NUMERICAL SIMULATION OF SCOUR BY A WALL JET

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Abstract: The time consuming and expensive nature of experimental research on scouring processes caused by flowing water makes it attractive to develop numerical tools for the prediction of the interaction of the fluid flow and the movable bed. In this paper the numerical simulation of scour by a wall jet is presented. The flow is assumed to be two-dimensional, and the alluvium is cohesionless. The solution process, repeated at each time step, involves simulation of a turbulent wall jet flow, solution of the convection-diffusion of sand concentration, and prediction of the bed deformation. For simulation of the jet flow, the governing equations for momentum, mass balance and turbulent parameters are solved by the finite volume method. The SIMPLE scheme with momentum interpolation is used for pressure correction. The convection-diffusion equation is solved for sediment concentration. A boundary condition for concentration at the bed, which takes into account the effect of bed-load, is implemented. The time rate of deposition and scour at the bed is obtained by solving the continuity equation for sediment. Comparison of simulation results with experimental data shows favorable agreement on the time evolution of the scour. The shape and position of the scour hole and deposition of the bed material downstream of the hole appear realistic.

Key Words: Scour, Numerical Simulation, Turbulent Wall Jet, Sediment

1. INTRODUCTION

In recent years, and with ever increasing computer capabilities, several authors have focused on the numerical simulation of local scour.

Ushijima et al. (1992) considered 2-D and 3-D scouring by warmed jets. The unsteady continuity and momentum equations along with the k-ε turbulence closure model were solved using a finite difference method. Both bed-load and suspended- load were considered in the solution of the continuity equation for sediment at the bed. The convection-diffusion equation for suspended-load concentration was solved numerically. Sediment concentration at the bed was calculated with the aid of the Lane and Kalinske (1941) relation. Although the results were, in general, satisfactory, the computed bed profiles exhibited some undulations which were not present in the experimental results. To solve these problems, Ushijima (1996) subsequently used an arbitrary Lagrangian-Eulerian approach in which three-dimensional body-fitted coordinates were generated for the sand bed profile which was unsteadily deformed by the flow.

Hoffman and Booij (1993) presented a model for the flow in a trench based on the solution of the 2-D Reynolds equation and the convection-diffusion equation. Bed-load and suspended-load were computed by the stochastic method of van Rijn (1987). The results were satisfactory, even though it was necessary to calibrate some parameters in the sediment transport formula for different flow zones.

Olsen and Melaaen (1993) simulated the scour process around a cylinder. Sediment concentration for the bed elements was calculated with van Rijn's (1987) deterministic formula. Based on continuity for the bed elements, erosion and deposition were calculated. Comparison of the numerical results with experimental data showed fairly good agreement.

In the present paper the numerical simulation of the 2-D scour process by a wall jet is presented. Numerical solution of the governing equations is obtained on a non-orthogonal curvilinear mesh which is updated at each time step as the scour profile develops.

2. GOVERNING EQUATIONS

The main equations governing fluid flow are the Reynolds equations, i.e.:

$$U_j \partial U_i / \partial x_j = -(1/\rho)\partial(P\delta_{ij})/\partial x_j - \partial(\overline{u_i u_j})/\partial x_j \tag{1}$$

where U_j is time averaged velocity, P is dynamic pressure and $-\rho\overline{u_i u_j}$ is turbulent shear stress. The turbulent stresses are usually modeled with the aid of the Boussinesq relations (Rodi, 1993) as follows:

$$-\rho\overline{u_i u_j} = \rho \nu_T(\partial U_i/\partial x_j + \partial U_j/\partial x_i) - 2/3k\delta_{ij} \tag{2}$$

The turbulent eddy viscosity (ν_T) is modeled using the k-ε turbulence model. In this model the eddy-viscosity is obtained from

$$\nu_T = c_\mu k^2/\varepsilon \tag{3}$$

in which k is the turbulent kinetic energy and ε is the dissipation rate of k. The equations for k and ε are :

$$U_j \partial\varepsilon/\partial x_j = \partial(\nu_T/\sigma_\varepsilon \partial\varepsilon/\partial x_j)/\partial x_j + c_{\varepsilon 1}\varepsilon/kP_k - c_{\varepsilon 2}\varepsilon^2/k \tag{4}$$

$$U_j \partial k/\partial x_j = \partial(\nu_T/\sigma_k \partial k/\partial x_j)/\partial x_j + P_k - \varepsilon \tag{5}$$

where P_k is production of k and is defined as

$$P_k = \nu_T \frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right) \tag{6}$$

c_μ, $c_{\varepsilon 1}$, $c_{\varepsilon 2}$, σ_ε and σ_k are the coefficients of the k-ε model with the following values: c_μ=0.09, $c_{1\varepsilon}$=1.44,
$c_{2\varepsilon}$=1.92, σ_ε=1.24 and σ_k=1.0.

The partial differential equation for the sediment flow can be written as a convection-diffusion equation. For steady flow this becomes:

$$U_j \frac{\partial C}{\partial x_j} + w_s\left(\frac{\partial C}{\partial x_z}\right) = \frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial C}{\partial x_j}\right) \tag{7}$$

where C is the concentration, w_s is the fall velocity of the sediment particles, and Γ is the diffusion coefficient which is proportional to the eddy-viscosity.

3. BOUNDARY CONDITIONS

For the outflow boundary, a zero-gradient boundary condition is used and velocities are set equal to the values in the elements closest to the outflow. At the water surface, zero-gradient conditions are used for velocity components parallel to the free surface. Sediment concentration, ε, k, and velocity component perpendicular to the free surface are set to zero.

For the bed, it is assumed that the center of the wall element lies within the log-layer of the wall and therefore the logarithmic distribution for the velocity is used, i.e.,

$$\frac{U}{u_*} = 1/\kappa \, ln \frac{E y u_*}{\nu} \qquad (8)$$

where, in the above equation, $E = 9.0$ is used, u_* is shear velocity, κ is von Karman constant and ν is kinematic viscosity.

At the wall, based on the assumption of a balance between production and dissipation of k near the wall, ε for the closest elements to the wall is calculated from the following equation:

$$\varepsilon = \frac{C_\mu^{3/4}}{\kappa \delta_{np}} k_p^{3/2} \qquad (9)$$

in which δ_{np} is the distance from the wall to the center of the bed element and subscript p refers to the value at the centre of the bed element.

The sediment concentration near the bed is an important boundary condition for the sediment calculation. Two different formulae for the bed concentration proposed by van Rijn (1987), namely deterministic and stochastic formulae, can be used. In the deterministic formula, C_a is calculated from

$$C_a = 0.015 \frac{d_{50}}{a} \frac{T^{1.5}}{D_*^{0.3}} \qquad (10)$$

in which

$$T = \frac{\tau - \tau_c}{\tau_c}$$

$$D_* = d_{50} \left[\frac{(\rho_s - \rho_w) g}{\nu^2} \right]^{1/3}$$

where in this work, we have taken $a=\delta_{np}$, d_{50} is the median size of the sediment, τ_c is the critical shear stress, ρ_s and ρ_w respectively refer to sediment and water density and g is the acceleration due to gravity.

In turbulent flow, the actual shear stress oscillates around a mean value. Therefore, it may be more appropriate to compute sediment transport based on the difference between flow shear stress and critical shear stress of the bed material by taking into account the stochastic behavior. To do this, the following stochastic formula for C_a (proposed by van Rijn (1987)) can be adopted:

$$C_a = 0.03 \frac{d_{50}}{a} \frac{E_1(1.5) + E_2(1.5)}{D_*^{0.3}} \qquad (11)$$

in which

$$E_1(\gamma) = \int_0^{\tau} \left[T(\tau_0')\right]^{\gamma} P(\tau_0') d\tau_0'$$

$$E_2(\gamma) = \int_{-\tau}^{0} \left[T(\tau_0')\right]^{\gamma} P(\tau_0') d\tau_0'$$

$$P(\tau_0') = \frac{1}{\sqrt{2\pi}\,\sigma_t} \exp\left[- \left[\frac{\tau_0' - \overline{\tau_0'}}{\sqrt{2}\,\sigma_t} \right] \right]$$

$$\overline{\tau_0'} = \mu \tau_0 = \mu \rho u_*^2 \qquad (\mu = 1.0)$$

$$T(\tau_0') = \frac{-\tau_0' + \tau_2}{\tau_c} \qquad -\infty \langle \tau_0' \langle \tau_2$$

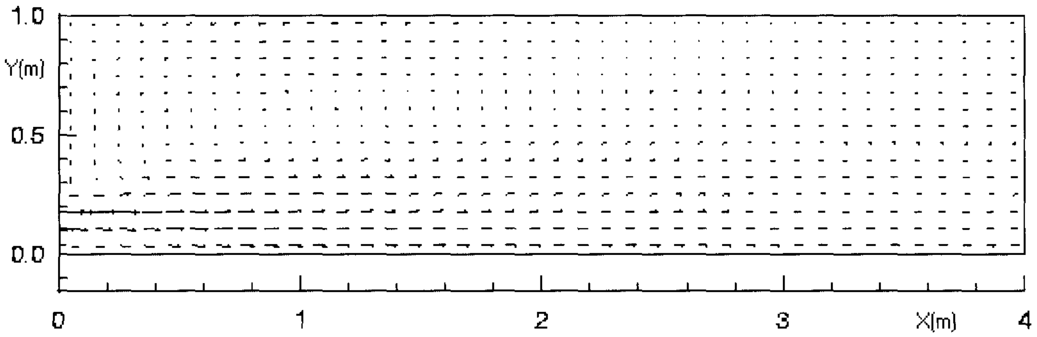

Fig. 1. Flow field of the wall jet at the beginning of the scour process

$$T(\tau_0^{'}) = 0 \qquad \tau_2 \langle \tau_0^{'} \langle \tau_1$$

$$T(\tau_0^{'}) = \frac{\tau_0^{'} - \tau_1}{\tau_c} \qquad \tau_2 \langle \tau_0^{'} \langle \infty$$

Here, σ_t is standard deviation which should be determined based on the physical characteristics of the flow.

4. NUMERICAL SOLUTION

The partial differential equations for the flow and sediment concentration are integrated over an element using a finite volume technique. The resulting equation for any variable Φ (velocity components, sediment concentration, k or ε) is written as (Patankar, 1980):

$$a_P \phi_P = \sum_{nb} a_{nb} \phi_{nb} + source \qquad (12)$$

The subscript *nb* indicates neighboring elements.

The difference schemes calculate the a_{nb} and a_P coefficients based on the convective and orthogonal diffusive fluxes in the equation. The non-orthogonal fluxes are discretized separately and incorporated into the source term. Also, the remaining terms are calculated separately and considered in the source term. For velocity-pressure coupling, the SIMPLE algorithm has been followed. Since a non-staggered grid is used in this study, the Rhie and Chow (1983) interpolation method is used to avoid instabilities in the calculation of the velocities and pressure.

5. APPLICATION OF THE MODEL

To check the ability of the present model for prediction of scour, the model is applied to the case of a wall jet. The details of the flow field in this example are as follows (see Fig. 1):

Original flow field dimensions : depth = 1 m, length = 4 m

Inlet jet velocity: U_0 = 2.0 m/s, V_0 = 0.0 m/s

Width of the incoming jet : 0.2 m

Sediment density (ρ_s): 2650 kg/m^3

Sediment diameter : 0.82 mm

Sediment porosity : 40%

Grid size : 15×41

The result of application of the present model using the deterministic sediment transport formula of van Rijn (Eq. 10), is shown in Fig. 2 and Fig. 3, corresponding to the durations of 1*s* and 640*s* from the beginning of the scouring process, respectively. These figures show that the computed scouring pattern is in agreement with physical observations. Also, they indicate that the deposition of sediment downstream of

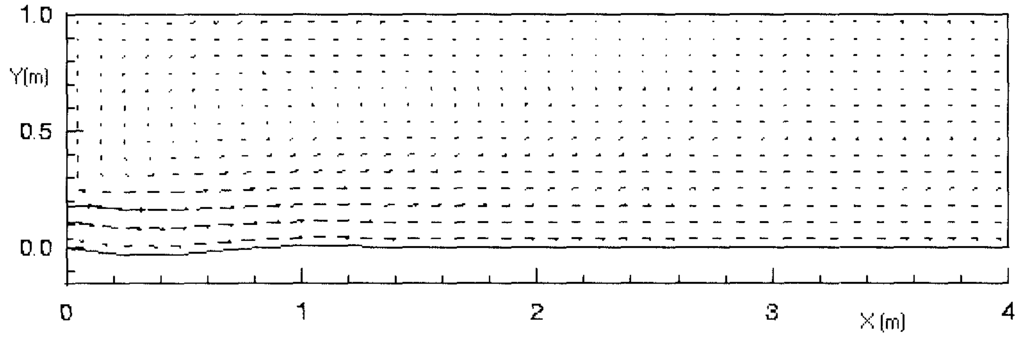

Fig. 2. Flow pattern and scour hole profile after 1 sec.

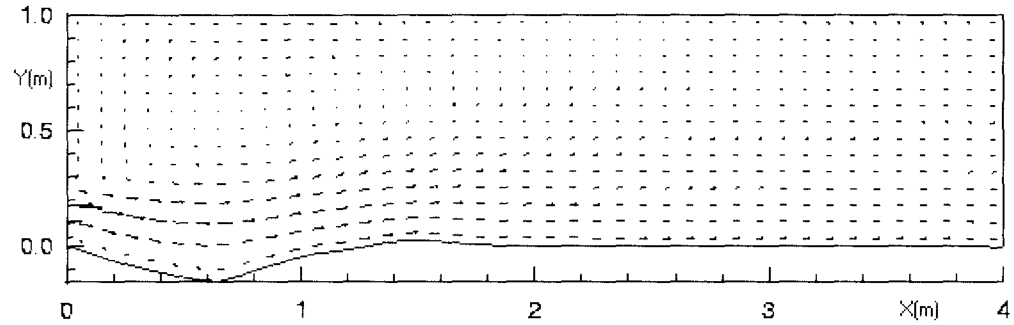

Fig. 3. Flow pattern and scour hole profile after 640 sec.

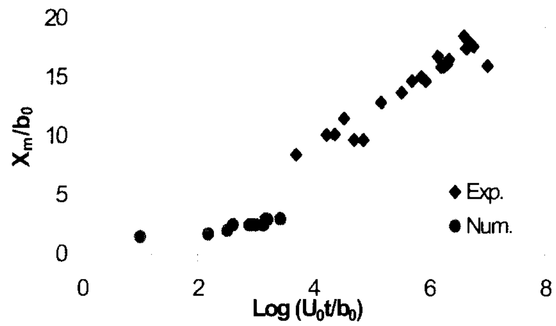

Fig. 4 Development of maximum scour position with time (Exp. Data of Salehi Neyshabouri(1988))

the scour hole can be realistically simulated. In Fig. 4, the numerical results are plotted with the experimental results of Salehi Neyshabouri (1988) on the scour process by a wall jet. In this figure, X_m, b_0 and t stand for the horizontal distance of maximum scour position from the jet inlet, width of the incoming jet, and time, respectively. One can observe that the time interval for the numerical predictions and the experiments do not overlap in Fig. 4. Nevertheless, it is well-known (Salehi Neyshabouri (1988)) that the overall profile should behave as illustrated in this figure. For small time, X_m/b_0 should be small and should increase slowly, as seen from the numerical results. As time increases, eventually the horizontal position of the maximum scour begins to increase linearly with $\log(U_0t/b_0)$. The experimental data shows this behaviour. Thus by combining the numerical results with the experimental results, one is able to obtain a more complete picture of the movement of the scour hole over a wider time range.

6. CONCLUSIONS

In this paper the validity of a numerical simulation of the scour process caused by a wall jet is presented. Based on the application of the numerical model, it is found that the flow pattern and, in particular, the shear stress at the bed, which is the main factor influencing the scour process, is correctly predicted by the hydrodynamic component of the model. The other important factor for correct simulation of the scour process is the sediment transport. For the results presented in the present work, the deterministic formula of van Rijn is used to capture this effect.

In general, the numerical modeling of the scour process needs further consideration. For example, in regions where reverse flow develops, commonly used deterministic sediment transport formulae fail to predict the sediment load correctly. In this case, the stochastic model must be implemented.

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