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Analysis of Structural Reliability under Model and Statistical Uncertainties: a Bayesian Approach

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ABSTRACT

A framework for reliability analysis of structural components and systems under conditions of statistical and model uncertainty is presented. The Bayesian parameter estimation method is used to derive the posterior distribution of model parameters reflecting epistemic uncertainties. Point, predictive and bound estimates of reliability accounting for parameter uncertainties are derived. The bounds estimates explicitly reflect the effect of epistemic uncertainties on the reliability measure. These developments are enhancements of second-moment uncertainty analysis methods developed by A. H-S. Ang and others three decades ago.

Keywords: reliability, Bayesian methods, uncertainty analysis, aleatory uncertainty, epistemic uncertainty, model uncertainty, structural reliability, probability of failure, reliability index

1. Introduction

Two types of uncertainties prevail in the assessment of structural reliability: intrinsic or aleatory variabilities, and epistemic or knowledge-based uncertainties. The former are those arising from variabilities inherent in the nature of phenomena, such as the natural randomness in the material property values or in the magnitudes and durations of loads such as those arising from earthquakes, wind or traffic. These variabilities have been the main focus of attention in the structural reliability literature. Epistemic uncertainties arise primarily from the inexact nature of the mathematical models used to idealize structural behavior and limit states, and from the finite size and accuracy of data samples upon which estimates of the model parameters are made. Whereas the intrinsic variabilities are beyond our control (short of changing the nature of the phenomenon itself), the epistemic uncertainties can be influenced by our decisions. Specifically, we can reduce the epistemic uncertainties by employing more refined models or collecting larger and more accurate data. This fundamental difference suggests the desirability of separately treating the aleatory and epistemic uncertainties so that the influence of the latter on the reliability estimate can be assessed and appropriate actions can be taken to reduce it.

A. H-S. Ang has been a pioneer in dealing with the issue of uncertainties in structural reliability theory since the late 1960's. In a series of pioneering and important papers, he and his coworkers laid out a framework for reliability analysis that properly accounted for model and statistical uncertainties, in addition to intrinsic variabilities, within a first-order, second-moment reliability analysis context. Particularly noteworthy are the papers by Ang and Amin (1968, 1969), Ang (1972, 1973) and Ang and Cornell (1974). These concepts were applied to development of design criteria and code calibration in a number of papers, most notably in Ellingwood and Ang (1974) for reinforced concrete structures. Later papers extended these concepts to structural system reliability, e.g., Bennet and Ang (1986) and Quek and Ang (1990).

The first-order, second-moment structural reliability method described in the series of papers culminating with Ang and Cornell (1974), as well as earlier works by Freudenthal (1947), Cornell (1969) and others, laid out a consistent framework that formed the bases of a more refined theory developed by Hasofer and Lind (1974). The latter proposed linearization of the limit-state function at a

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point that preserved the invariance of the second-moment reliability measure with respect to the formulation of the problem. Later advances by Rackwitz and Fiessler (1978), Ditlevsen (1981), Hohenbichler and Rackwitz (1981), Der Kiureghian and Liu (1986) and Winterstein and Bjerager (1987) allowed incorporation of information on the probability distribution or higher statistical moments in reliability analysis.

This paper attempts to propose a Bayesian framework for reliability analysis that is capable of incorporating all the relevant information that is available to the analyst. Furthermore, it properly and fully accounts for all the prevailing uncertainties. In this sense, it is a full-distribution version of what Ang and others did in a second-moment context three decades ago.

While second-moment methods have played a crucial role in the development of structural reliability theory, the time has come to move on. The idea that we know the first two (or any finite number of) statistical moments of a random variable and nothing else is a fabrication of our imagination or an idealization for the sake of convenience. In the real world, information about a random variable is available in the form of observed values or bounds, possibly affected by measurement error, from which estimates of the mean, the standard deviation and higher moments of the random variable can be obtained with decreasing order of accuracy. Furthermore, mathematical models used to describe structural behavior and limiting states are often idealizations of complex physical reality that need to be assessed, both in terms of their bias and prevailing error, by statistical analysis of observed laboratory or field data. The Bayesian parameter estimation method provides an ideal framework for processing of information and analysis of uncertainties under these conditions.

This framework is used here to propose a new and comprehensive approach to the assessment of structural reliability under model and statistical uncertainties.

2. Formulation of the Reliability Problem

Attention in this paper is restricted to time- and space-invariant reliability problems that are defined in terms of a set of random variables (not random processes) $\boldsymbol{x}$ describing quantities with intrinsic variabilities. Let $f(\boldsymbol{x}|\theta_f)$ denote the distribution of $\boldsymbol{x}$, where θ_f denotes the set of distribution parameters. In many cases, θ_f denotes the set of first and second moments of the random variables. In general, these parameters are unknown and must be estimated on the basis of available data.

As is usual, we will define the state of the structural system in the outcome space of the random variables in terms of a set of limit-state functions $g_i(\boldsymbol{x}, \theta_g), i = 1, 2, \ldots,$ where the index i denotes the components of the structural system, and θ_g denotes the set of parameters entering the limit-state functions. By convention, these functions are formulated such that the event $\{g_i(\boldsymbol{x}, \theta_g) \le 0\}$ denotes the failure of component i. In general, the limit-state parameters θ_g are unknown and must be estimated by assessing the limit-state models against relevant laboratory or field observations. For the sake of convenience, we introduce $\theta = (\theta_f, \theta_g)$ as the set of all model parameters of the reliability problem.

Having defined the distribution function and the limit-state functions, the reliability of the structural system is expressed in the most general form as

$$P_f(\theta) = \int_{\bigcup_k \bigcap_{i \in C_k} \{g_i(x, \theta_g) \le 0\}} f(\boldsymbol{x}|\theta_f) d\boldsymbol{x} \tag{1}$$

where the integration is over the failure domain of the structural system. In this expression, $C_k, k = 1, 2, \ldots$, denote the cut sets of the system. Each cut set denotes a collection of components, whose joint failures constitute the failure of the structural system. The intersection is over all the components of each cut set, and the union is over all the cut sets. This formulation includes the special case of a series system, where each cut set contains only one component, as well as the special case of a parallel system, where there is only one cut set. It is also useful to define the generalized reliability index by the transformation

$$\beta(\theta) = \Phi^{-1}[1 - P_f(\theta)] \tag{2}$$

where $\Phi^{-1}[\cdot]$ denotes the inverse of the standard normal probability function.

In the above expressions, we have explicitly shown the dependence of the failure probability and the generalized reliability index on the assumed values of the model parameters. Clearly, if θ is uncertain, either due to uncertainty in the distribution parameters, the limit-state parameters, or both, the corresponding estimates of the failure probability and the generalized reliability index will also be uncertain. The above reliability values should then be regarded as conditional reliability measures for given values of the parameters. Under such conditions, depending on how the uncertainties in the model parameters are handled, one can obtain different measures of reliability, as described later in this paper.

In the above formulation, we have employed the distribution model $M_f \equiv f(\boldsymbol{x}, \theta_f)$ and the set of limit-state models $M_f \equiv \{g_i(\boldsymbol{x}, \theta_g), i = 1, 2, \ldots\}$. The choice for these models may not be obvious. Geyskens and Der Kiureghian (1996) describe a decision theoretic framework for model selection. Alternatively, models may be selected on the basis of their respective error estimates, as described below.

3. Bayesian Parameter Estimation

The well known Bayesian updating rule is used to estimate both the distribution and limit-state parameters on the basis of relevant observations. For a set of parameters $\boldsymbol{\theta}$, this rule is stated as

$$f(\boldsymbol{\theta}) = cL(\boldsymbol{\theta})p(\boldsymbol{\theta}) \tag{3}$$

where $p(\boldsymbol{\theta})$ is the prior distribution representing our state of knowledge about $\boldsymbol{\theta}$ before making the observations, $L(\boldsymbol{\theta})$ is the likelihood function representing the objective information gained from the observations, $c = [\int L(\boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}]^{-1}$ is a normalizing factor, and $f(\boldsymbol{\theta})$ is the posterior distribution representing our updated state of knowledge about $\boldsymbol{\theta}$. The prior may incorporate any subjective information about $\boldsymbol{\theta}$ that is based on our engineering experience and judgment. The likelihood is a function proportional to the conditional probability of the observations for given $\boldsymbol{\theta}$. Specific cases of its formulation are described in the subsequent sections.

Once the posterior distribution of $\boldsymbol{\theta}$ is determined, one can proceed with computing its posterior statistics. We denote the posterior mean vector as $\boldsymbol{M}_{\theta}$ and the posterior covariance matrix as $\boldsymbol{\Sigma}_{\theta\theta}$. Computation of the normalizing factor and the posterior statistics is not a simple matter, as it requires evaluation of multifold integrals over the Bayesian kernel $L(\boldsymbol{\theta})p(\boldsymbol{\theta})$. Numerical algorithms for computing such integrals are developed by Geyskens *et al.* (1993). Alternatively, one can use importance sampling to evaluate these integrals (Ditlevsen and Madsen 1996). When large amounts of data are available, the maximum likelihood estimator (the value of $\boldsymbol{\theta}$ that maximizes the likelihood function), denoted $\boldsymbol{\theta}_{MLE}$, provides a good approximation of the posterior mean. Furthermore, the negative of the inverse of the Hessian of the logarithm of the likelihood function at the MLE point provides an approximation to the posterior covariance matrix. These approximations can be used to formulate an effective sampling density for importance sampling integration of the Bayesian kernel.

In the following two sections, we describe the formulation of the likelihood function for distribution and limit-state parameters.

3.1 Likelihood Function for Distribution Parameters

The most common type of observation consists of measured values $x_i, i, 1, 2, \ldots, N$, of the random variables. If the observations are statistically independent, then the likelihood function is given by the well known proportionality relation

$$L(\boldsymbol{\theta}_f) \propto \prod_{i=1}^{N} f(\boldsymbol{x}_i|\boldsymbol{\theta}_f) \tag{4}$$

Note that any scalar coefficient of the likelihood function can be incorporated into the normalizing factor c. Hence, we only need to determine the likelihood function in proportion. In many cases, the observed values of random variables contain measurement error. That is, instead of measuring the actual realization $\boldsymbol{x}_i$, we measure $\hat{\boldsymbol{x}}_i$ such that $\boldsymbol{e}_i = \boldsymbol{x}_i - \hat{\boldsymbol{x}}_i$ is the measurement error. In most cases $\boldsymbol{e}_i$ can be considered to be normally distributed with its mean vector and covariance matrix determined by calibration of the measurement devices or procedures. If the errors at successive measurements are statistically independent, the likelihood function remains the same as in (4), except that one has to use the joint probability density function of the sum $\hat{\boldsymbol{x}}_i + \boldsymbol{e}_i$ in place of $f(\boldsymbol{x}_i, \boldsymbol{\theta}_f)$.

In some cases, the available observations are not direct measurements of the random variables, but a set of events that involve the random variables. Most generally, these can be formulated as either inequality events $\{h_i(\boldsymbol{x}) \leq 0\}$ or equality events $\{h_i(\boldsymbol{x}) = 0\}$, for $(i = 1, 2, \ldots, N)$, where $h_i(\boldsymbol{x})$ are "limit-state" functions defining the observed events. For example, in estimating the compressive strength of concrete, one may make use of the observation that spalling in a specimen has occurred under a certain deterministic or random load. In that case, $h_i(\boldsymbol{x})$ denotes the limit-state function describing the event of spalling. If the inequality events are statistically independent, the likelihood function takes the form

$$L(\boldsymbol{\theta}_f) \propto \prod_{i=1}^{N} P[h_i(\boldsymbol{x}) \leq 0] = \prod_{i=1}^{N} \int_{h_i(\boldsymbol{x}) \leq 0} f(\boldsymbol{x}|\boldsymbol{\theta}_f)d\boldsymbol{x} \tag{5}$$

The probability integrals on the right-hand side are seen to be similar to integrals appearing in component reliability analysis and can be evaluated by similar techniques. One special inequality event is $h_i(\boldsymbol{x}) = a_i - x \leq 0$, where x is an element of $\boldsymbol{x}$. In that case we are observing a lower bound a_i for the random variable x. Likewise, $h_i(\boldsymbol{x}) = x - b_i \leq 0$, when we observe an upper bound b_i for the random variable x. In such cases, the probability terms in (5) can be expressed in terms of the cumulative distribution function of the random variable x. In Der Kiureghian (1999b), an application of the above likelihood function for estimating the distribution parameters of the capacity of certain electrical substation equipment is presented, where the inequality events represent observations of damage or no damage of the equipment in past earthquakes.

The likelihood function for statistically independent equality events can be written in the form

$$L(\theta_f) \propto \prod_{i=1}^{N} \lim_{\delta \to 0} P[0 < h_i(x) \leq \delta] \propto \prod_{i=1}^{N} \frac{\partial}{\partial \delta} P[h_i(x) - \delta \leq 0]_{\delta = 0} \tag{6}$$

Each term on the right-hand side is similar to the probability sensitivity in component reliability analysis and can be solved by techniques available for such analysis. The special case $h_i(x) = x - x_i \leq 0$ corresponds to the direct measurement of the random variables described above, for which the likelihood function is given by (4). One can account for errors in the formulation of the event functions $h_i(x)$ in the manner described in the following section.

The uncertainties inherent in the parameters θ_f are primarily statistical in nature. That is, they are due to the finite size of the observation data. However, they may also arise from the wrong selection of the distribution model $M_f \equiv f(x, \theta_f)$. If several contending models are available, one may select a combined model that paramertizes the choice (Der Kiureghian 1989). A decision theoretic framework for selection of the model is discussed in Geyskens *et al.* (1996). Alternatively, one may select the model that has the smallest error variance.

3.2 Likelihood Function for Limit-State Parameters

In general, the limit-state functions $g_i(x, \theta_g), i = 1, 2, \ldots$, represent idealized mathematical models describing the boundary between "fail" and "safe" domains of the components of the structural system. Often these models are themselves composed of sub-models that describe the behavior, capacity or load effect of the specific components of the system. Without loss of generality, we will focus our attention on a generic model $y = g(x, \theta_g)$ which may represent either a limit-state model itself, or a constituent sub-model.

A model such as $y = g(x, \theta_g)$ is a mathematical expression relating a set of observable variables (y, x) through a set of unobservable parameters θ_g. Most often the main purpose of a model is to provide a prediction of the dependent variable y for given values of the independent (not in the statistical sense) variables x. This is the case, for example, when y represents the capacity of a component and x is the set of material properties and member dimensions. In virtually all cases, the model is an imperfect representation of reality. The imperfection in the model may arise from its inexact form, or due to "missing variables," i.e., variables that have an influence on the dependent variable, but which are not included in the model either for the sake of simplicity or due to our ignorance of their effect. To signify the imperfect nature of the model, we use the notation $\hat{g}(x, \theta_g)$. The perfect model then takes the form

$$y = \hat{g}(x, \theta_g) + \varepsilon \tag{7}$$

where ε denotes the model error. With a proper formulation of the model, it is justifiable to assume that ε has the normal distribution (normality assumption) with a constant but unknown standard deviation σ (homoskedasticity assumption). Furthermore, with the objective of obtaining an unbiased model, the mean of ε is set to zero. The normality and homoskedasticity assumptions can be, at least approximately, satisfied through a suitable transformation of $\hat{g}(x, \theta_g)$ and y (Box and Tiao, 1992). For example, if the quantities of interest are non-negative and the model error is proportional to the mean of the quantity, then the logarithmic transformation might be suitable. Also, setting the mean of ε equal to zero forces the model parameters to take values such that the corrected model is unbiased. In situations where an existing (biased) deterministic model $\hat{g}(x)$ is to be employed, the corrected model may take the form

$$y = \hat{g}(x) + \gamma(x, \theta_g) + \varepsilon \tag{8}$$

where the term $\gamma(x, \theta_g)$ now corrects the bias in the deterministic model as a function of x and the parameters θ_g. This is the form used, for example, by Gardoni *et al.* (2002) in developing predictive capacity models for reinforced concrete columns based on state-of-the-art deterministic models. In the following discussion of the likelihood function, we use the more compact form in (7).

The likelihood function for estimating the unknown model parameters θ_g and σ depends on the nature of the available information. Various forms of this function are presented in Der Kiureghian (1990), including forms that account for measurement error. Here we will only consider the case where accurately measured values or bounds of the dependent variable y are given for a set of observations $x_i, i = 1, 2, \ldots, N$, of the independent variables. Suppose these measurements consist of the values $y_i, i = 1, \ldots, m$, lower bounds $a_i < y, i = m+1, \ldots, m+n$, and upper bounds $y > b_i, i = m+n+1, \ldots, N$. Owing to the normal distribution of ε, assuming the model error terms at successive observations are statistically independent, the likelihood function takes the form

$$L(\theta_g, \sigma) \propto \prod_{i=1}^{m} \varphi\left(\frac{y_i - \hat{g}(x_i, \theta_g)}{\sigma}\right) \prod_{i=m+1}^{m+n} \Phi\left(-\frac{a_i - \hat{g}(x_i, \theta_g)}{\sigma}\right) \prod_{i=m+n+1}^{N} \Phi\left(\frac{b_i - \hat{g}(x_i, \theta_g)}{\sigma}\right) \tag{9}$$

where $\varphi(\cdot)$ denotes the standard normal probability density function and $\Phi(\cdot)$ denotes the standard normal cumulative probability function. If measurement errors are present, y_i and x_i must be replaced by $\hat{y}_i + e_i$ and $\hat{\theta}_i + \theta_i$, respectively, where $\hat{y}_i$ and $\hat{\mathbf{x}}_i$ are the measured values and e_i and $\mathbf{e}_i$ are the respective measurement errors. The corresponding likelihood function then involves the probability distribution of $\hat{g}(\hat{x}_i + \mathbf{e}_i, \theta_g) + \varepsilon - e_i$. A first-order approximation of $\hat{g}(\hat{x}_i + \mathbf{e}_i, \theta_g)$ around the mean of $\boldsymbol{e}_i$ can be used to linearize this expression so that the distributions remains normal.

4. Reliability Estimates Under Epistemic Uncertainties

Having described methods for assessing the posterior distribution of the model parameters $\boldsymbol{\theta} = [\theta_f, \theta_g]$, we now turn our attention to the estimation of reliability under parameter uncertainties. As mentioned earlier, when $\boldsymbol{\theta}$ are uncertain, the reliability measures in (1) and (2) should be regarded as conditional measures. Depending on how we treat the parameter uncertainties, different measures of reliability are obtained, as described below.

4.1 Point Estimate of Reliability

A simple estimate of the reliability is obtained by use of a point estimate $\hat{\theta}$ of θ in (1) and (2). This could be either the posterior mean, $\hat{\theta} = M_\theta$, or the maximum likelihood estimator, $\hat{\theta} = \theta_{MLE}$. The corresponding point estimates of the failure probability, $P_f(\hat{\theta})$, and the generalized reliability index, $\beta(\hat{\theta})$, obviously do not account for the epistemic uncertainties inherent in the model parameters. It is worth noting that $P_f(M_\theta)$ and $\beta(M_\theta)$ represent first-order approximations of the means of the conditional failure probability and the reliability index, respectively.

4.2 Predictive Reliability

One way to account for the epistemic uncertainties in assessing the reliability is to incorporate the uncertainty in the model parameters directly in the calculation of the failure probability. In essence, we treat the epistemic uncertainties in the same manner as the intrinsic variabilities. The corresponding estimate of the failure probability is obtained as

$$\tilde{P}_f = \int_{\bigcup_k \bigcap_{i \in C_k} \{g_i(x, \theta_g) \le 0\}} f(x|\theta_f) f(\theta_f) f(\theta_g) d\theta_f d\theta_g dx \tag{10a}$$

$$= \int_{\bigcup_k \bigcap_{i \in C_k} \{g_i(x, \theta_g) \le 0\}} \tilde{f}(x) f(\theta_g) d\theta_g dx \tag{10b}$$

$$= E_\theta[P_f(\theta)] \tag{10c}$$

And the corresponding reliability index is

$$\tilde{\beta} = \Phi^{-1}[1 - \tilde{P}_f] \tag{11}$$

where $f(\theta_f)$ and $f(\theta_g)$ are the posterior distributions of θ_f and θ_g, respectively, and

$$\tilde{f}(x) = \int f(x|\theta_f) f(\theta_f) d\theta_f \tag{12}$$

is the predictive (Bayesian) distribution of the random variables. Equation (10) shows that the predictive failure probability is the mean value of the conditional failure probability over the space of the model parameters. Furthermore, (10a) and (10b) offer two alternative ways of computing the predictive failure probability: (a) solve the reliability problem in the combined space of the random variables (x, θ_f, θ_g) using the joint distribution $f(x|\theta_f)f(\theta_f)f(\theta_g)$, or (b) solve the reliability problem in the reduced space of the random variables (x, θ_g) with the joint distribution $\tilde{f}(x)f(\theta_g)$. The second alternative is useful when the predictive distribution of x is available in closed form. This is the case, for example, for the so called conjugate distributions (see Ang and Tang, 1975). It is noted from (11) that the predictive reliability index corresponds to the mean of the failure probability over the space of the model parameters. Thus, $P_f(M_\theta)$ is a first-order approximation of $\tilde{P}_f$.

Whereas the above predictive reliability measures incorporate the effect of epistemic uncertainties in the reliability estimate, they do not provide a direct measure of the influence of these uncertainties. As their name suggests, these estimates are useful for prediction purposes, as they account for all the prevailing uncertainties. They are also useful in expected utility decision making. However, in many applications, it is necessary to provide an explicit measure of the uncertainty in the reliability estimate that arises from the epistemic uncertainties. A practical approach for this purpose is suggested below.

4.3 Bounds on Reliability

As described in Der Kiureghian (1989), it is possible to determine the probability distribution of the conditional failure probability, $P_f(\theta)$, and reliability index, $\beta(\theta)$, that reflect the uncertainties in these measures arising from the epistemic uncertainties. However, such analysis requires nested reliability calculations that can be cumbersome and costly. A simpler approach is to estimate the statistical moments of these measures by first-order approximation. Owing to the less nonlinear dependence of the reliability index on the model parameters θ, it is better to apply this

approximation on the function $\beta(\theta)$. Accordingly, we obtain the
approximate mean and variance of the conditional reliability index from

$$\mu_\beta \cong \beta(\boldsymbol{M}_\theta) \tag{13}$$

$$\sigma_\beta^2 \cong \nabla_\theta \beta^T \Sigma_{\theta\theta} \nabla_\theta \beta \tag{14}$$

where $\nabla_\theta \beta$ denotes the gradient of the reliability index with respect to the model parameters, evaluated at the posterior mean $\boldsymbol{M}_\theta$. The standard deviation σ_β is a direct measure of the uncertainty in the reliability measure arising from the epistemic uncertainties. It is important to note that the calculation of μ_β and σ_β from the above expressions requires a single reliability analysis using the posterior mean values of the model parameters together with the reliability sensitivities. Note that the reliability sensitivity measures are easily computed in the first-order reliability method, FORM (Ditlevsen and Madsen 1996).

Roughly speaking, the reliability index interval $\mu_\beta \pm \mu_\beta$ corresponds to a 70% confidence interval on the reliability index. This interval can be transformed back to estimate the corresponding interval of the failure probability. However, since $\tilde{\beta}$ corresponds to the true mean of the failure probability, it is better to compute the probability interval by transforming $\tilde{\beta} \pm \sigma_\beta$. Hence, an approximation of the 70% confidence interval of the failure probability around its mean $\tilde{P}_f$ is given by

$$\langle P_f \rangle_{70\%} = \Phi[-(\tilde{\beta} \pm \sigma_\beta)] \tag{15}$$

This interval reflects the influence of the epistemic uncertainties on the failure probability.

Specific applications of the above Bayesian reliability formulations can be found in Der Kiureghian (1999a, 1999b), Gardoni *et al.* (2002) and Sasani and Der Kiureghian (2001).

5. Summary and Conclusions

A Bayesian, full-distribution theory of structural reliability under conditions of statistical and model uncertainty is described. This theory is an enhancement of the second-moment theory for such analysis presented by A. H-S. Ang and others in late 1960's and early 1970's. The formulation presented here is richer in the sense that it can incorporate all kinds of information and it properly accounts for all types of uncertainties. Furthermore, explicit measures of uncertainty in the reliability estimate that reflect the effect of epistemic uncertainties is given. No doubt, these developments would not have been possible without the pioneering contributions of A. H-S. Ang and others three decades ago.

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