◆ Research Paper

# Development of Parts Sequencing Rule in a Two-machine Robotic Cell

**Gee Ju Moon***
**Sang Seop Yun***

## ABSTRACT

This paper suggests a new heuristic to improve robotic cells efficiency with a finite buffer. An efficient heuristic for parts sequencing with this configuration is developed. Analyzing robot's movements and defining the events for the completion times of robot's segmental activities enable us to develop a mathematical model that can be used to estimate the completion time in the robotic cell. The robotic cell is consisting of two computer numerical control machines and a finite buffer to manufacture multiple parts. The developed heuristic can be used to determine an optimum or near optimum parts sequences for this configuration. Numerical examples are followed to show the validity of the heuristic.

## 1. Introduction

Many companies are urged to enhance their productivities for the competitive world markets. It is common to see robots that can load a part on a computer numerical control(CNC) machine and unload a finished part from it in a manufacturing environment. In this paper a configuration of a robotic-cell consisting of two CNCs and one robot is given and a heuristic is developed to find efficient parts sequences. The problem of minimizing total completion time by considering parts sequences and robot's moving sequences simultaneously requires much efforts and time. The problem of processing multiple parts is NP-Complete problem in a robotic cell served by one robot.

In this paper we developed a model of robotic cell with an intermediate buffer that can be assumed to decrease robot's waiting time on a CNC and suggested a new efficient heuristic, which can be used to determine parts sequence in this configuration.

## 2. A robotic cell modeling

### 2.1 Robotic cell configuration

If a robotic cell has several CNCs, there would be many possible moving sequences for the robot in the cell. When the sequence is predetermined, the problem of parts sequencing can be classified into two cases. One is to determine robot's moving sequence which

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* Dept. of Industrial & Systems Engineering, Dong-A University

results minimum completion time in the case of all parts are identical, and the other is to determine robot's activities and parts sequences when non-identical parts are to be processed.

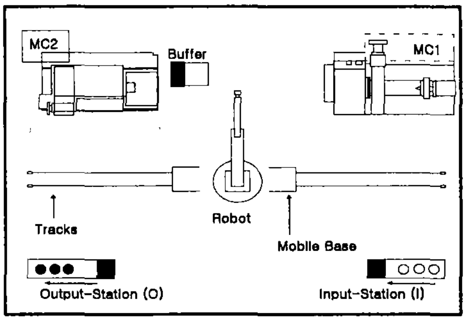

Fig. 1. Robotic cell with two machines

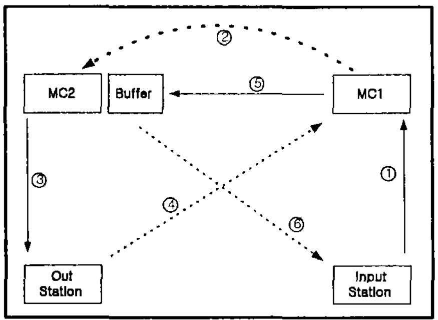

Fig. 2. An example moving sequence

The robot's segmental activities are divided into two cases, loaded move and unloaded move. The configuration of a mobile robotic cell with two machines and one robot is shown in Fig. 1. In this configuration robot is mounted on a mobile base and can move vertically along the tracks.

In this paper we develop parts sequence determination heuristic under the assumption of robot's moving sequence as in Fig. 2. This robotic cell has two CNCs and the $MC_2$ has a finite storage capacity buffer for one.

The buffer has an own feeder which can load a new part on $MC_2$ when a previous part is finished and unloaded from $MC_2$.

## 2.2 Notations and segmentation of robot movement

One cycle of robot movements consist of 18 events.

Each Event descriptions in detail along with the time taken to perform each are shown in parenthesis.

$E_1$ : robot picks up part $i$ from input station I ( $\varepsilon_1$)

$E_2$ : robot moves the arm to $MC_1$ ( $\delta_1$)

$E_3$ : robot loads part $i$ on $MC_1$ ( $\varepsilon_2$)

$E_4$ : robot slides to the left ( $\delta_0$)

$E_5$ : robot moves the arm to $MC_2$ ( $\delta_3$)

$E_6$ : robot waits until part ( $i-2$) is processed on $MC_2$ ( $w_{2,i-2}$) and sets itself ready to pick up part ( $i-2$)

$E_7$ : robot unloads part (i -2) from $MC_2$ ( $\varepsilon_4$)

$E_8$ : robot moves to the output station(O) ( $\delta_4$)

$E_9$ : robot drops off part (i -2) at output station (O) ( $\varepsilon_7$)

$E_{10}$ : robot slides to the right ( $\delta_0$)

$E_{11}$ : robot moves to the $MC_1$ ( $\delta_3+\delta_4-\gamma$)
$E_{12}$ : robot waits until part $i$ is processed on $MC_1$ ( $w_{1,i}$)
$E_{13}$ : robot unloads part $i$ from $MC_1$ ( $\varepsilon_3$)
$E_{14}$ : robot slides to the left ( $\delta_0$)
$E_{15}$ : robot moves to the buffer ( $\delta_2$)
$E_{16}$ : robot loads part $i$ on the buffer ( $\varepsilon_5$)
$E_{17}$ : robot slides to the right ( $\delta_0$)
$E_{18}$ : robot moves to the input station (I), ( $\delta_1+\delta_2-\gamma$)

where, $\varepsilon_4$ is the necessary time by robot to road a part onto Buffer, and $\varepsilon_5$ is the necessary time by feeder in the buffer to unload a part from machine $MC_2$. $\gamma$ is the time saved by not stopping at an intermediate point during the movement between two nonconsecutive machines. $w_{m,i}$ is the waiting time of robot to unload part $i$ from machine $m$. $i$ is the $i$th part in the scheduled part sequence.

## 3. Mathematical model for evaluating the cycle time

We let $s(E_q)$ as the starting time of event $E_q$ and $f(E_q)$ as the finishing time of event $E_q$. The robot's moving cycle time can be measured from the start time of event $E_1(s(E_1))$ to the finishing time of the event $E_{18}(f(E_{18}))$ in this system and $T_i$ as the completion time which robot consumes to pick up part $i$ at input station(I) and prepares to pick up part ( $i+1$), therefore we can express as equation (1)

$$T_i=f(E_{18})-s(E_1) \tag{1}$$

where, $s(E_q)=f(E_{q-1})$ and $E_0$, when q=0, means the event $E_{18}$ of the previous robot moving cycle.

### 3.1 Evaluation of $w_{1,i}$ and $w_{2,i-2}$

$MC_1$ processes part $i$ at time $f(E_3)$ and robot prepares to remove part $i$ from machine $MC_1$ at time $f(E_{11})$. In the period of $D_1=[f(E_{11})-f(E_3)]$, part $i$ can be processed on machine $MC_1$. The waiting time of robot to remove part calculated by using equation (2) from The waiting time of robot to remove part $i$ from $MC_1$ can be calculated by using equation (2)

$$w_{1,i}=\max\{0,\ a_i\ -\ D_1\} \tag{2}$$

Let $f^{-1}(E_q)$ as the finish time of event $E_q$ in the previous robot moving cycle. In the period of $D_1=[f(E_5)-\{f^{-1}(E_8)+\varepsilon_5\}]$, part ( $i-2$) can be processed on the machine $MC_2$.

The waiting time $w_{2,i-2}$ could be calculated by using equation (3)

$$w_{2,i-2}=\max\{0,\ b_{i-2}\ -\ D_2\} \tag{3}$$

In the above equations $a_i$ is the necessary part time to process part $i$ on machine $MC_1$ and $b_i$ is the necessary time to process part $i$ on machine $MC_2$, where $i$ is part number.

### 3.2 Evaluation of cycle Time in the robotic cell with multiple parts

Let $T_{i-2,i}$ as the necessary time to perform following events : The robot is about to unload part ( $i-2$) from machine $MC_2$ and ready to remove part $i$ from machine $MC_2$.

$T_{i-2,i} = t_1 + t_2 + t3 + w_{1,i} + w_{2,i-2}$ or $T_{i-2,i} = t_1 + t_2 + t_3 + \max\{0,\ a_i - D_1\} + \max\{0,\ b_{i-2} - D_2\}$ where, $t_1 = \varepsilon_1 + \varepsilon_2 + \delta_0 + \delta_1 + \delta_3$ , $t_2 = \varepsilon_6 + \varepsilon_7 + \delta_0 + \delta_3 + 2\delta_4 - \gamma$ , $t_2 = \varepsilon_3 + \varepsilon_4 + 2\delta_0 + \delta_1 + 2\delta_2 - \gamma$ .

When this robotic cell processes multiple part t22types, the total processing time $T$ could be obtained by equation (5)

$$T = n(t_1 + t_2 + t_3) + \sum_{i=0}^{n}\{\max\{0,\ a_i - D_1\} + \max\{0,\ b_{i-2} - D_2\}\} \tag{5}$$

Since $n(t_1 + t_2 + t_3)$ is constant in the equation (5), the objective function turns out to be $\min\{\max\{0,\ a_i - D_1\} + \max\{0,\ b_{i-2} - D_2\}\}$.

## 4. Development of Parts sequencing heuristic

The main idea of this heuristic is to assign a part having longer processing time on $MC_1$ and shorter processing time on $MC_2$ repeatedly. Letting $R_1$ be the minimum necessary time for the robot to complete the following activities without any waiting time : loads part $i$ on machine $MC_1$ and returns to the same machine to remove part $i$. Similarly, $R_2$ is the minimum traveling time related to $MC_2$. The proposed heuristic can be described in detail as :

Step 1 : calculate
$R_1 = 2\delta_0 + 2\delta_3 + 2\delta_4 + \varepsilon_6 + \varepsilon_7 - \gamma$.
Step 2 : calculate $A_i = a_i - R_1$
Step 3 : calculate $R_2 = 4\delta_0 + 2\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 - 2\gamma$
Step 4 : calculate $B_i = b_i - R_2 + \varepsilon_5$
Step 5 : add $A_i$ and $B_i$ to the table and regenerate the table.
Step 6 : find the large value in the $A_i$ and $B_i$ column. In case of tie, select arbitrarily.
Step 7 : If the selected value is a component of column $A_i$ , assign it to the first position in the unscheduled part sequence. And if the value is a component of column $B_i$ place it to the last position.
Step 8 : remove all information related to the selected value.
Step 9 : select the second largest value and repeat step 7.
Step 10 : repeat above steps until all parts are assigned.

## 5. Analysis of the heuristic

### 5.1 Comparison study with L&S

Efficiency of the proposed heuristic is shown by comparing with the model introduced in Logendran and Sriskandrajah's study[6].

They used Gilmore and Gomory's algorithm to determine the parts sequence in robotic cell without a buffer. The data used in their study are :

$\delta_0=0.55$, $\delta_1=0.40$, $\delta_2=0.45$, $\delta_3=0.50$, $\gamma=0.20$, $\varepsilon_1=0.10$, $\varepsilon_2=0.30$, $\varepsilon_3=1.10$, $\varepsilon_4=0.25$, $\varepsilon_5=0.15$, $\varepsilon_6=0.05$(min).

**Table 1. Processing Time used in Logendran's model (min)**

| $P_i$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ |
|---|---|---|---|---|---|---|---|---|
| $a_i$ | 2.0 | 2.0 | 2.5 | 3.0 | 3.5 | 3.5 | 4.0 | 4.0 |
| $b_i$ | 3.5 | 3.5 | 1.0 | 2.0 | 4.0 | 4.0 | 3.5 | 3.5 |

The optimal part sequence which produce minimum cycle time by using Gilmore and Gomory's algorithm in Logendrans study is [1,4,8,6,7,5,2,3] and the cycle time ( $T$) is 55.6 min. The processing times for each part on $MC_1$ and $MC_2$ are shown in Table 1.

To make the total traveling time in the robotic cell with the buffer be equal to the robotic cell without buffer, above data are transformed as :

$\delta_0=0.55$, $\delta_1=0.40$, $\delta_2=0.40$, $\delta_3=0.45$, $\delta_4=0.50$, $\gamma=0.20$ $\varepsilon_1=0.10$, $\varepsilon_2=0.30$, $\varepsilon_3=0.10$, $\varepsilon_4=0.10$, $\varepsilon_5=0.20$, $\varepsilon_6=0.15$, $\varepsilon_7=0.05$

First of all we determine the part sequence using the suggested heuristic in this paper and estimate the cycle time by using a simulation model constructed GPSS/PC EMS.

The newly generated part sequence is [7,8,5,6,4,3,1,2] and cycle time is 51.8 min. Therefore we can improve robotic cell's efficiency by 6.834% using a buffer. The above improvement of system's performance can not be used to prove for the efficiency of the suggested heuristic. The buffer makes any other part sequence to generate the same cycle time as 51.8min with this Logendran and Srikskandrajah's example. To examine the efficiency of the proposed heuristic, we need another example of different data and part family. This will be shown in next section.

### 5.2 Evaluation of the heuristic performance

To test the heuristic performance, we generated a new set of data as Table 2. And let the data associated with robot's activities be the same as the previous numerical example.

**Table 2. Newly generated processing times and transformed times (min)**

| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ |
|---|---|---|---|---|---|---|---|---|
| $a_i$ | 1.78 | 9.46 | 4.09 | 3.86 | 2.95 | 4.93 | 6.37 | 4.76 |
| $b_i$ | 5.89 | 6.52 | 9.99 | 9.47 | 2.95 | 2.36 | 7.66 | 2.18 |
| $A'$ | -1.22 | 6.46 | 1.09 | 0.86 | -0.05 | 1.93 | 3.37 | 1.76 |
| $B'$ | 1.24 | 1.87 | 5.34 | 4.82 | -1.70 | -2.29 | 3.01 | -2.47 |

When we use the newly generated data and processing times, we can determine the parts sequence as [2,7,6,8,5,1,4,3] and the cycle time is 64.27min.

The method could be used to determine parts sequence in this type of problem having a buffer in robotic cell isn't found in any other conducted research or study. But we can't ensure that the part sequence obtained from the heuristic always generates the optimum or near optimum solution, so we use statistical method to test the proposed heuristic performance.

First, determine the part sequence by using the proposed heuristic and generate other 49 part sequences randomly. Then analyze the distribution of cycle times for 50 part sequences.

Fig. 3 shows the histogram of cycle time data. To test normality we use the normal probability paper as Fig. 4.

As dots form linear shape, we can assert that the population of cycle time data form normal distribution as $T \sim N(68.963, 2.399^2)$. The required number of cycle time data sufficient to represent the population is shown in equation (6).

$$n = \left(\frac{Z \cdot \sigma}{\mu \cdot \alpha}\right)^2 \tag{6}$$

When the confidence level $\alpha = 0.05$, the required number of cycle time data is $n = 1.94$. Even more $\alpha = 0.025$, n = 7.44. So the fifty generated part sequences are sufficient to represent the population of cycle times for 8! possible part sequences. Therefore the proposed heuristic can be used to determine the part sequence in robotic cell having an intermediate buffer efficiently.

Whether this heuristic can be used in more generalized problem or not, we generated new problem having 10 different parts. The required data are presented in Table 3 and the results are shown in table 4.

As seen in Table 4, the part sequence generated by the proposed heuristic is [2,7,6,8,5,10,1,9,4,3] and the cycle time is 79.44 (min).

The cycle time of 79.44 min is less than that of forty nine randomly generated part sequences, so we can conclude that this heuristic can be used to determine the part sequence in robotic cells having two machines and a buffer for processing multiple parts.

**Table 3. Processing times (10 parts) (min)**

| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_i$ | 1.78 | 9.46 | 4.09 | 3.86 | 2.95 | 4.93 | 6.37 | 4.76 | 1.45 | 3.28 |
| $b_i$ | 5.89 | 6.52 | 9.99 | 9.47 | 2.95 | 2.36 | 7.66 | 2.18 | 8.86 | 5.49 |

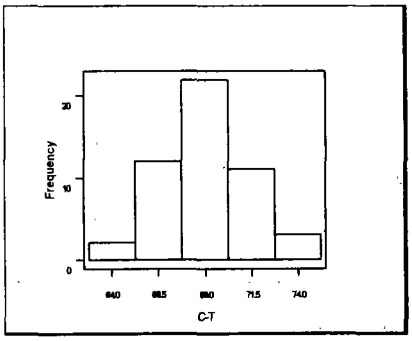

Fig. 3. Histogram for 50 cycle times

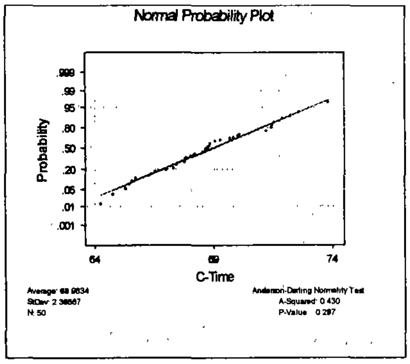

Fig. 4. Normality Test

## 6. Conclusion

This paper suggested an efficient heuristic for part sequencing. The robotic cell was composed of two computer numerical control machines and a buffer to manufacture multiple parts. Many researches done in this field can be classified into two cases. In the case of two machine robotic cell without buffer a sequence of minimum makespan is the main purpose. However, In case of three machine robotic cell without buffer only the robot's moving sequence is the main target under the assumption of manufacturing identical parts. This type of problem is intractable even if a buffer is added. A mathematical model which can be used to estimate the completion time in the robotic cell is formulated by analyzing robot's movements and defining the events which mean the completion times of robot's segmental activities. To show the validity of proposed heuristic two case studies are examined. The proposed heuristic generated minimal completion time compared with other possible sequences

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**Table 4. Completion Times (10 parts)** **(min)**

| | Part Sequence | Delay Time | | | | Completion Time | Desc. |
|---|---|---|---|---|---|---|---|
| | | $MC_1$ | | $MC_2$ | | | |
| | | Mean | STD | Mean | STD | | |
| 1 | 2 7 6 8 5 10 1 9 4 3 | 0.55 | 0.74 | 1.30 | 1.58 | 79.44 | Proposed |
| 2 | 3 9 1 10 5 7 6 8 4 2 | 1.47 | 2.03 | 1.02 | 1.73 | 85.84 | |
| 3 | 1 9 4 8 6 7 10 5 2 3 | 1.27 | 2.08 | 0.60 | 1.21 | 79.66 | |
| 4 | 6 7 10 1 8 2 4 9 3 5 | 1.24 | 2.12 | 1.57 | 1.98 | 89.09 | |
| 5 | 7 8 2 10 1 9 3 4 5 6 | 1.12 | 1.96 | 1.50 | 1.87 | 87.61 | |
| 6 | 9 1 7 6 10 4 3 5 2 8 | 0.66 | 0.74 | 1.35 | 1.83 | 81.61 | |
| 7 | 6 9 3 4 7 5 10 1 2 8 | 1.10 | 1.95 | 1.44 | 1.92 | 86.33 | |
| 8 | 1 10 5 3 9 8 7 6 2 4 | 0.95 | 1.86 | 1.47 | 1.95 | 85.21 | |
| 9 | 7 9 6 2 5 1 10 3 4 8 | 0.48 | 0.91 | 1.63 | 1.79 | 82.08 | |
| 10 | 10 1 6 3 8 5 7 4 9 2 | 1.09 | 1.15 | 1.20 | 1.48 | 83.89 | |
| 11 | 6 3 10 4 5 9 1 2 7 8 | 1.05 | 1.25 | 1.34 | 1.96 | 84.96 | |
| 12 | 8 9 6 4 1 10 3 7 2 5 | 1.12 | 1.77 | 0.94 | 1.36 | 81.61 | |
| 13 | 4 5 10 3 7 9 1 2 6 8 | 1.18 | 1.25 | 1.18 | 1.69 | 84.62 | |
| 14 | 3 1 8 7 5 9 6 4 10 2 | 0.85 | 1.11 | 1.39 | 1.80 | 93.34 | |
| 15 | 3 1 6 4 2 10 7 9 5 8 | 1.03 | 2.05 | 1.26 | 1.76 | 83.97 | |
| | | ⋮ | | | | | |
| 35 | 1 5 9 6 3 2 7 8 10 4 | 1.44 | 2.11 | 0.74 | 1.36 | 82.77 | |
| 36 | 4 3 1 5 9 8 2 10 6 7 | 1.17 | 1.60 | 1.43 | 1.78 | 87.01 | |
| 37 | 3 4 1 10 6 9 5 2 8 7 | 1.03 | 1.27 | 1.48 | 1.97 | 86.04 | |
| 38 | 1 2 9 5 8 6 10 4 7 3 | 0.83 | 1.15 | 1.29 | 1.74 | 82.24 | |
| 39 | 2 9 7 6 3 1 5 4 10 8 | 0.90 | 1.96 | 1.48 | 1.71 | 84.82 | |
| 40 | 5 2 4 9 1 7 3 6 10 8 | 0.95 | 2.02 | 1.59 | 1.95 | 86.41 | |
| 41 | 10 5 4 3 7 1 2 9 8 6 | 0.66 | 1.3 | 1.51 | 1.73 | 82.64 | |
| 42 | 9 4 1 10 7 6 5 3 2 8 | 1.48 | 2.00 | 0.83 | 1.49 | 84.07 | |
| 43 | 8 3 10 4 1 9 2 7 5 6 | 1.23 | 1.94 | 1.22 | 1.83 | 85.49 | |
| 44 | 1 3 7 2 10 4 8 9 6 5 | 1.32 | 1.72 | 0.73 | 0.98 | 81.46 | |
| 45 | 6 2 1 7 3 8 9 5 10 4 | 1.04 | 1.34 | 1.31 | 1.66 | 84.53 | |
| 46 | 4 3 5 8 7 10 2 9 6 1 | 1.05 | 1.53 | 1.45 | 1.76 | 86.01 | |
| 47 | 8 4 3 1 9 5 7 10 2 6 | 1.02 | 1.36 | 1.45 | 1.85 | 85.70 | |
| 48 | 7 6 5 10 1 8 9 2 4 3 | 0.97 | 1.99 | 1.16 | 1.80 | 82.34 | |
| 49 | 3 7 10 1 5 9 4 8 2 6 | 0.94 | 1.20 | 1.19 | 1.51 | 82.37 | |
| 50 | 9 2 4 3 7 10 5 8 1 6 | 1.14 | 2.00 | 1.16 | 1.80 | 83.99 | |