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ENTRAINMENT OF SEDIMENT PARTICLES FROM SCOUR HOLE BY TURBULENT VORTICES DOWNSTREAM OF HYDRAULIC STRUCTURE

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Abstract: This study presents the estimation of the entrainment velocity of sediment particles from the scour hole. Sediment particles are entrained from the scour hole downstream of hydraulic structures by the turbulent vortices. Mathematical form of the entrainment velocity of sediment particles from the scour hole was obtained using the impulse-momentum equation with given value of the vertical component of turbulent velocity of the line vortex. Also, its probability density distribution was obtained with the results that the probability density distribution of the vertical turbulent velocity followed the normal distribution. Experimental results of the entrainment velocity of sediment particles showed relatively good agreements with theoretical ones.

Key Words: entrainment velocity, scour hole, turbulent vortices, impulse-momentum equation

1. INTRODUCTION

Vortex erosion occurs at the outlet of the hydraulic structure by the high shear stress of the turbulent vortices, and a resulting large scour hole is formed (Kim and Choe, 2000). Once the scour hole is formed, a line vortex occurs in it and this vortex causes additional erosion problems. Sand particles are entrained from the scour hole by the line vortex. The macroturbulence of the line vortex consists of continuous and strong upward flows. It is compared with that formed at the lee-side of the sand ripple in river whose turbulence is not so strong and intermittent, which supports the existing concept that the local concentration of suspended sediments is related to the intensity of the vertical component of turbulent vortices (Ikeda and Asaeda, 1983).

Sand particles lifted from the scour hole are circulated with the same direction of the line vortex. They contribute the formation of a hump by deposition or swept away downstream by the main flow (Kim, 1992). Granular jump analogous to the hydraulic jump occurs in the scour hole, since the upstream slope of the scour hole is relatively steep. The volume fraction of the suspended sediments from the scour hole is relatively high on the steep slope since it has the high erosion potential. Momentum transfer is made through the collisions of sediment particles and the energy dissipation evidently takes place within the granular jump. It is assumed that the grain-inertia regime on a fluid-granule mixed flow is formed (Savage, 1984).

It is important to estimate the entrainment velocity of sand particles from the scour hole for determining the amount of vortex erosion. To estimate the entrainment velocity of sand particles from the scour hole, it is necessary to determine the impulse equation between the vertical component of fluid velocity and particle movement.

The main purposes of this study are to derive the mathematical formulation of the entrainment velocity of sediment particles from the scour hole and to verify it through the experiments.

2. THEORETICAL APPROACH ON SEDIMENT ENTRAINMENT

2.1 Probability density distribution for the vertical velocity of turbulent vortices

It is well known that the sand particles are entrained into the water from the trailing edge of the scour hole by the vertical component of the turbulent vortices (Asaeda et al., 1989). The commencement of the sediment particles is caused by the vertical velocity fluctuation near the surface of the sand bed. To obtain the initial velocity of the sand particles, it is necessary to determine the vertical velocity of the turbulent vortices. The magnitude of the vertical fluid velocity, v fluctuates and follows the normal distribution with a mean $\bar{v}$ and a standard deviation v_{*l}. Thus, the probability density function is given by (Asaeda et al., 1989),

$$f(v) = \frac{1}{\sqrt{2\pi}v_{*l}}\exp[-\frac{1}{2}(\frac{v-\bar{v}}{v_{*l}})^2] \qquad (1)$$

Here v_{*l} is known to be the local friction velocity at the trailing edge of the scour hole (Asaeda et al., 1989).

It can be estimated that the probability density distribution for the vertical velocity of the turbulent vortices in the scour hole also follows the same normal distribution as eq. (1), since the characteristics of the vortex circulation and the vortex scale are almost the same as those of Tanaka (1986)'s experiments in the rippled model.

If the standardized variable x is used instead of v, eq. (1) becomes,

$$f(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{1}{2}x^2] \qquad (2)$$

where $x = \frac{v-\bar{v}}{v_{*l}}$ following the standardized normal distribution : N(0,1).

2.2 Governing equation for the entrainment of the sediment particles

The commencement of sediment particles is caused by vertical velocity fluctuation near the surface of sand bed. For a given value of the vertical component of fluid velocity, an initial entrainment velocity of a disturbed particle can be given by the impulse equation as,

$$\left[\frac{\pi}{4}\rho C_L W^2 d^2 - \frac{\pi}{6}(\sigma-\rho)gd^3\right]t = \frac{\pi}{6}\sigma d^3 W_0 \qquad (3)$$

where W is the vertical component of a fluid velocity, W_0 is the initial entrainment velocity of a disturbed particle, C_L is a coefficient of lift force, d is a diameter of sand particle, σ is a specific gravity of the sand particle, and t is the duration of lift force. According to Asaeda et al. (1989), t is assumed to be d/W_0. Thus eq. (3) becomes

$$W_0 = \left[\frac{3\rho}{2\sigma} C_L W^2 - \frac{\sigma - \rho}{\sigma} gd\right]^{1/2} \tag{4}$$

2.3 Probability density distribution for the entrainment velocity of sand particles

The entrainment velocity of sand particles can be represented by the impulse-momentum equation for a given value of the vertical fluid velocity. As was stated in 2.1, the vertical fluid velocity follows the normal distribution given by eq. (1). From eq. (4),

$$W_0 = u(W) = \sqrt{aW^2 - b}, \quad a = \frac{3\rho}{2\sigma} C_l, \quad b = \frac{\sigma - \rho}{\sigma} gd \tag{5}$$

Since W follows the normal distribution $N(\overline{W}, \sigma_s^2)$, the distribution of W_0 can be estimated from eq. (5). Let the inverse of $W_0 = u(W)$ be denoted by $W = v_1(W_0)$ and let the derivative dW/dW_0 be denoted by $v_1'(W_0)$, then by the transformation of variables,

$$P_1(W_0) = f\{v_1(W_0)\} \cdot |v_1'(W_0)|$$
$$= \frac{1}{\sqrt{2\pi} v_{*1}} \exp[-\frac{1}{2}(\frac{\sqrt{(W_0^2 + b)/a} - \overline{W}}{v_{*1}})^2] \cdot \frac{W_0}{aW}$$
$$= \frac{1}{\sqrt{2\pi a(W^2 + b)} \cdot v_{*1}} \exp[-\frac{1}{2}(\frac{\sqrt{(W_0^2 + b)/a} - \overline{W}}{v_{*1}})^2] \tag{6}$$

where $J = v_1'(W_0)$ is the Jacobian of the inverse transformation $W = v_1(W_0)$. If the dimensionless form of eq. (6) is considered,

$$P(y) = P_1\{v(y)\} \cdot |v'(y)|$$
$$= \frac{\overline{W}_0 y}{\sqrt{2\pi a(y^2 \overline{W}_0^2 + b)} \cdot v_{*1}} \exp[-\frac{1}{2}(\frac{\sqrt{(\overline{W}_0^2 y^2 + b)/a} - \overline{W}}{v_{*1}})^2] \tag{7}$$

where $y = W_0 / \overline{W}_0$

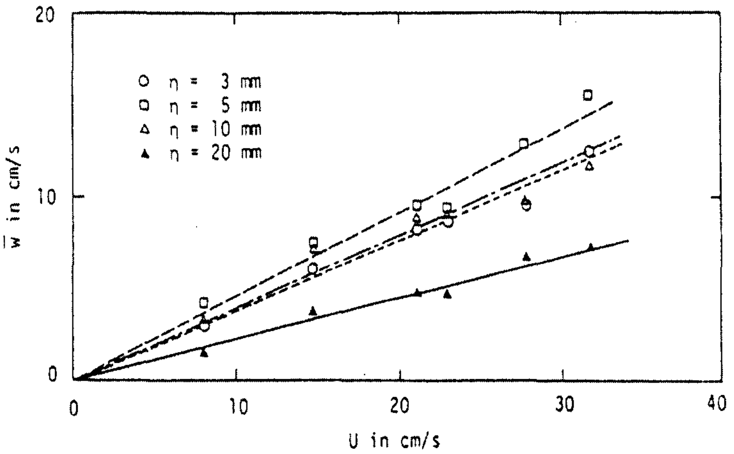


Fig. 1. Relationships Between $\overline{W}$ and U (Ikeda and Asaeda, 1983)

2.4 Estimation of the entrainment velocity using outer parameters

It is useful to estimate the entrainment velocity of sediment particles, using outer parameters related. According to Ikeda and Asaeda (1983), the mean value of the vertical component of turbulent velocity, $\overline{W}$ and the longitudinal average fluid velocity, U near the surface of the sand bed have linear relationships as

$$\overline{W} = \beta' U \quad (8)$$

where β' is the coefficient of proportionality. These relationships are depicted in Fig. 1.

Based on the eq. (8), the relationship between the mean value of vertical fluid velocity from the scour hole and the average upstream overflow velocity can be considered as,

$$\overline{W} = \alpha_1 \overline{u}_0 \quad (9)$$

where α_1 is the coefficient of proportionality.

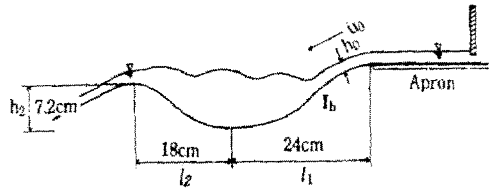
3. EXPERIMENTAL APPRATUS AND PROCEDURE

To measure the entrainment velocity, experiments were done in a recirculating channel of 0.4m wide and 10m long. The artificial sand bed was made to simulate the actual profile of the scour hole using the clean and fluorescent sand particles. The sidewall of the channel was made of glass. A transparent scale was attached to the sidewall for the quantitative measurement of entrainment velocity. Since the length between the upstream end of the channel and the location of the gate was relatively short, a thick screen was installed at the entrance of the channel.

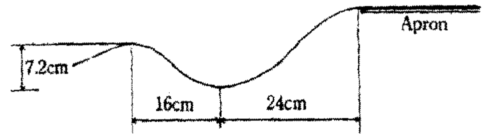
A video camera and a time-lapsed video tape recorder were used to measure the incipient entrainment velocity. The test section was illuminated by strong vertical light sheet through a slide projector in dark condition for clean view. Velocity field of the turbulent vortices in the scour hole was measured using aluminum particles and a motor driven camera (Kim and Choe, 2000). As a tracer method, aluminum particles mixed with a neutral detergent were put into the channel at the upstream end. Photos were taken using the motor driven camera with the vertical light sheet. Then, the velocity could be measured from the length of a trajectory of aluminum particle divided by exposure time. A slit width of the slide projector and exposure time of the camera was adjusted considering concentration and velocity of the aluminum particles. By a trial and error, 0.5～0.7mm of the slit width and 1/15～1/30 second of the exposure time were suitable values.

Experimental conditions are shown in Fig. 2. Here, h_0 is an entering flow depth into the scour hole, u_0 is the mean velocity of the entering flow I_b is the slope of an artificial bed and T is an exposure time of the motor driven camera. The type 1 and the type 2 of the artificial bed correspond to those of the instantaneous sand bed profile when the large macroturbulence occurred at a movable bed experiment.

Photo development was done under the condition of ISO up to 3200 in order to obtain clear trajectory in pictures taken in dark condition. This photo was used for obtaining a digitized velocity vector, which is shown in Fig. 3. Fig. 4 shows the procedure for obtaining the velocity field.



Type 1



Type 2

Fig. 2. Sand Bed Model for Scour Hole

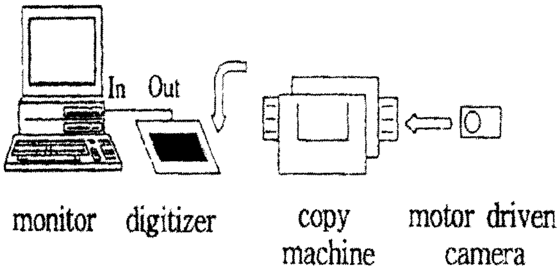


Fig. 3. Data System for Obtaining Digitized Velocity Vector

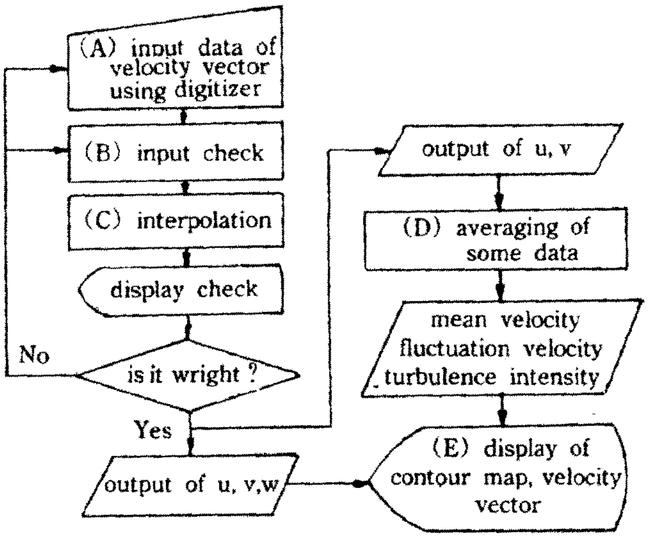


Fig. 4. Flow Chart for Data System (Kim and Choe, 2000)

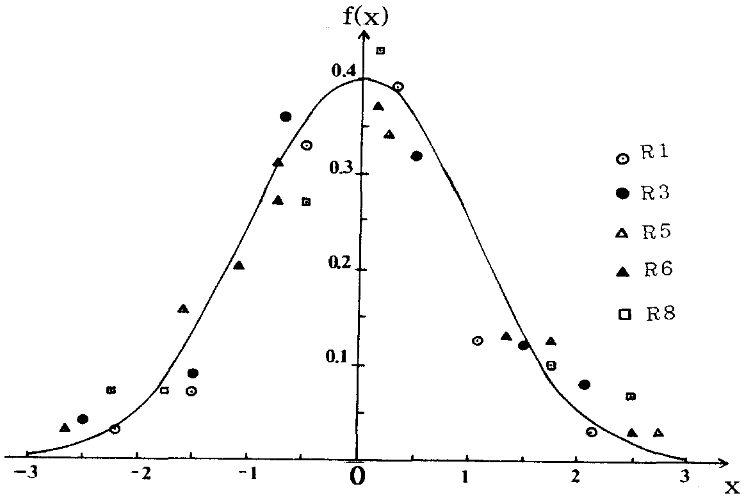


Fig. 5. Probability Density Distribution for Vertical Velocity of Turbulent Vortices

4. EXPERIMENTAL RESULTS

Experimental results of the probability density distribution for vertical velocity of turbulent vortices are shown in Fig. 5 with the theoretical values of eq. (2). Here R is the run number of the experiments.

A fairly good agreement is seen between these two values. To check the goodness of fit, χ^2-test was done. Calculated deviation is χ^2=6.296 if the significant level is $\alpha = 5\%$ and the interval number is k=6. For given values of k-1 degree of freedom and the significant level α, the value C=11.07 as a solution of $P(\chi^2 < C) = 95\%$ from the table of χ^2 distri-

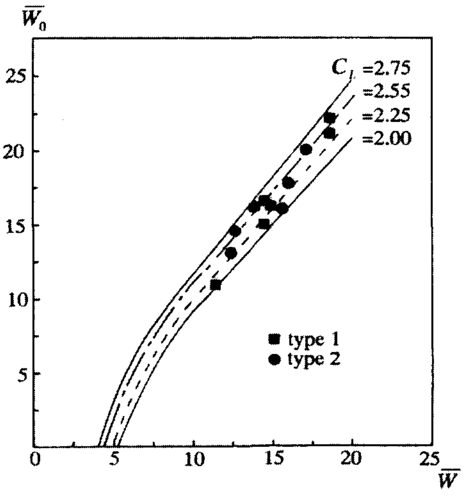


Fig. 6. Relationship Between $\overline{W}_0$ and $\overline{W}$

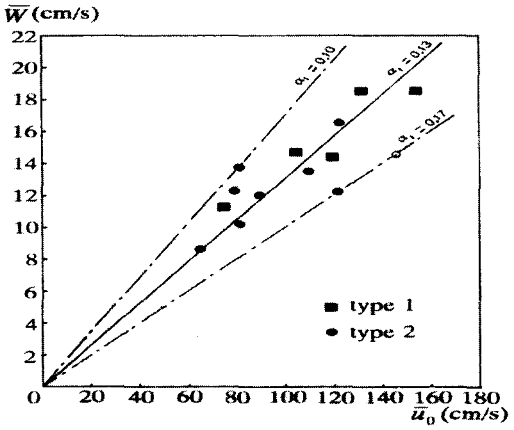


Fig. 7. Relationship Between $\overline{W}$ and $\overline{u}_0$

bution is obtained. Since $\chi_0^2 \prec C$, the hypothesis that a vertical turbulent velocity follows the normal distribution is not rejected.

Fig. 6 shows the relationship between the mean value of the particle entrainment velocity, $\overline{W}_0$ and that of the vertical component of fluid velocity, $\overline{W}$ for some different values of the lift coefficient, C_L.

Here, the particle velocity was measured using the tape recording taken by a video camera and a time-lapsed video tape recorder. Note that Fig. 6 takes a mean value since the vertical fluid velocity fluctuates following the normal distribution and the particle entrainment velocity also fluctuates as a result. The procedure for obtaining the vertical fluid velocity was explained in Fig. 4. In Fig. 6, experimental values are in good agreements with the theoretical ones for a lift coefficient is 2.25~2.55. It is similar to Ashida and Fujita (1986)'s results.

The relationship between the mean value of vertical fluid velocity and the average upstream overflow velocity in eq. (9) is depicted in Fig. 7. Reasonable linearity is revealed, where α_1 is found to be between 0.10 and 0.17 with the average value being 0.13. Therefore, the entrainment velocity of sand particles can be derived from eqs. (5) and (9) as

$$W_0^2 = a\alpha_1^2\overline{u}_0^2 - b \qquad (10)$$

The entrainment velocity of sand particles from the scour hole can be determined with eq. (10) if the mean value of the upstream overflow velocity is given.

5. CONCLUSION

Mathematical form of the entrainment velocity of sediment particles from the scour hole was obtained by applying the impulse-momentum equation with given value of the vertical component of turbulent vortices. Thus, if the mean value of the upstream overflow velocity is given, the entrainment velocity of sand particles from the scour hole was estimated. The probability density distribution of the entrainment velocity was obtained with the results that the vertical turbulent velocity follows the normal distribution. A video camera and a time-lapsed video tape recorder were used to measure the incipient entrainment velocity. Experimental results of the entrainment velocity of sand particle showed relatively good agreements with the lift coefficient 2.25~2.55.

5. ACKNOWLEDGEMENT

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