ANALYSIS OF LOOPED WATER DISTRIBUTION NETWORK

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Abstract: There are three methods for analyzing flow and pressure distribution in looped water distribution networks (the loop method, the node method, the element method) taking into consideration hydraulic parameters chosen as unknown. For all these methods the non-linear system of equations can be solved by iterative procedures.

The paper presents a different approach to this problem by using the method of variational formulations for hydraulic analysis of water distribution networks. This method has the advantage that it uses a specialized optimization algorithm which minimizes directly an objective multivariable function without constraints, implemented in a computer program. The paper compares developed method to the classic Hardy–Cross method. This shows the good performance of the new method.

Key Words: looped water distribution networks, method of variational formulations, Hardy–Cross method

1. INTRODUCTION

There are three methods for analyzing flow and pressure distribution in looped water distribution networks (the loop method, the node method, the element method) taking into consideration hydraulic parameters chosen as unknown.

For all these methods the non-linear system of equations can be solved by iterative procedures: Hardy–Cross method (Abramov, 1976; Divénot, 1980) or Newton–Raphson method (Chandrashekar and Stewart, 1975; Gofman and Rodeh, 1981; Sârbu, 1987; Wood and Rayes, 1981).

This paper shows a different approach to this problem by using the method of variational formulations for functional analysis of water distribution networks. This method has the advantage that it uses a specialized optimization algorithm which minimizes directly an objective multivariable function without constraints, implemented in a computer program.

2. THE FUNDAMENTALS OF HYDRAULIC COMPUTATION

In the case of a complex topology for a looped network, with reservoirs and pumps at the nodes, the total number of independent loops (closed-loops, possibly containing pumps mounted in the pipes, and pseudoloops) M is given by the following formula:

$$M = T - N + N_{RP} \qquad (1)$$

in which: T is the number of pipes in network; N – number of nodes in network; N_{RP} – number of reservoirs and pump at the nodes, equal to the number of nodes with know hydraulic grade.

Each open-loop (pseudoloop) makes the connection between a node with a known piezometric head (reservoir) or with a determined relation discharge – piezometric head (pump station), and another node with a known piezometric head or a determined relation discharge – piezometric head.

In classical analysis of looped networks in permanent water flow regime, fundamental equations of the computing model express:

– discharge continuity at nodes:

$$\sum_{\substack{i=1 \\ i \neq j}}^{N} Q_{ij} + q_j = 0 (j = 1,..., N - N_{RP}) \tag{2}$$

in which: Q_{ij} is the discharge through pipe ij, with the sign (+) when entering node j and (–) when leaving it; q_j – concentrated discharge at node j with the sign (+) for node inflow and (–) for node outflow.

– energy conservation in loops:

$$\sum_{\substack{ij \in m \\ ij=1}}^{T} \varepsilon_{ij} h_{ij} - f_m = 0 (m = 1,..., M) \tag{3}$$

in which: h_{ij} is the head loss of the pipe ij; ε_{ij} – orientation of flow through the pipe, having the values (+1) or (–1) as the water flow sense is the same or opposite to the path sense of the loop m, and (0) value if $ij \notin m$; f_m – pressure head introduced by the potential elements of the loop m, given by the relations:

- simple closed-loops:

$$f_m = 0 \tag{4}$$

- closed-loops containing pumps mounted in the pipes:

$$f_m = \sum_{\substack{ij \in m \\ ij=1}}^{T} \varepsilon_{ij} H_{p.ij} \tag{5}$$

- open-loops with pumps and/or reservoirs at nodes:

$$f_m = Z_I - Z_E \tag{6}$$

where: Z_I, Z_E are piezometric heads at pressure devices at the entrance or exit from the loop; $H_{p.ij}$ – the pumping head of the pump mounted in the pipe ij, for the discharge Q_{ij}, approximated by parabolic interpolation on the pump curve given by points:

$$H_{p.ij} = AQ_{ij}^2 + B\left|Q_{ij}\right| + C \tag{7}$$

the coefficients A, B, C can be determined from three points of operating data.

The head loss is given by the Darcy–Weisbach functional relation:

$$h_{ij} = \frac{8}{\pi^2 g} \lambda_{ij} \frac{L_{ij}}{D_{ij}^r} Q_{ij}^2 \tag{8}$$

in which: g is the gravitational acceleration; λ_{ij} – friction factor of pipe ij which can be calculated using the Colebrook-White formula; D_{ij}, L_{ij} – diameter and the length of pipe ij;

r – exponent having the value 5.0.

Equation (8) is difficult to use in the case of pipe networks and therefore it is convenient to write it in the following general form:

$$h_{ij} = R_{ij} Q_{ij}^{\beta} \tag{9}$$

where R_{ij} is the hydraulic resistance of pipe ij, having the succeeding relation:

$$R_{ij} = \frac{8\lambda_* \; L_{ij}}{\pi^2 g \; D_{ij}^r} \tag{10}$$

The variation of hydraulic parameters λ_* and β has been determined for different pipe materials and water temperatures, using a computer program (Sârbu, 1997).

Formula (9) can be written as follows:

$$h_{ij} = Z_i - Z_j = R_{ij} Q_{ij} \left| Q_{ij} \right|^{\beta-1} \tag{11}$$

or:

$$Q_{ij} = R_{ij}^{-\frac{1}{\beta}} h_{ij}^{\frac{1}{\beta}} = R_{ij}^{-\frac{1}{\beta}} \left(Z_i - Z_j + \Pi_{ij} \right) \left| Z_i - Z_j + \Pi_{ij} \right|^{\frac{1-\beta}{\beta}} \tag{12}$$

in which: Z_i and Z_j are the piezometric heads at nodes i and j; Π_{ij} – active pressure introduced by the intermediate pump on the pipe ij.

If it is associated to each loop m a circulation flow ΔQ_m and if is choosed initial flow distribution $Q_{ij}^{(0)}$ which have to satisfy equation (2), then it can be written:

$$Q_{ij} = Q_{ij}^{(0)} + \sum_{\substack{ij \in m \\ m=1}}^{M} \varepsilon_{ij} \Delta Q_m \; (ij = 1, \ldots, T) \tag{13}$$

and for simple loops ($f_m = 0$) the system (2), (3), (9), (13) is equivalent to the following:

$$\sum_{(m)} \varepsilon_{ij} R_{ij} \left(Q_{ij}^{(o)} + \sum_{\substack{ij \in m \\ m=1}}^{M} \varepsilon_{ij} \Delta Q_m \right) \left| Q_{ij}^{(o)} + \sum_{\substack{ij \in m \\ m=1}}^{M} \varepsilon_{ij} \Delta Q_m \right|^{\beta-1} = 0 \quad (m = 1, \ldots, M) \tag{14}$$

Substituting equation (12) in equation (2) one gets a system of N–N_{RP} equations at nodes with N–N_{RP} unknown:

$$\sum_{\substack{i \neq j \\ i=1}}^{N} R_{ij}^{-\frac{1}{\beta}} \left(Z_i - Z_j + \Pi_{ij} \right) \left| Z_i - Z_j + \Pi_{ij} \right|^{\frac{1-\beta}{\beta}} + q_j = 0 \quad (j = 1, \ldots, N - N_{RP}) \tag{15}$$

Specific consumption of energy for water distribution w_{sd}, in kWh/m^3, is obtained by referring the hydraulic power dissipated in pipes to the sum of node discharges:

$$w_{sd} = 0.00272 \frac{\sum_{ij=1}^{T} R_{ij} \left| Q_{ij} \right|^{\beta+1}}{\sum_{\substack{j=1 \\ q<0}}^{N} \left| q_j \right|} \tag{16}$$

where q_j is the outflow at the node j.

3. THE VARIATIONAL FORMULATION OF THE PROBLEM

The hydraulic analysis of looped networks can be achieved, according to the element method, by using a conditioned optimization model named "the content model" (Coolins and Cooper, 1978; Coolins, 1979).

The discharges at equilibrium, in loop method can be determined by using the criterion of minimization of the energy content in time (power) for the whole of the network. For networks containing potential elements (reservoirs and pumps at nodes, pumps mounted in the pipes) objective function can be expressed as follow:

$$F_e = \sum_{ij=1}^{T}\left[\int_0^{Q_{ij}}(Z_i - Z_j)dQ_{ij}\right] - \sum_{ij=1}^{T}\left(\int_0^{Q_{ij}} H_{p,ij}dQ_{ij}\right) - \sum_{j=1}^{N_{RP}}\left(\int_0^{q_j} Z_j^* dq_j\right) \to min \quad (17)$$

where Z^*_j is the piezometric head of node source j.

Equation (17) is subject to continuity constraints (2) and to constraints of non-negativity for discharges ($Q_{ij} \geq 0$).

The functionals in the first term of equation (17) represent energy loss through the network pipes when the network carries the discharges that satisfy continuity requirements, and the functionals in the second and third terms represent the external power input to the system.

Thus substituting equations (11) and (13) in the objective function (17) as well as considering equation (7) after one calculates all the integrals one eliminate all the constraints. In order to determine the discharges at which hydraulic equilibrium network occurs, we have only to find the minimum of a function with M variables (ΔQ_m) without constraints:

$$F_e = \frac{1}{\beta+1}\sum_{ij=1}^{T} R_{ij}\left|Q_{ij}^{(o)} + \sum_{\substack{ij\in m\\ m=1}}^{M}\varepsilon_{ij}\Delta Q_m\right|^{\beta+1} - \sum_{ij=1}^{T}\left(\frac{1}{3}A_{ij}\left|Q_{ij}^{(o)} + \sum_{\substack{ij\in m\\ m=1}}^{M}\varepsilon_{ij}\Delta Q_m\right|^3 + \frac{1}{2}B_{ij}\left|Q_{ij}^{(o)} + \sum_{\substack{ij\in m\\ m=1}}^{M}\varepsilon_{ij}\Delta Q_m\right|^2 + C_{ij}\left|Q_{ij}^{(o)} + \sum_{\substack{ij\in m\\ m=1}}^{M}\varepsilon_{ij}\Delta Q_m\right|\right) - \sum_{j=1}^{N_{RP}} Z_j^* q_j \to min \quad (18)$$

By using the extremum requirements in (18), $\partial F_e/\partial \Delta Q_m = 0$ ($m = 1,...,M$), one gets the system (14) for a simple network in the classical formulation of the problem.

This formulation is advantageous because it can make use of a minimization algorithm for function (18) like the "conjugate gradient algorithm" recommended by literature (Sârbu, 1994; Todini and Pilati, 1987). The objective function form is independent from the initial solution, for a certain loop system. Process convergence depends of loop configuration mode and the minimum value that remains unchanged. Function has a much favorable form if common loop part is shorter.

The discharges through pipes Q_{ij} are obtained using equation (13), in which are introduced the circulation flow ΔQ_m obtained as those values that minimize the objective function. After the head losses have been calculated with equation (9), the piezometric heads Z_j can be determined starting from a node of known piezometric head. Then the residual pressure head H_j at the node j is calculated from the relation:

$$H_j = Z_j - ZT_j \quad (19)$$

where ZT_j is the elevation head at the node j.

One can find a variational formulation for the hydraulic analysis of looped networks also in node method, using the following objective function:

$$F_e = \sum_{ij=1}^{T}\left(\int_0^{h_{ij}} Q_{ij}dh_{ij}\right) - \sum_{j=1}^{N}\left(\int_0^{Z_j} q_j dZ_j\right) \to min \quad (20)$$

and the energy conservation in loops constraints (3).

After substituting the functional relation (12) in (20) and after the integrals have been calculated the constraints can be eliminated and the problem can be simplified to the finding of the minimum of a function with N–N_{RP} variables (Z_i, Z_j) without constraints:

$$F_e = \frac{\beta}{\beta+1}\sum_{ij=1}^{T} R_{ij}^{-\frac{1}{\beta}}\left|Z_i - Z_j + \Pi_{ij}\right|^{\frac{\beta+1}{\beta}} - \sum_{j=1}^{N} q_j \qquad Z_j \to min. \qquad (21)$$

which can be achieved by using the conjugate gradient algorithm.

Using the extremum requirements $\partial F_e/\partial Z_j = 0$ ($j = 1,\ldots,N$–N_{RP}) one gets the system of node equations (15).

In order to determine an initial approximation of the piezometric head at nodes one must solve the following linear associated system:

$$\sum_{\substack{i\neq j\\ i=1}}^{N} R_{ij}^{-1}(Z_j^{(o)} - Z_i^{(o)} - \Pi_{ij}) = q_j^{\beta}\,(j = 1,\ldots, N - N_{RP}) \qquad (22)$$

Once the piezometric head at nodes have been determined one can calculate the residual pressure head using equation (19) and afterwards the discharges through pipes with equation (12) as well as other hydraulic parameters of the network.

Two computer programs ACIREV and ANOREV were elaborated (Sârbu, 1997), based on the optimization models developed above. It are realized in the FORTRAN 5.1 program-ming language and implemented on IBM–PC compatible computers.

4. NUMERICAL APPLICATION

The looped distribution network with the topology from Figure 1 is considered. It is made of cast iron and is supplied with a discharge of 0.50 m^3/s. The following data are known: pipe length L_{ij}, in m, pipe diameter D_{ij}, in m, elevation head ZT_j, in m, industrial concen-trated

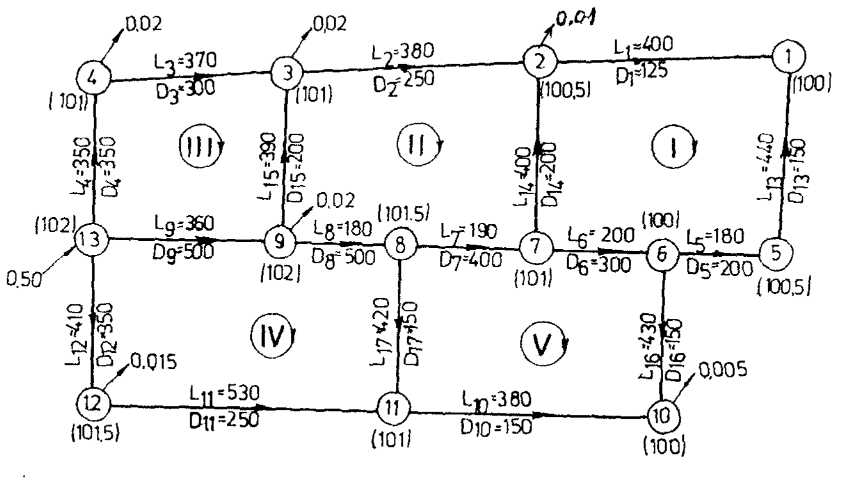

Fig.1 Schema of the analyzed distribution network

Table 1. The discharges and head losses trough pipes

Pipe $i-j$	Computation model					
	HARDY-CROSS		ACIREV		ANOREV	
	Q [m^3/s]	h [m]	Q [m^3/s]	h [m]	Q [m^3/s]	h [m]
0	1	2	3	4	5	6
2–1	0.01209	3.949	0.01219	4.007	0.01204	3.915
3–2	0.03608	0.890	0.03616	0.894	0.03605	0.889
4–3	0.06917	1.203	0.06902	1.198	0.06916	1.203
13–4	0.11373	1.359	0.11358	1.356	0.11371	1.356
6–5	0.03771	1.443	0.03761	1.437	0.03776	1.447
7–6	0.08199	0.906	0.08174	0.900	0.08206	0.907
8–7	0.13520	0.521	0.13496	0.519	0.13531	0.521
9–8	0.17547	0.262	0.17542	0.261	0.18034	0.276
13–9	0.25299	1.067	0.25317	1.069	0.25266	1.065
11–10	0.01597	2.518	0.01613	2.568	0.01610	2.558
12–11	0.04802	2.160	0.04799	2.158	0.04806	2.163
13–12	0.09508	1.123	0.09505	1.123	0.09501	1.122
5–1	0.01656	3.127	0.01647	3.094	0.01661	3.147
7–2	0.02626	1.588	0.02628	1.590	0.02625	1.587
9–3	0.02579	1.495	0.02602	1.521	0.02579	1.495
6–10	0.01666	3.092	0.01649	3.034	0.01661	3.075
8–11	0.01332	1.958	0.01351	2.014	0.01327	1.945

discharges in nodes q_j, in m^3/s, piezometric head at the "critical node" Z_l = 124 m, and the exponent β = 1.936.

It is required to determine the pumping head, discharges and pressures distribution using the classic HARDY–CROSS procedure and the two optimization models (ACIREV, ANOREV) developed above. Results of the numerical solution performed by means of an IBM–AT 586 computer, referring to the hydraulic characteristics of the pipes and nodes are presented in tables 1 and 2.

Table 1 shows the discharges and head losses through pipes established by using the three mentioned models of computation (the iterative tolerance imposed is 10^{-5}). It can be seen that the results are very close.The difference between the discharges obtained with HARDY–CROSS and these given by ACIREV vary between 0.08 % (pipe 3–2) to 1.8 % (pipe 3–7), and the difference between discharges obtained with HARDY–CROSS and ANOREV varies from 0 % (pipe 9–3) to 2.7% (pipe 9–8). Specific consumption of energy for water distribution is 0.00705 kWh/m^3, for all three models of computation used.

Table 2 presents the values for the piezometric head Z_j and the residual pressure head H_j at nodes determined by using the classic procedure and the two new models of computation. The

Table 2. The piezometric head and the residual pressure head at nodes

Node j	Computation model					
	HARDY–CROSS		ACIREV		ANOREV	
	Z_j [m]	H_j [m]	Z_j [m]	H_j [m]	Z_j [m]	H_j [m]
0	1	2	3	4	5	6
1	124.000	24.000	124.000	24.000	124.000	24.000
2	127.949	27.449	128.007	27.507	127.915	27.415
3	128.839	27.839	128.901	27.901	128.804	27.804
4	130.042	29.042	130.099	29.099	130.007	29.007
5	127.127	26.627	127.094	26.594	127.147	26.647
6	128.571	28.571	128.530	28.530	128.594	28.594
7	129.537	28.537	129.597	28.597	129.502	28.502
8	130.058	28.558	130.116	28.616	130.023	28.523
9	130.335	28.335	130.386	28.386	130.299	28.299
10	125.478	25.478	125.497	25.497	125.520	25.520
11	128.118	27.118	128.175	27.175	128.078	27.078
12	130.278	28.778	130.332	28.832	130.241	28.741
13	131.402	29.402	131.455	29.455	131.363	29363

piezometric head at the node 13 has the following values: 131.402 m, 131.455 m and 131.363 m which give a residual pressure head of 29.402 m, 29.455 m and 29.363 m that is sufficient for the supply water to the consumers. The divergence of piezometric line on network contour is 0.122 m for HARDY–CROSS, 0.109 m for ACIREV and only 0.001 m for ANOREV.

5. CONCLUSIONS

The mathematical model expressed by the objective functions (18) and (21) constitutes a new way of hydraulic analysis of complex looped networks based on unconditioned optimization techniques.

The new method replaces the solving of the non-linear system of equations (2), (3), (9) with the direct minimization of a multivariable function, without constraints that express the energy consumption across the network.

The computer programs ACIREV and ANOREV include this particular aspect and contain the conjugate gradient algorithm, which give it efficiency especially in functional analyzing of complex distribution networks. This new method is computationally more efficient and consequently helps the designer to get the best design of water distribution systems with fewer efforts.

REFERENCES

Abramov N. N., *Rascet vodoprovodnîi setei*, Stroizdat, Moskva, 1976.

Chandrashekar M., and Stewart K., *Sparsity oriented analysis of large pipe networks*, Journal of the Hydraulics Division, ASCE, nr. HY4, 1975.

Coolins M., and Cooper L., *Solving the pipe network analysis problem using optimization techniques*, Management Science, nr. 7

1978.

Collins M. et al., *Multiple operating points in complex pump network*, Journal of the Hydraulics Division, ASCE, nr. HY3, 1979.

Divénot A., *Une nouvelle méthode de calcul des réseaux maillés*, La Houille Blanche, nr.6, 1980.

Gofman E., and Rodeh M., *Loop equation with unknown pipe characteristics*, Journal of the Hydraulics Division, ASCE, nr. HY9, 1981.

Lekane T., *Modele de calcul de l'écoulement en régim permanent dans réseau d'eau maillés*, Journal de Recherches Hydrauliques, nr.2, 1979.

Sârbu I., *Model de calcul al regimului hidraulic în rețele complexe de distribuție a apei*, Rev. Hidrotehnica, nr.8, 1987.

Sârbu I., *Energetical Optimization of Water Distribution Systems*, Ed. Academiei Române, Bucureşti, 1997.

Sârbu I., *Numerical and Optimizing Methods in Building Equipment Design*, Ed. Tehnică, Bucureşti, 1994.

Sima V., and Varga A., *Practica optimizării asistată de calculator*, Ed. Tehnică, Bucureşti, 1986.

Todini E., and Pilati S., *A Gradient Method for the solution of looped Pipe Networks*, Int. Conf. on Computer Applications in Water Supply and Distribution, V(1), 1987.

Wood D., and Rayes A., *Reliability of algorithms for pipe network analyses*, Journal of the Hydraulics Division, ASCE, nr. HY10, 1981.

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