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Durbin-Watson Type Unit Root Tests for the Deterministic Trend Models†

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ABSTRACT

We have developed a "Durbin-Watson type" test statistics for regular and seasonal unit roots in the deterministic trend models. The limiting distributions of the proposed test statistics are the functionals of standard Brownian motions. Finite distributions of the test statistics for selected seasonal periods, if any, are numerically obtained using the Imhof routine. The powers and sizes of the test statistics are examined for finite samples and compared with those of the DF-type tests. Simulation results showed that the DW-type tests have good behaviors against the DF-type tests for all models considered.

Keywords: Generalized Durbin-Watson statistics; Regular unit root; Seasonal unit root; Deterministic trend; Standard Brownian motions; Imhof routine

1. INTRODUCTION

Consider a time series model of the form

$$Y_t = \mathbf{x}_t'\beta + u_t, \tag{1.1}$$

where $\{\mathbf{x}_t\}$ is a deterministic sequence. If $\{u_t\}$ satisfies

$$u_t = \phi u_{t-1} + \epsilon_t, \tag{1.2}$$

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where ϵ_t's are independently and identically distributed (i.i.d.) random variables with mean zero and variance σ_ϵ^2, the models (1.1) and (1.2) are jointly represented by

$$Y_t = \phi Y_{t-1} + (\mathbf{x}_t - \phi \mathbf{x}_{t-1})'\beta + \epsilon_t. \tag{1.3}$$

The traditional Dickey-Fuller (DF) regular unit root test procedures test H_0 : $\phi = 1$ in (1.3). For the seasonal time series one may consider $\{u_t\}$ which satisfies

$$u_t = \Phi u_{t-s} + \epsilon_t, \tag{1.4}$$

where s is the seasonal period. The models (1.1) and (1.4) can be jointly represented by

$$Y_t = \Phi Y_{t-s} + (\mathbf{x}_t - \Phi \mathbf{x}_{t-s})'\beta + \epsilon_t, \tag{1.5}$$

and DF seasonal unit root test procedures test $H_0 : \Phi = 1$ in (1.5).

Bhargava (1986), Nabeya and Tanaka (1990), and Tanaka (1996) developed regular unit root tests for the deterministic trend models using the generalized Durbin-Watson (DW) statistics

$$d_k = \sum_{t=k+1}^{n} \frac{(\hat{u}_t - \hat{u}_{t-k})^2}{\sum_{t=1}^{n} \hat{u}_t^2}, \quad k = 1, \ldots, n-1,$$

where $\hat{u}_t$ are the residuals of the regression model (1.1). Kim and Cho (1998) also considered the DW-type unit root tests for the regular and seasonal time series models.

In this paper we develop the DW-type regular and seasonal unit root tests for the deterministic trend models and compare the performances with those of the DF-type unit root tests proposed by Dickey and Fuller (1979) and Cho *et al.* (1995), respectively. The advantages of the DW-type unit root tests over DF-type tests are that the calculation of the exact finite distributions and powers are easy and the extension to the general models and wide class of tests are much flexible. These advantages are presented by an example.

2. Regular unit root tests

Let $\{Y_t\}$ satisfy the following deterministic trend models

$$Y_t = \mu + \alpha t + u_t, \quad u_t = \phi u_{t-1} + \epsilon_t, \tag{2.1}$$

or

$$Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \gamma t + u_t, \quad u_t = \phi u_{t-1} + \epsilon_t, \tag{2.2}$$

where $\epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$ and δ_{jt}'s are the indicator functions such that $\delta_{jt} = 1$ if t is in j-th season or 0 otherwise. Model (2.2) implies the possibility of a regular unit root for a seasonal time series. Note that models (2.1) and (2.2) can be represented by

$$Y_t = (1-\phi)\mu + \alpha\phi + (1-\phi)\alpha t + \phi Y_{t-1} + \epsilon_t, \tag{2.3}$$

and

$$Y_t = \sum_{j=1}^{s} (\beta_j - \phi\beta_{j-1})\delta_{jt} + \gamma\phi + (1-\phi)\gamma t + \phi Y_{t-1} + \epsilon_t. \tag{2.4}$$

Models (2.3) and (2.4) are used for the DF-type tests.

Assume that $n = ms$ for simplicity where m is a integer. For the test of

$$H_0 \; : \; \phi = 1, \qquad H_1 \; : \; |\phi| < 1,$$

we define DW-type test statistics for models (2.1) and (2.2) as

$$\begin{aligned} R_{31} &= \frac{\sum_{t=2}^{n}(Y_t - Y_{t-1} - \hat{\alpha})^2}{\sum_{t=1}^{n}(Y_t - \hat{\mu} - \hat{\alpha}t)^2}, \\ R_{32} &= \frac{\sum_{t=2}^{n}(Y_t - Y_{t-1} - \sum_{j=1}^{s}(\hat{\beta}_j - \hat{\beta}_{j-1})\delta_{jt} - \hat{\gamma})^2}{\sum_{t=1}^{n}(Y_t - \sum_{j=1}^{s}\hat{\beta}_j\delta_{jt} - \hat{\gamma}t)^2}, \end{aligned}$$

where $\hat{\alpha} = \frac{12}{n(n^2-1)}\sum_{t=1}^{n}(t - \frac{n+1}{2})Y_t$, $\hat{\mu} = \bar{Y} - \hat{\alpha}\frac{n+1}{2}$, $\hat{\gamma} = \frac{12}{s^3 m(m^2-1)}\sum_{t=1}^{n}(t - \sum_{j=1}^{s}\bar{t}_j\delta_{jt})Y_t$, $\hat{\beta}_j = \bar{Y}_j - \hat{\gamma}\bar{t}_j$, and $\hat{\beta}_0 = \hat{\beta}_s$, where $\bar{Y}_j = \frac{1}{m}\sum_{l=1}^{m} Y_{(l-1)s+j}$ and $\bar{t}_j = (m-1)s/2 + j$. Note that R_{31} and R_{32} are independent of μ, α, β_j's, and γ under both H_0 and H_1, and independent of Y_1 under H_0. In the followings, "$\Rightarrow$" means convergence in distribution.

Theorem 1. *Under $H_0 : \phi = 1$, nR_{31} and nR_{32} have the same limiting distribution*

$$nR_{31}, nR_{32} \Rightarrow \frac{1}{\int_0^1 W^2(r)dr - \{\int_0^1 W(r)dr\}^2 - 12\{\int_0^1 (r - \frac{1}{2})W(r)dr\}^2},$$

where $W(r)$ is a standard Brownian motion.

The limiting distribution of nR_{31} are given in Tanaka (p. 340, 1996) and the derivation of the limiting distribution of nR_{32} is given in the Appendix. Note that R_{31} and R_{32} are $O_p(n^{-1})$ under H_0 and $O_p(1)$ under H_1. By rewriting R_{31} and R_{32} in quadratic forms, the exact distributions and the powers of nR_{31} and nR_{32} can be obtained following Imhof (1961) with the additional assumption of normality for $\{\epsilon_t\}$.

In Table 1 and 2 we provide the exact distributions of nR_{31} and nR_{32} numerically calculated by the Imhof routine for various sample sizes under $H_0 : \phi = 1$ for models (2.1) and (2.2). Without loss of generality, we assume that μ, α, β_j's, γ, and Y_1 are all zeros.

We obtain the exact powers of the DW-type test statistic nR_{31} and the DF-type $\hat{\rho}_\tau$ test statistic for model (2.1) in Table 3. For power comparisons we consider the DF-type test statistics $\hat{\rho}_\tau = n(\hat{\phi}_\tau - 1)$ following Dickey and Fuller (1979), where $\hat{\phi}_\tau$ is the ordinary least squares estimate (OLSE) of ϕ in model (2.3). In this paper, we do not consider the pivotal statistics (t-statistics), since, for example, the pivotal statistic τ_τ is less powerful than $\hat{\rho}_\tau$ as shown in Dickey *et al.* (1986), and so on. Without loss of generality we assume that $\mu = 0$, $\alpha = 0$, and $Y_1 \sim N(0, 1/(1 - \phi^2))$. The significant level is 0.05 and the sample sizes considered are $n = 25, 50, 100, 200$, and 500. The critical values of the DW-type test statistic are obtained from Table 1. The distributions and powers of the DF-type test statistic $\hat{\rho}_\tau$ are also calculated using the Imhof routine. The obtained critical values are -17.956, -19.695, -20.666, -21.178, and -21.496, respectively.

Table 4 compares the powers of the DW-type test statistic nR_{32} and the DF-type test statistic $\hat{\rho}_{\tau s} = n(\hat{\phi}_{\tau s} - 1)$ for model (2.2), where $\hat{\phi}_{\tau s}$ is the OLSE of ϕ in model (2.4). Without loss of generality, we assume that $\beta_j = 0$ for all j, $\gamma = 0$, and $Y_1 \sim N(0, 1/(1 - \phi^2))$. The critical values of the DW-type test statistic are obtained from Table 2, and the powers of this statistic are obtained using the Imhof routine. The distributions and powers of the DF-type test statistic $\hat{\rho}_{\tau s}$ are obtained using the simulation method with 50,000 and 10,000 iterations, respectively. The critical values of $\hat{\rho}_{\tau s}$ are -18.233, -19.223, -19.806, -20.194, -20.985, and -21.494 for $s = 4$ and -19.287, -19.678, -20.154, -20.565, -20.960, and -21.219 for $s = 12$ corresponding to the sample sizes in Table 4, respectively.

We have observed from table 3 and 4 that the DW-type tests performs better than the DF-type tests for models (2.1) and (2.2). It should be remarked that though the OLSE are used for the comparison in this paper we may be able to use other types of the DF-type test statistics based on the generalized least squares (GLS) or maximum likelihood (ML) estimates as indicated in Pantula *et al.* (1994).

3. Seasonal unit root tests

Let $\{Y_t\}$ follow seasonal deterministic trend models with period s,

$$Y_t = \sum_{j=1}^{s} (\mu_j + \gamma_j \tau)\delta_{jt} + u_t, \quad u_t = \Phi u_{t-s} + \epsilon_t, \tag{3.1}$$

or

$$Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \gamma t + u_t, \quad u_t = \Phi u_{t-s} + \epsilon_t, \tag{3.2}$$

where $\epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$, δ_{jt}'s are the seasonal indicator functions, and $\tau = [(t-1)/s+1]$ with [x] denoting the largest integer no larger than x. In model (3.1) the means and trends parameters, μ_j's and γ_j's, are separately determined according to the corresponding seasons. The trend parameter is common for all seasons in model (3.2) which may be more practical than model (3.1) in most cases. Note that models (3.1) and (3.2) can be represented by

$$Y_t = \sum_{j=1}^{s} \{(1-\Phi)\mu_j + \Phi\gamma_j + (1-\Phi)\gamma_j)\tau\}\delta_{jt} + \epsilon_t, \tag{3.3}$$

and

$$Y_t = \sum_{j=1}^{s} (1-\Phi)\beta_j\delta_{jt} + s\gamma\Phi + (1-\Phi)\gamma t + \Phi Y_{t-s} + \epsilon_t. \tag{3.4}$$

Models (3.3) and (3.4) are used for the DF-type tests.

Assume that $n = ms$ for simplicity where m is a integer. For the test of

$$H_0 \; : \; \Phi = 1, \qquad H_1 \; : \; |\Phi| < 1,$$

we define DW-type test statistics for (3.1) and (3.2) as, respectively,

$$S_{31} = \frac{\sum_{t=s+1}^{n} (Y_t - Y_{t-s} - \sum_{j=1}^{s} \hat{\gamma}_j \delta_{jt})^2}{\sum_{t=1}^{n} (Y_t - \sum_{j=1}^{s} (\hat{\mu}_j + \hat{\gamma}_j \tau)\delta_{jt})^2}, \quad S_{32} = \frac{\sum_{t=s+1}^{n} (Y_t - Y_{t-s} - s\hat{\gamma})^2}{\sum_{t=1}^{n} (Y_t - \sum_{j=1}^{s} \hat{\beta}_j \delta_{jt} - \hat{\gamma} t)^2},$$

where $\hat{\gamma}_j = \frac{12}{m(m^2-1)} \sum_{l=1}^{m} (l - \frac{m+1}{2}) Y_{(l-1)s+j}$, $\hat{\mu}_j = \bar{Y}_j - \hat{\gamma}_j \frac{m+1}{2}$, $\hat{\gamma} = \frac{12}{s^3 m(m^2-1)} \sum_{t=1}^{n} (t - \sum_{j=1}^{s} \bar{t}_j \delta_{jt}) Y_t$, and $\hat{\beta}_j = \bar{Y}_j - \hat{\gamma} \bar{t}_j$, where $\bar{Y}_j = m^{-1} \sum_{l=1}^{m} Y_{(l-1)s+j}$ and $\bar{t}_j = (m-1)s/2 + j$. Note that S_{31} and S_{32} are independent of μ_j's, γ_j's, β_j's, and γ, respectively, under both H_0 and H_1. And these statistics are independent of $Y_1, \ldots, Y_s$ under H_0.

Theorem 2. *Under $H_0 : \Phi = 1$, nS_{31} and nS_{32} have the limiting distributions, respectively,*

$$nS_{31} \Rightarrow \frac{s^2}{\sum_{j=1}^{s} \left[\int_0^1 W_j^2(r)dr - \{\int_0^1 W_j(r)dr\}^2 - 12\{\int_0^1 (r - \frac{1}{2}) W_j(r)dr\}^2 \right]},$$

$$nS_{32} \Rightarrow \frac{s^2}{\sum_{j=1}^{s} \left[\int_0^1 W_j^2(r)dr - \{\int_0^1 W_j(r)dr\}^2 \right] - 12s^{-1} \left[\sum_{j=1}^{s} \int_0^1 (r - \frac{1}{2}) W_j(r)dr \right]^2},$$

where $W_j(r)$'s are the mutually independent standard Brownian motions.

The first part of Theorem 2 is an immediate consequence of Theorem 1 and the second part is obtained from Cho *et al.* (1997). Note that S_{31} and S_{32} are $O_p(n^{-1})$ under H_0 and $O_p(1)$ under H_1, respectively. With the additional assumption of normality for $\{\epsilon_t\}$ we may rewrite S_{31} and S_{32} in quadratic forms, so we can obtain the exact distributions and powers of nS_{31} and nS_{32} using the Imhof routine.

Table 5 and 6 show the exact distributions of nS_{31} and nS_{32} numerically calculated by the Imhof routine for various sample sizes under $H_0 : \Phi = 1$ for models (3.1) and (3.2). Without loss of generality, we assume that μ_j's, γ_j's, β_j's, γ, and $Y_1, \ldots, Y_s$ are all zeros.

We obtain the exact powers of the DW-type test statistic, nS_{31}, and the DF-type test statistic $\hat{\rho}_{31}$ for model (3.1) in Table 7. For the power comparisons we consider the DF-type test statistics $\hat{\rho}_{31} = n(\hat{\Phi}_{31} - 1)$ due to Cho *et al.* (1995), where $\hat{\Phi}_{31}$ is the OLSE of Φ in model (3.3). Without loss of generality we assume that $\mu_j = 0$ and $\gamma_j = 0$ for all j and that for $1 \leq j \leq s$, $Y_j \sim N(0, 1/(1 - \Phi^2))$ independently. The significant level is 0.05 and the critical values of the DW-type test statistic are obtained from Table 5. The distributions and powers of the DF-type test statistic $\hat{\rho}_{31}$ are also obtained using the Imhof routine with the critical values −41.500, −44.601, −46.329, −47.424, −49.750, and −50.984 for $s = 4$ and −93.460, −105.508, −111.130, −114.179, −116.088, and −120.081 for $s = 12$ corresponding to the sample sizes in Table 7, respectively.

Table 8 compares the exact powers of the DW-type test statistic nS_{32} and the DF-type test statistic $\hat{\rho}_{32} = n(\hat{\Phi}_{32} - 1)$ for model (3.2), where $\hat{\Phi}_{32}$ is the OLSE of Φ in model (3.4). Without loss of generality, we assume that $\beta_j = 0$ for all j, $\gamma = 0$, and for $1 \le j \le s$, $Y_j \sim N(0, 1/(1 - \Phi^2))$. The critical values of the DW-type test statistic are obtained from Table 2. The distributions and powers of the DF-type test statistic $\hat{\rho}_{32}$ are obtained using the Imhof routine with the critical values -28.470, -29.646, -30.272, -30.661, -31.463, and -31.876 for $s = 4$ and -56.326, -59.002, -60.038, -60.577, -60.903, and -61.571 for $s = 12$ corresponding to the sample sizes in Table 8, respectively.

As in the regular unit root cases it is observed that the DW-type tests performs better than the DF-type tests for the seasonal models (3.1) and (3.2).

4. Example and discussions

An example is presented in order to clarify some of the concepts involved. The example consists of simulated data generated by model (2.2).

Example We simulate a quarterly seasonal data of sample size 100 from (2.2), where $s = 4, \beta_1 = 2, \beta_2 = 4, \beta_3 = 6, \beta_4 = 4, \phi = 1, \gamma = 0.5$, and $u_0 = 0$. Figure 4.1 is the time plot of the simulated data.

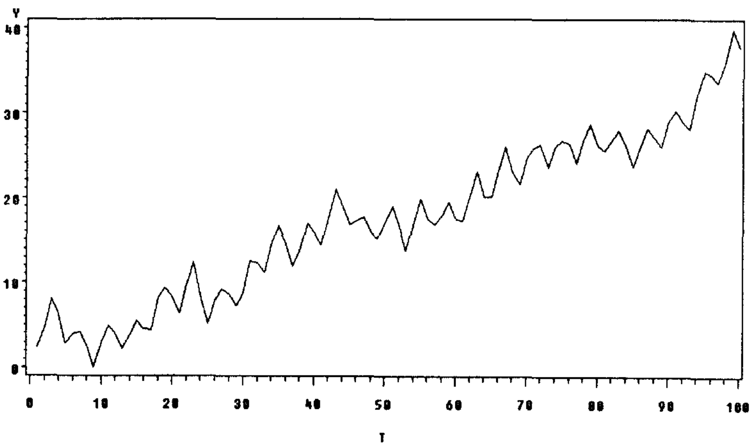

Figure 4.1: Time plot of Y_t generated from (2.2).

Figure 4.1 shows the increasing time trend and the seasonal patterns. For the unit root tests, we may consider the regular unit root test based on (2.2) and the seasonal unit root test based on (3.2), if we do not know the true model. Since

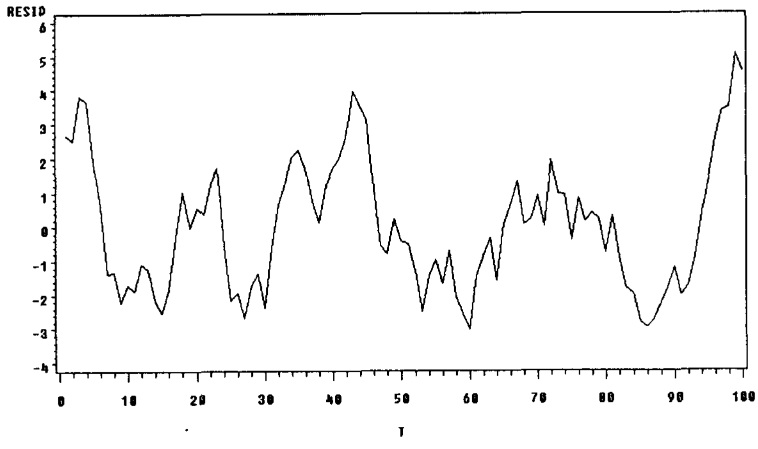

Figure 4.2: Time plot of the residuals of the regression model $Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \gamma t + u_t$.

this series shows the seasonality and the common time trend, we do not consider (2.1) nor (3.1). We calculate nR_{32} and nS_{32}, and the p-values using the Imhof routine. The obtained value of nR_{32} is 25.53 with the p-value 0.229, and nS_{32} is 117.78 with the p-value 0.000. Thus, in this case, the regular unit root test based on (2.2) is more suitable than the seasonal unit root test based on (3.2), which is supported by Figure 4.2. Figure 4.2 is the time plot of the residuals of the regression model $Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \gamma t + u_t$, which shows the non-seasonality and the strong first-order autocorrelation.

Figure 4.1 may be the typical time plot of the seasonal time series, especially for a economic time series data. However, in many cases, we do not know the true model. In the analysis of a seasonal time series similar to Figure 4.1, it is recommended to consider not only model (3.2) but also model (2.2) for the unit root test.

For this example, the DF-type test statistic $\hat{\rho}_{32}$ is -65.05 with p-value 0.000, while $\hat{\rho}_{Ts}$ is -11.20. We do not provide the p-value of $\hat{\rho}_{Ts}$, since it requires a messy calculation by hand to obtain the distribution of the p-value of DF-type test using the Imhof routine. This is because the quadratic form of the test statistic is complicated and the design matrix includes the lagged variable of Y_t. Therefore, the distribution has to be calculated by hand for each model. In General, the more the considered model is complicated the more the calculation of the p-value of the DF-type test becomes difficult. On the other hand, the

DW-type test statistics can be easily expressed in the quadratic form. Therefore, the distribution tables of the DW-type tests are not necessary and we can obtain (automatically) the p-values as well as the distributions and the powers. This is one of the advantages of the DW-type tests over DF-type tests.

APPENDIX : Proof of Theorem 1

For simplicity, we denote $\int_0^1 W(r)dr$ as $\int W$ and assume $n = ms$ where s is a seasonal period. And we assume that β_j's and γ are all zeros, and that $\phi = 1$. First, note that

$$\begin{aligned} n^{-5/2}\sum_{t=1}^{n}(t-\sum_{j=1}^{s}\bar{t}_j\delta_{jt})Y_t &= n^{-5/2}\sum_{t=1}^{n}(t-\frac{n+1}{2}+O(1))Y_t \\ &= n^{-5/2}\sum_{t=1}^{n}(t-\frac{n+1}{2})Y_t + O_p(n^{-1}) \\ &\Rightarrow \sigma_\epsilon\int(r-\frac{1}{2})W. \end{aligned}$$

Thus,

$$\begin{aligned} n^{1/2}\hat{\gamma} &= n^{1/2+5/2}\times\frac{12}{s^3m(m^2-1)}n^{-5/2}\sum_{t=1}^{n}(t-\sum_{j=1}^{s}\bar{t}_j\delta_{jt})Y_t \\ &\Rightarrow \sigma_\epsilon[12\int(r-\frac{1}{2})W]. \end{aligned}$$

The numerator part of R_{32} is

$$\sum_{t=2}^{n}(Y_t-Y_{t-1}-\sum_{j=1}^{s}(\hat{\beta}_j-\hat{\beta}_{j-1})\delta_{jt}-\hat{\gamma})^2$$

$$\begin{aligned} &= \sum_{t=2}^{n}(\epsilon_t-\sum_{j=1}^{s}(\bar{Y}_j-\bar{Y}_{j-1})\delta_{jt}-\hat{\gamma}\sum_{j=1}^{s}(1-\bar{t}_j+\bar{t}_{j-1})\delta_{jt})^2 \\ &= \sum_{t=2}^{n}(\epsilon_t+O_p(n^{-1/2})+O_p(n^{-1/2}))^2, \end{aligned}$$

since, for $j > 1$

$$\begin{aligned} \bar{Y}_j-\bar{Y}_{j-1} &= \frac{1}{m}(\epsilon_j+\epsilon_{s+j}+\cdots+\epsilon_{(m-1)s+j}) \\ &= O_p(n^{-1/2}), \end{aligned}$$

and for $j = 1$

$$\begin{aligned} \bar{Y}_j - \bar{Y}_{j-1} &= \frac{1}{m}(Y_1 + \epsilon_{s+1} + \cdots + \epsilon_{(m-1)s+1} - Y_n) \\ &= O_p(n^{-1/2}). \end{aligned}$$

Therefore,

$$n^{-1}\sum_{t=2}^{n}(Y_t - Y_{t-1} - \sum_{j=1}^{s}(\hat{\beta}_j - \hat{\beta}_{j-1})\delta_{jt} - \hat{\gamma})^2 \to \sigma_\epsilon^2 \ \textit{in probability.} \quad (A1)$$

The denominator part of R_{32} is

$$\begin{aligned} \sum_{t=1}^{n}(Y_t - \sum_{j=1}^{s}\hat{\beta}_j\delta_{jt} - \hat{\gamma}t)^2 &= \sum_{t=1}^{n}(Y_t - \sum_{j=1}^{s}(\bar{Y}_j - \hat{\gamma}\bar{t}_j)\delta_{jt} - \hat{\gamma}t)^2 \\ &= \sum_{t=1}^{n}(Y_t - \sum_{j=1}^{s}\bar{Y}_j\delta_{jt})^2 - \hat{\gamma}^2\frac{s^3m(m^2-1)}{12} \\ &= \sum_{t=1}^{n}Y_t^2 - m\sum_{j=1}^{s}\bar{Y}_j^2 - \hat{\gamma}^2\frac{s^3m(m^2-1)}{12}. \end{aligned}$$

Note that $\bar{Y}_j - \bar{Y} = \frac{1}{n}\sum_{t=1}^{n} a_t\epsilon_t$, where $-s < a_t < s$ such that $\bar{Y}_j - \bar{Y} = O_p(n^{-1/2})$. Since $n^{-1/2}\bar{Y} \Rightarrow \sigma_\epsilon \int W$, we can obtain that $n^{-1/2}\bar{Y}_j \Rightarrow \sigma_\epsilon \int W$ for all $1 \leq j \leq s$. Therefore,

$$n^{-2}\sum_{t=1}^{n}(Y_t - \sum_{j=1}^{s}\hat{\beta}_j\delta_{jt} - \hat{\gamma}t)^2 \Rightarrow \int W^2 - \{\int W\}^2 - 12\{\int (r - \frac{1}{2})W\}^2 . \quad (A2)$$

From (A1) and (A2), we obtain the limiting distribution of nR_{32} as

$$nR_{32} \Rightarrow \frac{1}{\int W^2 - \{\int W\}^2 - 12\{\int (r - \frac{1}{2})W\}^2} .$$

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Table 1. Distributions of nR_{31} for various sample sizes [a]

	Probability of a smaller value									
n	0.01	0.025	0.05	0.10	0.20	0.80	0.90	0.95	0.975	0.99
25	4.856	5.776	6.772	8.197	10.390	24.393	29.452	33.957	38.043	42.937
50	4.721	5.704	6.765	8.289	10.651	26.389	32.412	37.952	43.143	49.582
100	4.656	5.669	6.763	8.337	10.788	27.485	34.068	40.227	46.089	53.506
200	4.624	5.652	6.763	8.363	10.858	28.061	34.946	41.444	47.682	55.639
500	4.605	5.641	6.762	8.378	10.901	28.415	35.490	42.203	48.680	56.992

[a] The entries are obtained using the Imhof routine.

Table 2. Distributions of nR_{32} for various sample sizes [a]

	Probability of a smaller value									
n	0.01	0.025	0.05	0.10	0.20	0.80	0.90	0.95	0.975	0.99
	seasonal period $s = 4$									
40	4.500	5.423	6.420	7.848	10.045	24.539	30.072	35.162	39.936	45.866
60	4.535	5.497	6.535	8.024	10.326	25.753	31.743	37.307	42.562	49.148
80	4.550	5.533	6.592	8.113	10.471	26.416	32.669	38.505	44.054	51.051
100	4.559	5.555	6.627	8.167	10.560	26.832	33.256	39.271	45.012	52.271
200	4.578	5.595	6.695	8.277	10.741	27.711	34.504	40.915	47.074	54.931
400	4.585	5.616	6.728	8.332	10.835	28.175	35.167	41.797	48.184	56.371
	seasonal period $s = 12$									
72	4.176	5.093	6.086	7.513	9.723	24.985	31.209	37.137	42.862	50.190
120	4.350	5.314	6.354	7.846	10.159	26.041	32.432	38.491	44.325	51.797
180	4.434	5.423	6.489	8.020	10.400	26.769	33.350	39.584	45.590	53.273
240	4.475	5.477	6.557	8.110	10.526	27.184	33.890	40.244	46.369	54.203
300	4.499	5.509	6.598	8.164	10.604	27.452	34.240	40.678	46.882	54.832
600	4.547	5.572	6.680	8.275	10.763	28.026	35.006	41.635	48.037	56.254

[a] The entries are obtained using the Imhof routine.

Table 3. Powers of nR_{31} and $\hat{\rho}_T$ at the 5% level [a]

n	Test statistics	ϕ .99	.98	.95	.90	.85	.80
25	nR_{31}	.050	.051	.053	.063	.077	.098
	$\hat{\rho}_T$	.050	.051	.053	.062	.075	.095
50	nR_{31}	.051	.052	.063	.097	.157	.248
	$\hat{\rho}_T$	.051	.052	.062	.095	.151	.237
100	nR_{31}	.052	.059	.097	.240	.491	.759
	$\hat{\rho}_T$	.052	.058	.094	.231	.471	.738
200	nR_{31}	.059	.080	.237	.742	.982	1.00
	$\hat{\rho}_T$	.058	.079	.227	.721	.978	1.00
500	nR_{31}	.096	.234	.908	1.00	1.00	1.00
	$\hat{\rho}_T$	.094	.225	.894	1.00	1.00	1.00

[a] The entries are obtained using the Imhof routine.

Table 4. Powers of nR_{32} and $\hat{\rho}_{Ts}$ at the 5% level [a]

Test statistics	n	ϕ .99	.98	.95	.90	.85	.80	n	ϕ .99	.98	.95	.90	.85	.80
	seasonal period $s = 4$							seasonal period $s = 12$						
nR_{32}	40	.050	.052	.058	.080	.117	.172	72	.051	.055	.075	.144	.274	.451
$\hat{\rho}_{Ts}$		.049	.053	.059	.083	.114	.168		.049	.050	.070	.130	.250	.408
nR_{32}	60	.051	.053	.068	.117	.204	.337	120	.053	.062	.115	.324	.645	.893
$\hat{\rho}_{Ts}$		.052	.055	.068	.116	.193	.331		.051	.057	.116	.308	.620	.872
nR_{32}	80	.052	.056	.080	.169	.332	.552	180	.057	.075	.199	.640	.951	.998
$\hat{\rho}_{Ts}$		.050	.059	.077	.162	.311	.533		.058	.078	.191	.623	.942	.997
nR_{32}	100	.052	.059	.096	.240	.489	.757	240	.062	.093	.321	.885	.998	1.00
$\hat{\rho}_{Ts}$		.048	.060	.093	.224	.464	.726		.057	.091	.305	.866	.998	1.00
nR_{32}	200	.059	.080	.236	.741	.982	1.00	300	.068	.115	.473	.980	1.00	1.00
$\hat{\rho}_{Ts}$		.058	.080	.221	.718	.976	1.00		.062	.112	.433	.973	1.00	1.00
nR_{32}	400	.080	.166	.733	1.00	1.00	1.00	600	.115	.318	.979	1.00	1.00	1.00
$\hat{\rho}_{Ts}$		.076	.157	.710	.999	1.00	1.00		.118	.303	.972	1.00	1.00	1.00

[a] The Imhof routine for nR_{32} and the simulation method for $\hat{\rho}_{Ts}$ are used.

Table 5. Distributions of nS_{31} for various sample sizes [a]

n	Probability of a smaller value									
	0.01	0.025	0.05	0.10	0.20	0.80	0.90	0.95	0.975	0.99
	seasonal period $s = 4$									
40	30.568	33.092	35.524	38.639	42.864	63.127	69.266	74.496	79.116	84.534
60	30.554	33.453	36.237	39.799	44.637	68.430	75.903	82.392	88.220	95.182
80	30.574	33.656	36.618	40.411	45.575	71.346	79.596	86.830	93.383	101.292
100	30.594	33.785	36.853	40.789	46.156	73.191	81.947	89.670	96.706	105.253
200	30.641	34.055	37.342	41.570	47.362	77.117	86.991	95.806	103.930	113.927
400	30.668	34.193	37.594	41.974	47.988	79.209	89.702	99.130	107.864	118.680
	seasonal period $s = 12$									
60	105.846	109.086	112.097	115.819	120.666	141.868	147.793	152.705	156.955	161.860
120	111.791	117.425	122.562	128.807	136.833	171.892	182.048	190.693	198.358	207.426
180	114.355	120.809	126.695	133.866	143.106	184.028	196.103	206.468	215.734	226.798
240	115.723	122.602	128.884	136.543	146.438	190.609	203.773	215.126	225.319	237.558
300	116.563	123.711	130.234	138.200	148.504	194.741	208.606	220.597	231.398	244.387
600	118.326	126.001	133.028	141.635	152.797	203.452	218.839	232.229	244.346	259.017

[a] The entries are obtained using the Imhof routine.

Table 6. Distributions of nS_{32} for various sample sizes [a]

n	Probability of a smaller value									
	0.01	0.025	0.05	0.10	0.20	0.80	0.90	0.95	0.975	0.99
	seasonal period $s = 4$									
40	11.166	12.652	14.181	16.287	19.408	37.745	43.982	49.467	54.430	60.379
60	11.154	12.762	14.417	16.689	20.051	40.020	47.012	53.263	59.009	66.019
80	11.156	12.827	14.541	16.896	20.381	41.234	48.646	55.329	61.517	69.137
100	11.160	12.866	14.618	17.023	20.583	41.988	49.667	56.626	63.105	71.116
200	11.173	12.950	14.775	17.282	20.994	43.561	51.812	59.369	66.469	75.352
400	11.182	12.994	14.857	17.414	21.203	44.381	52.938	60.818	68.261	77.626
	seasonal period $s = 12$									
60	41.224	43.774	46.211	49.320	53.550	74.834	81.681	87.663	93.053	99.507
120	41.958	45.279	48.430	52.432	57.848	85.073	93.964	101.834	109.034	117.810
180	42.346	45.920	49.314	53.621	59.455	88.927	98.640	107.289	115.241	124.992
240	42.564	46.267	49.781	54.244	60.292	90.953	101.108	110.181	118.550	128.848
300	42.699	46.481	50.069	54.628	60.808	92.204	102.637	111.973	120.602	131.246
600	42.990	46.929	50.665	55.417	61.865	94.785	105.800	115.702	124.874	136.241

[a] The entries are obtained using the Imhof routine.

Table 7. Powers of nS_{31} and $\hat{\rho}_{31}$ at the 5% level [a]

Test statistics	n	Φ .99	.98	.95	.90	.85	.80	n	Φ .99	.98	.95	.90	.85	.80
	seasonal period $s = 4$							seasonal period $s = 12$						
nS_{31}	40	.050	.050	.052	.056	.063	.073	60	.050	.050	.051	.052	.055	.059
$\hat{\rho}_{31}$		.050	.050	.051	.055	.061	.070		.050	.050	.050	.052	.054	.056
nS_{31}	60	.050	.051	.054	.064	.081	.107	120	.050	.050	.053	.062	.078	.102
$\hat{\rho}_{31}$		.050	.051	.053	.063	.078	.102		.050	.050	.053	.061	.075	.097
nS_{31}	80	.050	.051	.056	.075	.108	.161	180	.050	.051	.057	.080	.123	.197
$\hat{\rho}_{31}$		.050	.051	.056	.073	.103	.151		.050	.051	.057	.077	.117	.183
nS_{31}	100	.050	.052	.060	.090	.146	.238	240	.051	.052	.063	.107	.200	.359
$\hat{\rho}_{31}$		.050	.052	.059	.087	.138	.222		.051	.052	.062	.103	.186	.330
nS_{31}	200	.052	.057	.090	.236	.517	.813	300	.051	.054	.071	.148	.315	.573
$\hat{\rho}_{31}$		.052	.056	.087	.221	.484	.780		.051	.053	.070	.139	.290	.530
nS_{31}	400	.057	.076	.234	.799	.995	1.00	600	.054	.064	.150	.569	.951	1.00
$\hat{\rho}_{31}$		.056	.074	.220	.767	.992	1.00		.053	.063	.141	.527	.931	.999

[a] The entries are obtained using the Imhof routine.

Table 8. Powers of nS_{32} and $\hat{\rho}_{32}$ at the 5% level [a]

Test statistics	n	ϕ .99	.98	.95	.90	.85	.80	n	ϕ .99	.98	.95	.90	.85	.80
	seasonal period $s = 4$							seasonal period $s = 12$						
nS_{32}	40	.053	.056	.066	.087	.117	.156	60	.053	.057	.068	.091	.121	.159
$\hat{\rho}_{32}$		.052	.055	.064	.084	.110	.144		.053	.056	.066	.088	.114	.147
nS_{32}	60	.054	.059	.076	.118	.182	.271	120	.057	.065	.096	.174	.292	.446
$\hat{\rho}_{32}$		.054	.058	.074	.111	.167	.245		.057	.064	.091	.158	.257	.390
nS_{32}	80	.056	.062	.089	.159	.272	.426	180	.061	.075	.133	.300	.543	.781
$\hat{\rho}_{32}$		.055	.061	.085	.147	.246	.383		.060	.073	.123	.262	.472	.702
nS_{32}	100	.057	.066	.103	.210	.384	.599	240	.066	.086	.180	.461	.784	.957
$\hat{\rho}_{32}$		.057	.065	.098	.192	.345	.544		.064	.082	.162	.399	.704	.916
nS_{32}	200	.066	.089	.211	.592	.913	.994	300	.071	.098	.238	.634	.931	.996
$\hat{\rho}_{32}$		.065	.086	.193	.539	.875	.988		.069	.093	.210	.555	.877	.988
nS_{32}	400	.089	.160	.588	.994	1.00	1.00	600	.099	.183	.637	.996	1.00	1.00
$\hat{\rho}_{32}$		.086	.149	.536	.986	1.00	1.00		.093	.164	.557	.987	1.00	1.00

[a] The entries are obtained using the Imhof routine.