, \tag{16}$$

where

$$\omega = \left[\left(F - H\frac{a_1}{l_1} \right) / EI \right]^{\frac{1}{2}}. \tag{17}$$

Within Eq. (17) E is Young's modulus of elasticity and I is second moment of area about the neutral axis in case of simple bending of fixed column 1. Displacement δ is in Eq. (16) not only on the left, but also on the right side, because H and consequently ω depends on δ, see Eqs. (15) and (17). Arbitrarily accurate solution of transcendental Eq. (16) we can obtain by iterations. The first approximation δ_1 to displacement δ we can determine such, that we neglect terms in which occur products $a_i, a_1, \delta \cdot a_1$. Then from the Eq. (15) we find that

$$H\frac{a_1}{l_l} = (W + EQ)\frac{a_1}{l_1} \tag{18}$$

and after substituting Eq. (18) into Eq. (17) (with ω_0 stand for ω)

$$\omega_o = \left[\left(F_1 - (W + WQ)\frac{a_1}{l_1} \right) / EI \right]^{\frac{1}{2}} \tag{19}$$

Denoting δ_1 instead of δ and ω_o instead of ω in Eqs. (15) and (16) arises after substituting Eq. (15) and (18) into Eq. (16) and after small adaptation first approximation to δ:

$$\delta_1 = \frac{\left(W + EQ + \sum_{i=1}^{n} \frac{a_i}{l_i} F_1 \right) \left(\frac{tg\,\omega_o l_1}{\omega_o l_1} - 1 \right) l_1}{F_1 - (W + EQ)\frac{a_1}{l_1} - \left(\sum_{i=2}^{n} \frac{F_i}{l_i} \right) \left(\frac{tg\,\omega_o l_1}{\omega_o l_1} - 1 \right) \cdot l_1} \tag{20}$$

First approximation to magnitude of horizontal force H, which must catch fixed column we can obtain from Eq. (15):

$$H_1 = W + EQ + \sum_{i=2}^{n} \frac{a_i}{l_i} F_1 + \left(\sum_{i=2}^{n} \frac{F_i}{l_i} \right) \delta_1 \tag{21}$$

By substituting H_1 into (17) is given more accurate approximation ω_1 to genuine ω:

$$\omega_1 = \left[\left(F - H_1 \frac{a_1}{l_1} \right) / (EI) \right]^{\frac{1}{2}} \tag{22}$$

Arbitrarily accurate solution of Eq. (16) can be gained after carrying out necessary number N of iterations for $k = 1, \ldots, N$:

Table 1. Verification of the accuracy of transformation models on structure with one fixed and three leaning socketed columns (see Fig. 5) by comparison with FEM calculations

Imperfections [mm]				W+EQ [kN]	Horizontal displacement [mm], Fig. 5			
α_1	α_2	α_3	α_4		δ (28)	δ_1 (20)	δ_{K+1}* (23)	FEM** calculation
30	45	15	27	0	40.926	40.926	40.923	40.900
30	45	15	27	25	168.82	168.78	168.76	168.64
30	45	15	27	89	496.23	496.00	495.67	493.11
300	450	150	270	0	409.26	409.26	405.95	408.01
300	450	150	270	25	537.15	536.47	531.27	529.43
300	450	150	270	89	864.56	860.63	848.83	847.49
10	15	5	9	0	13.642	13.642	13.642	13.585
10	15	5	9	25	141.54	141.53	141.52	141.42
10	15	5	9	89	468.94	468.87	468.78	464.32
–30	45	15	27	0	15.347	15.347	15.348	15.313
–30	45	15	27	25	143.24	143.26	143.29	143.28
–30	45	15	27	89	470.65	470.87	471.17	467.32
30	–45	–15	–27	0	–15.347	–15.347	–15.348	–15.313
–30	–45	–15	–27	25	112.55	112.53	112.52	112.40
30	–45	–15	–27	89	439.95	439.76	439.52	435.41

*δ_{K+1} (Eq. (23)) is such first magnitude of displacement, for which $(\delta_{K+1}-\delta_K)/\delta_K \le 10^{-5}$

**Magnitude of displacement was for selected configurations calculated by FEM, software Ansys. Model of fixed column was created by 100 beam elements.

$$\delta_{k+1} = \frac{\frac{a_1}{l_1}+\frac{H_k}{F_1}}{1-\frac{a_1 H}{l_1 F_1}}\left(\frac{tg\,\omega_k l_1}{\omega_k l_1}-1\right)\cdot l_1, \tag{23}$$

where for $k = 1$ is H_k, ω_k given by Eqs. (21), (22) and for $k = 2, 3, \ldots, N$ becomes

$$H_k = \sum_{i=2}^{n}\frac{a_i}{l_i}F_i+(W+EQ)+\left(\sum_{i=2}^{n}\frac{F_i}{l_i}\right)\delta_k \tag{24}$$

$$\omega_k = \left[\left(F-H_k\frac{a_1}{l_1}\right)/EI\right]^{\frac{1}{2}} \tag{25}$$

Bending moment carried by the critical (fixed) section of column is given by Eq. (9) after substituting $x = l_1, F = F_1$, and on the assumption that, when $a_1 << l_1$, then (see Fig. 4 respectively Fig. 5) $\cos\alpha = 1$, $\sin\alpha = a_1/l_1, y(l_1) = \delta/\cos\alpha = \delta$. From the mentioned relations we get

$$M(l_1) = \left(F_1-H\frac{a_1}{l_1}\right)\cdot\delta+\left(F_1\frac{a_1}{l_1}+H\right)l_1 \tag{26}$$

After neglecting term in which is product a_1, δ, we obtain from Eq. (26) bending moment carried by critical section of fixed column

$$M(l_1) = F_1(a_1+\delta)+H\cdot l_1 \tag{27}$$

Axial force carried by critical section of column is F_1. If the first approximation to displacement of upper end of column is sufficient, it is possible to substitute into (27) $\delta = \delta_1$ and $H = H_1$ from Eqs. (20) and (21). For giving more précision to δ and H we can use Eqs. (23) and (24).

2.3.2 Alternative Modifications of the Model

Eq. (7) is an alternative relation to Eq. (16). Substituting Eq. (15) into Eq. (7) we can gain alternative relation to (20):

$$\delta = \frac{\left(W+EQ+\sum_{i=1}^{n}\frac{a_i}{l_i}F_1\right)\left(\frac{tg\,\omega l_1}{\omega l_1}-1\right)\cdot l_1}{F_1-\left(\sum_{i=2}^{n}\frac{F_i}{l_i}\right)\left(\frac{tg\,\omega l_1}{\omega l_1}-1\right)l_1} \tag{28}$$

where ω is in accordance with (3)

$$\omega = (F_1/(EI))^{\frac{1}{2}}. \tag{29}$$

Relations Eq. (28) and Eq. (29) tend to first approximation to displacement δ accordingly Eq. (20) and to ω_o accordingly Eq. (19). The difference between relations Eq. (20) and Eq. (28) is given by different simplification suppositions used during derivation of equations for the displacement δ.

2.3.3 Comparison of Analytical Models and the Model Based on FEM

Numerical results of calculation of the horizontal displacement δ of upper end of fixed column in Fig. 5 for loading $F_1 = 5 \cdot 10^5 N$, $F_2 = 3 \cdot 10^5 N$, $F_3 = 5 \cdot 10^5 N$, $F_4 = 3 \cdot 10^5 N$ lengths of columns $l_1 = 6m, l_2 = 9m, l_3 = 3m, l_4 = 5.4m$, second moment of area $I = 1.89 \cdot 10^{-4} m^4$, modulus of elasticity $E = 21 \cdot 10^5\ MPa$ and selected magnitudes of imperfections a_1, a_2, a_3, a_4 and horizontal forces W, EQ are given in Table 1. Results obtained from Eqs. (28), (20), and by iterations accordingly Eq. (23) should be compared with ones obtained by FEM and also presented in Table 1.

2.3 Leaning Socketed Columns Rested Against Several Fixed Columns

2.3.1 Transformation Model of the Frame

Fig. 1 shows the plane structure consist of n columns altogether, m of them with fixed end numbered 1 to m and $n-m$ socketed columns. The description in the introduction of paragraph 2.3.1 holds in adequate manner for structure shown in Fig. 1 too. It is possible to make use of knowledge gained in paragraph 2.3 for derivation of the transformation model of the structure in Fig. 1. In the following derivation Eqs. (15), (7) and (3) are used.

Let us agree upon the following symbolic:

$$\omega_j = (F_j/E_jI_j)^{\frac{1}{2}}, j = 1, \ldots, m \tag{30}$$

$$K_j = \frac{tg(\omega_j l_j)}{\omega_j l_j}, j = 1, \ldots, m \tag{31}$$

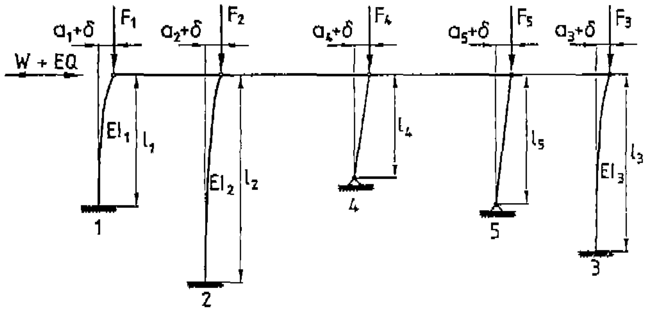

Fig. 6. Loading and scheme of deformed frame structure with three fixed and two socketed leaning columns. Initial imperfections are a_1, a_2, a_3, a_4, a_5, displacement of upper ends of columns due to loading is δ.

$$A = \left(W + EQ + \sum_{i=m+1}^{n} \frac{a_i}{l_i} F_i \right) \tag{32}$$

$$B = \sum_{i=m+1}^{n} \frac{F_i}{l_i} \tag{33}$$

After loading construction the horizontal displacement of upper ends of columns will be δ, and H_j the horizontal force, carried by jth fixed column, $j = 1, \ldots, m$. Displacement δ and forces H_j could be calculated by solving the following system of $(m+1)$ equations:

$$\delta - H_j \frac{K_j l_j}{F_j} = a_j K_j \tag{34}$$

$$B \cdot \delta - \sum_{j=1}^{m} H_j = -A \tag{35}$$

Remark: Indices j in Eq. (34) are not summation indices, it means that Eq. (34) represents m equations.

In critical section (fixed end) the jth column carries fixed-end moment

$$M_j = F_j(a_j + \delta) + H_j \cdot l_j \tag{36}$$

axial compressive force F_j and shear force H_j.

2.4.2 Comparison of Analytical Model and the Model Based on FEM

Numerical verification of the accuracy of analytical model which lead to Eqs. (34) and (35) is carried out on the structure shown in Fig. 6 by comparison with results obtained by FEM. The matter is particular form of the structure shown in Fig. 2, when number of fixed columns is $m = 3$ total number of columns is $n = 5$. Loading forces are $F_1 = 5 \cdot 10^5 N$, $F_2 = 3 \cdot 10^5 N$ $F_3 = 9 \cdot 10^5 N$, $F_4 = 5 \cdot 10^5 N$, $F_5 = 3 \cdot 10^5 N$. Magnitudes of *Wa EQ* forces a are given in Table 2. Lengths of columns are $l_1 = 6m$, $l_2 = 9m$, $l_3 = 6m$, $l_4 = 3m$, $l_5 = 5.4m$. Calculation is performed with imperfections $a_1 = 30mm$, $a_2 = 45mm$. $a_3 = 30mm$, $a_4 = 15mm$, $a_5 = 27mm$. Second moments of area of fixed columns are $I_1 = 1.49 \cdot 10^{-4} m^4$, $I_2 = 1.93 \cdot 10^{-4} m^4$, $I_3 = 3.08 \cdot 10^{-4} m^4$ Young's modulus of elasticity is E=2.1·10^5 MPa.

In the Table 2 are given calculated magnitudes of displacement δ of upper ends of columns and calculated magnitudes of horizontal forces H_1, H_2, H_3, which are carried by fixed columns 1, 2, 3 (see Fig. 6).

2.5 Evaluation of Proposed Transformation Models

On the basis of analysis of results obtained by application of transformation models we can make the following conclusion

Table 2. Verification of the accuracy of transformation model on structure shown in Fig. 6 with three fixed and two leaning socketed columns. Numerical results obtained by using Eqs. (34) and (35) and results calculated by FEM are presented

W+EQ [kN]	δ[mm]		H_1 [N]		H_2 [N]		H_3 [N]	
	(34) (35)	FEM	(34) (35)	FEM	(34) (35)	FEM	(34) (35)	FEM
0	13.128	13.051	1888.5	1866.2	162.89	154.06	4924.1	4877.9
10	23.629	23.504	5399.4	5361.6	1493.2	1478.3	12463	12385
25	39.382	39.177	10666	10603	3488.7	3464.0	23772	23640
89	106.60	106.02	33135	32969	12003	11935	72024	71673
200	223.17	221.72	72105	71773	26769	26620	155710	155000
−10	2.6255	2.6042	−1622.3	−1627.2	−1167.4	−1169.5	−2615.2	−2624.6
−25	−13.128	−13.069	−6888.5	−6867.8	−3162.9	−3155.0	−13924	−13879
−89	−80.340	−79.950	−29358	−29232	−11677	−11628	−62176	−61909
−200	−196.91	−195.80	−68328	−68029	−26443	−26310	−145862	−145220

1. Proposed analytical transformation models, which serve to calculation of horizontal displacement δ and horizontal forces carried by fixed columns of structure, are sufficiently accurate in a large range of displacements. By reason of this are these models suitable for using in connection with SBRA method (Marek, Guštar and Anagnos, 1995; Marek , Guštar, Teplý, Novák and Keršner , 1997) to reliability (that is safety and serviceability) assessment of plane frames, scheme of which is in Fig. 1. Proposed analytical model has in comparison with the model based on FEM specific advantages. It does not need such powerfull computer and pertinent FEM software equipment, which must be capable solve problems with geometrical nonlinearities. The response of structure to loading is obtained (when analytical model is used) for one simulation by direct substituting into formulae (structure with one fixed and arbitrary number of socketed leaning columns), or in case of m fixed columns by solving system of $m+1$ linear equations. With respect to the fact, that number of simulations (separate steps which differs from each other by applying random input of loading, imperfections, variability of areas,etc.) in dependance upon solved problem varies from a tens of thousands to millions, the use of FEM in connection with solved problem of plane frame is at present too cumbersome. Analytic solution of this problem is more convenient, as finite elements can offer no advantage where closed solution exist.
2. From comparison of numerical results in Table 1 which are obtained by use of (20) and (23) follows, that in case of large displacements it is possible to gain more accurate results by iterations using (23), but the difference is not too significant in most cases, so in thumping majority of technical important cases procedure based on (23) is not necessary.

3. Probability Assessment of Reliability

3.1 Application of the SBRA Method

The fully probabilistic method SBRA (Marek, Guštar and Anagnos, 1995) is applied on the structure shown in Fig. 5. The individual loadings are expressed by their max-imum values (see numerical values) and corresponding load duration curves represented by nondimensional his-tograms, see Table 3. For the corresponding histograms see [2]. Forces $F_1,\dots,F_4$ in vertical direction are given as sums of non-correlated dead, long-lasting and short-lasting loads. The force W represents effect of wind. Loading EQ produced by earthquake depends on magnitude of vertical loading of imposed structure on the moment of earth-quake, as shown in Table 3. Properties of three different areas (second moment of area I and section modulus S are correlated), Young's modulus of elasticity E and the yield stress FY for two different steels of fixed column with per-tinent histograms are given in Table 4. Lengths of columns and extreme magnitudes of mutually independent equivalent geometrical imperfections with relevant histograms are in Table 5.

3.2 Safety Assessment (carrying capacity limit state)

Load carrying capacity of structure shown in Fig. 5 is in the next evaluated with respect to spending elastic area working of section $x-x$. The load effect Q in the cross-section $x-x$ of fixed column is expressed by the stress corresponding to the combination of compressive axial

Table 3. Loading of the structure

Column	Force	Loading	Symbol*	Extreme magnitude [kN]	Histogram
		dead	DF1	100	DEAD 1
1	F_1	random long-lasting	LF1	100	LONG 2
		random short-lasting	SF1	100	SHORT 2
		dead	DF2	150	DEAD 1
2	F_2	random long-lasting	LF2	200	LONG 2
		random short-lasting	SF2	150	SHORT 2
		dead	DF3	150	DEAD 1
3	F_3	random long-lasting	LF3	200	LONG 2
		random short-lasting	SF3	150	SHORT 2
		dead	DF4	100	DEAD 1
4	F_4	random long-lasting	LF4	100	LONG 2
		random short-lasting	SF4	100	SHORT 2
1	W	wind	WIN	50	WIND 1
1	EQ	earthquake	EQV	0.02 ΣF_i	EARTH

*histograms representing load duration curves

Table 4. Properties of area and material properties of fixed column

Fixed column 1	Bar cross section				
	HE 300B	HE 280B	HE 260B		
Quantity, designation, units	Nominal value	Nominal value	Nominal value	Variability	Histogram
Section area A [mm²]	14900	13100	11800	±4%	N-04
Second mom. of area I [mm⁴]	$252 \cdot 10^6$	$193 \cdot 10^6$	$149 \cdot 10^6$	±8%	N1-08
Section modulus S [mm³]	$1680 \cdot 10^3$	$1380 \cdot 10^3$	$1150 \cdot 10^3$	±8%	N1-08
Young's modulus E [Nmm⁻²]		$2.1 \cdot 10^5$		constant	-----
Fe 360, yield stress FY [Nmm⁻²]		235		------	A 36-m
Fe 510, yield stress FY [Nmm⁻²]		355		------	A 572-m

*Material proparties of fixed column are the same for every section.

force and fixed-end moment. The limit of resistance R of the column 1 is defined by the onset of yielding in the critical cross-section $x-x$ and the resistance R is expressed by variable representing the yield stress $R = FY$ by histogram. The safety function, $SF = R-Q$, is analyzed using Monte Carlo simulation by M-Star™ computer program, see (Marek, Guštar and Anagnos, 1995). As the target probability of failure P_d corresponding to the structures of special importance is P_d=0.000008 (CSN 73 1401; 1998), the number of simulations is used 1250000=10.(1/P_d) for each of 20 alternatives mentioned in Table 6, where calculated probabilities of failure $P_{f,c}$ are reviwed. The output from the M-Star™ program (variability of imperfections, earthquake, cross section of column 1-HEB 300, steel grade Fe360) is shown in Fig. 7.

Table 5. Lengths and imperfections of columns

Lengths of columns		Imperfections of columns		
Designation	Magnitude [mm]	Designation	Variability [mm]	Histogram
l_1	6000	a_1	±30	Normal 2
l_2	9000	a_2	±45	Normal 2
l_3	3000	a_3	±15	Normal 2
l_4	5400	a_4	±27	Normal 2

3.3 Serviceability Assessment (serviceability limit state)

Serviceability assessment refers to the lateral displacement limit value δ_{tol}=30 mm of upper ends of columns. The serviceability function SF=($\delta_{tol}-\delta$) is analyzed by M-Star™ program. The number of iterations is 50000 for

Table 6. Calculated probabilities of failure P_f

IM*	EQ	Cross section of column 1			Steel Grade		Carrying capacity limit state		Serviceability limit state	
		HEB 300	HEB 280	HEB 260	Fe 360	Fe 510	$P_{f,c}$	Reliability level	$P_{f,s}$	Reliability level
var.	yes	x			x		$1.1 \cdot 10^{-5}$	standard	$11.6 \cdot 10^{-2}$	reduced
var.	yes	x				x	$<1 \cdot 10^{-6}$	elevated		
var.	yes		x		x		$1.5 \cdot 10^{-3}$	insufficient	$17.1 \cdot 10^{-2}$	insufficient
var.	yes		x			x	$5.4 \cdot 10^{-5}$	standard		
var.	yes			x	x		$1.8 \cdot 10^{-2}$	insufficient	$26.6 \cdot 10^{-2}$	insufficient
var.	yes			x		x	$6.8 \cdot 10^{-3}$	insufficient		
var.	no	x			x		$5.0 \cdot 10^{-6}$	elevated	$11.3 \cdot 10^{-2}$	reduced
var.	no	x				x	$<1 \cdot 10^{-6}$	elevated		
var.	no		x		x		$1.4 \cdot 10^{-3}$	insufficient	$16.6 \cdot 10^{-2}$	insufficient
var.	no		x			x	$2.6 \cdot 10^{-5}$	standard		
var.	no			x	x		$1.8 \cdot 10^{-2}$	insufficient	$25.8 \cdot 10^{-2}$	insufficient
var.	no			x		x	$6.6 \cdot 10^{-3}$	insufficient		
nom.	yes	x			x		$3.0 \cdot 10^{-5}$	standard	$12.0 \cdot 10^{-2}$	reduced
nom.	yes	x				x	$<1 \cdot 10^{-6}$	elevated		
nom.	yes		x		x		$2.1 \cdot 10^{-3}$	insufficient	$19.8 \cdot 10^{-2}$	insufficient
nom.	yes		x			x	$1.4 \cdot 10^{-4}$	reduced		
nom.	no	x			x		$1.6 \cdot 10^{-5}$	standard	$11.8 \cdot 10^{-2}$	reduced
nom.	no	x				x	$<1 \cdot 10^{-6}$	elevated		
nom.	no		x		x		$2.0 \cdot 10^{-3}$	insufficient	$19.3 \cdot 10^{-2}$	insufficient
nom.	no		x			x	$1.3 \cdot 10^{-4}$	reduced		

*Imperfections: var.-variable values, nom-nominal (extreme) values, see Table 5.

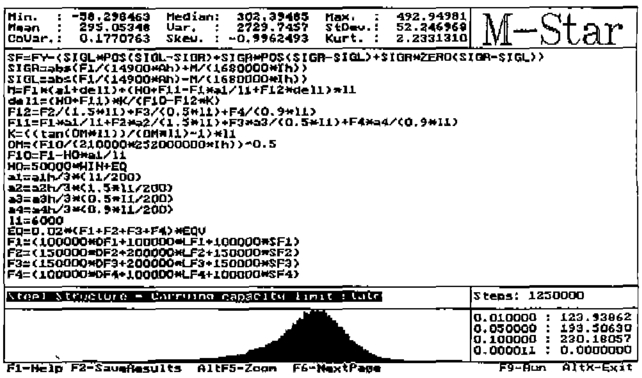

Fig. 7. M-Star™ program output, carrying capacity limit state of steel structure.

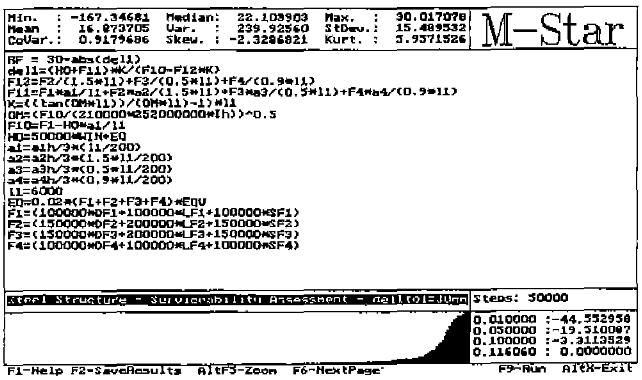

Fig. 8. M-Star™ program output, serviceability limit state of steel structure.

each of 10 alternatives mentioned in Table 6, where calculated probabilities of failure $P_{f,s}$ are reviwed. The target probabilities of failure P_d for three reliability levels from the point of view serviceability limit state are in (CSN 73 1401; 1998). The output from the M-Star™ program (variability of imperfections, earthquake, cross section of column 1-HEB 300) is shown in Fig. 8.

4. Summary and Conclusions

The substance of a probabilistic structural reliability assessment method SBRA (CSN 73 1401; 1998) is explained and the strategy of its application is otlined using safety and serviceability assessments of a planar steel frame as an example. Special attention is given to the definition of the transformation model serving for determining the response of the structure to the loading considering effects of several mutually dependent and several mutually independent loads, erection imperfections, effects corresponding to the second order analysis of structural components including leaning columns. The structural response to the loading is obtained using two different transformation models. First, the response is calculated using FEM, next an analytical model is developed and applied. Resulting responses determined according to these two models are compared. Both results are of very good agreement. Considering the simulation based reliability assessment SBRA concept, however, the analytical model seems to be in this particular case more convenient and practical compared to FEM model. The SBRA concept allows for determining the resulting probability of failure considering interaction of several different variables affecting the reliability, such as the effect of erection imperefections and the effect of correlation of some of the loads (earthquake load is depending on the magnitude of gravity loads). The applied concept leads to a good understanding of the significance of individual variables affecting the resulting probability of failure (see, e.g., (Marek P, Guštar M, Teplý B, Novák D, Keršner Z , 1997).

The safety and the serviceability are obtained by comparing the calculated probabilities of failure ($P_{f,c}$ and $P_{f,s}$) and the target probabilities ($P_{d,c}$ and $P_{d,s}$) contained in specifications (see CSN 73 1401; 1998) Appendix A). The designer is taking active part on the calculation of the reliability. His/her involvement is not limited to interpretation of factors, equations and black boxes contained in the specifications, as it is the case in the design according to the current reliability asessment method based on Partial Safety Factors.

Acknowledgements

Funding and support of research have been provided partly by the Grant Agency of the Czech Republic (103/99/0215), ITAM Academy of Sciences of the Czech Republic, and VŠB Technical University of Ostrava, Czech Republic.

References

Gere JM, Timoshenko SP (1990) Mechanics of Materials, PWS-KENT Publishing Company, Boston.

Marek P, Guštar M, Anagnos T (1995) Simulation-based ReliabilityAssessment for Structural Engineers, CRC Press, Inc., Boca Raton, FL.

Marek P, Guštar M, Teplý B, Novák D, Keršner Z (1997) Sensitivity Analysis in Simulation-Based Reliability of Structures, ICOSSAR97. November, Kyoto, Japan.

Timoshenko SP (1971) Stability of bars, plates and shells, Nauka, Moscow.(in Russian).

Marek P, Václavek L (1998) Probabilistic Analysis of Leaning Column Problem, Proceedings of the SSRC Conference, Atlanta, GA.

CSN 73 1401 (1998) Structural Steel Design, Czech Standards Institute, Prague, Czech Republic.