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LM Tests in Nested Serially Correlated Error Components Model with Panel Data

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ABSTRACT

This paper considers a panel data regression model in which the disturbances follow a nested error components with serial correlation. Given this model, this paper derives several Lagrange Multiplier(LM) tests for the presence of serial correlation as well as random individual effects, nested effects, and for existence of serial correlation given random individual and nested effects.

Keywords: Panel data model, Nested error component, Autocorrelation, LM tests.

1. INTRODUCTION

Breusch and Pagan(1980), Engle(1984) and Godfrey(1989) demonstrated the wide applicability of Lagrange Multiplier(LM) test to various model specifications in econometrics. The LM test is based on the estimation of the model under null hypothesis and its computation requires only ordinary least squares residuals. In the context of error components model, most researcher have been provided the LM test for testing the existence of the various error components in panel data model. Breusch and Pagan(1980) seem to be the first to derive a simple LM test for testing whether the variance components are both zero or individually zero with panel data. Recently, Baltagi and Li(1991) proposed a joint test for existence of serial correlation and random individual effects in an error component model with first-order serial correlation remainder disturbances

In many economic studies, the panel data may contain nested groupings. For example, data on firms may be grouped by industry, data on state by region and data on individuals by profession. In this case, one can control for

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unobserved industry and firms effects using a nested error components model. See Montemarquette and Mahseredjian(1989) for an empirical application of the nested error component model to study whether schooling matters in educational achievements in Montreal's Francophone public elementary schools. In addition, we allow the remainder disturbances to be serially correlated. This allows for decaying effects of the remainder shocks over time in addition to the equicorrelation due to the random effects which is persistent over time, see Lillard and Willis(1978)and Pantula and Pollock(1985). For this error component model, we derive a LM test which jointly tests for the presence of serial correlation as well as random individual effects and nested effects.

This joint test suffers from the problem of overtesting, see Bera and Jarque(1982). This paper proposes conditinal LM test for existence of serial correlation assuming that the random individual and nested effects are given.

2. THE MODEL

We consider the following panel data regression model

$$y_{ijt} = x'_{ijt}\beta + u_{ijt}, \quad i = 1, \cdots, M, \ j = 1, \cdots, N \text{ and } t = 1, \cdots, T, \tag{1}$$

where y_{ijt} be an observation on a dependent variable for the jth firm in the ith industry for the tth time period. x_{ijt} denotes a nonstochasticregressor vector of k independent variables. The disturbances of (1) are given by

$$u_{ijt} = \mu_i + \nu_{ij} + \varepsilon_{ijt}, \quad i = 1, \cdots, M, \ j = 1, \cdots, N \text{ and } t = 1, \cdots, T, \tag{2}$$

where the μ_i's denote the industry specific effects which are assumed to be $i.i.d.$ $(0, \sigma_\mu^2)$ and the ν_{ij}'s denote the nested effects which are $i.i.d.$ $(0, \sigma_\nu^2)$. The ε_{ijt} are the remainder disturbances which are also assumed to be $\varepsilon_{ijt} = \rho\varepsilon_{ijt-1} + e_{ijt}$, $|\rho| < 1$ with $E(e_{ijt}) = 0$, $Var(e_{ijt}) = \sigma_e^2$ and $Cov(e_{ijt}, e_{ijs}) = 0$, $s \neq t$. The μ_i's, ν_{ij}'s and the ε_{ijt}'s are independent. This is a panel data model with nested serially correlated error components (Pantula and Pollock, 1985). This assumes that there are M industries with N firms in each industry observed over T periods. The model (1) can be written in matrix notation as

$$y = X\beta + u, \tag{3}$$

where y is an $MNT \times 1$ observation vector, X is an $MNT \times k$ design matrix, β is a $k \times 1$ vector of regression coefficients and u is an $MNT \times 1$ disturbance vector. In vector form, (2) can be written as

$$u = (I_M \otimes \iota_N \otimes \iota_T)\mu + (I_M \otimes I_N \otimes \iota_T)\nu + \varepsilon, \quad (4)$$

where $\mu' = (\mu_1, \cdots, \mu_M)$, $\nu' = (\nu_{11}, \cdots, \nu_{MN})$, $\varepsilon' = (\varepsilon_{111}, \cdots, \varepsilon_{1NT}, \cdots, \varepsilon_{MNT})$, ι_N and ι_T are vectors of ones of dimension N and T, I_M and I_N are identity matrices of dimension M and N respectively, and $\otimes$ denotes the Kronecker product.

Under these assumptions, the disturbance covariance matrix $E(uu')$ can be written as

$$\begin{aligned} \Omega &= \sigma_\mu^2(I_M \otimes J_N \otimes J_T) + \sigma_\nu^2(I_M \otimes I_N \otimes J_T) + \sigma_\varepsilon^2(I_M \otimes I_N \otimes V) \\ &= I_M \otimes \left[\sigma_\mu^2(J_N \otimes J_T) + \sigma_\nu^2(I_N \otimes J_T) + \sigma_\varepsilon^2(I_N \otimes V)\right] \end{aligned} \quad (5)$$

where $\sigma_\varepsilon^2 = \sigma_e^2/(1-\rho^2)$, $J_N = \iota_N \iota_N'$ and $J_T = \iota_T \iota_T'$ are square matrices of dimension N and T with all elements to be 1 respectively, and V is the AR(1) correlation matrix of order T:

$$V = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{bmatrix}. \quad (6)$$

It is well established that

$$C = \begin{bmatrix} (1-\rho^2)^{1/2} & 0 & \cdots & 0 & 0 & 0 \\ -\rho & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -\rho & 1 & 0 \\ 0 & 0 & \cdots & 0 & -\rho & 1 \end{bmatrix} \quad (7)$$

transform the usual AR(1) model into a serially uncorrelated regression with independent observations. One can transform the model (3) by premultiplying it by $(I_M \otimes I_N \otimes C)$. The transformed regression disturbances are given in vector form

$$\begin{aligned} u^* &= (I_M \otimes I_N \otimes C)u \\ &= (I_M \otimes \iota_N \otimes C\iota_T)\mu + (I_M \otimes I_N \otimes C\iota_T)\nu + (I_M \otimes I_N \otimes C)\varepsilon. \end{aligned} \quad (8)$$

Using the fact that $C\iota_T = (1-\rho)\iota_T^\alpha$, where $\iota_T^{\alpha'} = (\alpha, \iota_{T-1}')$, one can rewrite (8) as

$$u^* = (1-\rho)(I_M \otimes \iota_N \otimes \iota_T^\alpha)\mu + (1-\rho)(I_M \otimes I_N \otimes \iota_T^\alpha)\nu + (I_M \otimes I_N \otimes C)\varepsilon. \quad (9)$$

Therefore, the covariance matrix of transformed disturbances is

$$\begin{aligned}\Omega^* &= E(u^* u^{*\prime}) \\ &= I_M \otimes \Big\{(1-\rho)^2\sigma_\mu^2(J_N \otimes J_T^\alpha) + (1-\rho)^2\sigma_\nu^2(I_N \otimes J_T^\alpha) + \sigma_e^2(I_N \otimes I_T)\Big\},\end{aligned} \tag{10}$$

since $(I_M \otimes I_N \otimes C)\ E(\varepsilon\varepsilon')\ (I_M \otimes I_N \otimes C') = \sigma_e^2(I_M \otimes I_N \otimes I_T)$. Alternatively, this can be rewritten as

$$\begin{aligned}\Omega^* = I_M \otimes \Big\{&(Nd^2(1-\rho)^2\sigma_\mu^2)(\bar{J}_N \otimes \bar{J}_T^\alpha) \\ &+ (d^2(1-\rho)^2\sigma_\nu^2)(I_N \otimes \bar{J}_T^\alpha) + \sigma_e^2(I_N \otimes I_T)\Big\},\end{aligned} \tag{11}$$

where $d^2 = \iota_T^{\alpha\prime}\iota_T^\alpha = \alpha^2 + (T-1)$. This replaces $\iota_T^\alpha\iota_T^{\alpha\prime}$ by its idempotent counterpart $\bar{J}_T^\alpha = \iota_T^\alpha\iota_T^{\alpha\prime}/d^2$ and $\bar{J}_N = \iota_N\iota_N'/N$. Replacing I_N by $E_N + \bar{J}_N$ and I_T by $E_T^\alpha + \bar{J}_T^\alpha$, where $E_N = I_N - \bar{J}_N$ and $E_T^\alpha = I_T - \bar{J}_T^\alpha$ and collecting terms with the same matrices (Wansbeek and Kapteyn, (1982, 1983)), one gets the spectral decomposition of Ω^* :

$$\Omega^* = \lambda_1 Q_1 + \lambda_2 Q_2 + \lambda_3 Q_3, \tag{12}$$

where $\lambda_1 = Nd^2(1-\rho)^2\sigma_\mu^2 + d^2(1-\rho)^2\sigma_\nu^2 + \sigma_e^2$, $\lambda_2 = d^2(1-\rho)^2\sigma_\nu^2 + \sigma_e^2$ and $\lambda_3 = \sigma_e^2$. Correspondingly, $Q_1 = I_M \otimes \bar{J}_N \otimes \bar{J}_T^\alpha$, $Q_2 = I_M \otimes E_N \otimes \bar{J}_T^\alpha$ and $Q_3 = I_M \otimes I_N \otimes E_T^\alpha$, respectively. The λ_i are the distinct characteristic roots of Ω^* of multiplicity M, $M(N-1)$ and $MN(T-1)$, respectively, and each Q_i is symmetric idempotent and its rank is equal to its trace. Moreover, the Q_i's are pairwise orthogonal and sum to the identity matrix. Therefore, Ω^{*^p} is obtained by

$$\Omega^{*^p} = \lambda_1^p Q_1 + \lambda_2^p Q_2 + \lambda_3^p Q_3, \tag{13}$$

where p is any arbitrary scalar. $p = -1$ obtains the inverse, while $p = -\frac{1}{2}$ obtains $\Omega^{*-1/2}$. $\Omega = E(uu')$ is related to Ω^* by $\Omega^* = (I_M \otimes I_N \otimes C)\ \Omega\ (I_M \otimes I_N \otimes C')$ and $|C| = \sqrt{1-\rho^2}$, $|I_M \otimes I_N \otimes C| = |C|^{MN}$ and $|\Omega^*| = (\lambda_1)^M\ (\lambda_2)^{M(N-1)}\ (\lambda_3)^{MN(T-1)}$. Therefore, the log likelihood function can be written as :

$$\begin{aligned}L(\beta, \sigma_e^2, \sigma_\mu^2, \sigma_\nu^2, \rho) &= const. + \frac{1}{2}MN\log(1-\rho^2) - \frac{1}{2}MN(T-1)\log\sigma_e^2 \\ &\quad -\frac{1}{2}M\log(\lambda_1) - \frac{1}{2}M(N-1)\log(\lambda_2) - u^{*\prime}\Omega^{*-1}u^*,\end{aligned} \tag{14}$$

where λ_1 and λ_2 are given by (12), and Ω^{*-1} is given by (13).

3. LM TESTS

3.1. A joint LM test for $H_0 : \sigma_\mu^2 = 0$, $\sigma_\nu^2 = 0$ and $\rho = 0$

Let us first consider the joint hypothesis $H_0 : \sigma_\mu^2 = 0$, $\sigma_\nu^2 = 0$ and $\rho = 0$. Following Breusch and Pagan(1980), we let $\theta = (\sigma_e^2, \sigma_\mu^2, \sigma_\nu^2, \rho)'$. Since the information matrix will be block diagonal between the θ and β parameters, the part of the information matrix corresponding to β will be ignored in computing the LM statistic, see equation (7) of Breusch and Pagan(1980, p. 241).

$$LM = \tilde{D}_1' \tilde{J}_{11}^{-1} \tilde{D}_1, \tag{15}$$

where $\tilde{D}_1 = (\partial L/\partial\theta)(\tilde{\theta})$ is a 4×1 vector of partial derivatives of the likelihood function with respect to each element of θ, evaluated at the restricted m.l.e. $\tilde{\theta}$. $J_{11} = E[-\partial^2 L/\partial\theta\partial\theta']$ is the part of the information matrix corresponding to θ, and $\tilde{J}_{11}$ is J_{11} when the null hypothesis is true, evaluated at the restricted m.l.e. $\tilde{\theta}$. Under the null hypothesis, the variance-covariance matrix reduces to $\Omega^* = \Omega = \sigma_e^2 I_{MNT}$ and the restricted m.l.e. of β is $\tilde{\beta}_{OLS}$, so that $\tilde{u} = y - X'\tilde{\beta}_{OLS}$ are the OLS residuals and $\tilde{\sigma}_e^2 = \tilde{u}'\tilde{u}/MNT$. Hemmerle and Hartly(1973) give a useful general formula to obtain $\tilde{D}_1$:

$$\partial L/\partial\theta_r = -\frac{1}{2} tr[\Omega^{-1}(\partial\Omega/\partial\theta_r)] + \frac{1}{2}[u'\Omega^{-1}(\partial\Omega/\partial\theta_r)\Omega^{-1}u], \tag{16}$$

for $r = 1, \cdots, 4$. Using the formula of Hemmerle and Hartly(1973), we obtain

$$\begin{aligned}
\frac{\partial L}{\partial \sigma_e^2} &= 0, \quad \frac{\partial L}{\partial \sigma_\mu^2} = \frac{MNT}{2\sigma_e^2}\Big(\frac{\tilde{u}'(I_M \otimes J_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}} - 1\Big) \\
\frac{\partial L}{\partial \sigma_\nu^2} &= \frac{MNT}{2\sigma_e^2}\Big(\frac{\tilde{u}'(I_M \otimes I_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}} - 1\Big) \\
\frac{\partial L}{\partial \rho} &= \frac{MNT}{2}\Big(\frac{\tilde{u}'(I_M \otimes I_N \otimes G)\tilde{u}}{\tilde{u}'\tilde{u}}\Big) = MNT\frac{\tilde{u}'\tilde{u}_{-1}}{\tilde{u}'\tilde{u}},
\end{aligned} \tag{17}$$

where G is the bidiagonal matrix of order T

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

and $\tilde{u}'\tilde{u}_{-1} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{t=2}^{T} \tilde{u}_{ijt}\tilde{u}_{ijt-1}$.

Therefore, the partial derivatives with respect to each element of θ, evaluated at the restricted m.l.e. is given by

$$\tilde{D}_1 = \begin{bmatrix} 0 \\ \frac{MNT}{2\tilde{\sigma}_e^2}\Big(\frac{\tilde{u}'(I_M \otimes J_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}} - 1\Big) \\ \frac{MNT}{2\tilde{\sigma}_e^2}\Big(\frac{\tilde{u}'(I_M \otimes I_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}} - 1\Big) \\ MNT\frac{\tilde{u}'\tilde{u}_{-1}}{\tilde{u}'\tilde{u}} \end{bmatrix}. \tag{18}$$

The information matrix for this model using the formula of Harville(1977) is

$$\tilde{J}_{11} = \frac{MNT}{2\tilde{\sigma}_e^4}\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & NT & T & \frac{2(T-1)}{T}\tilde{\sigma}_e^2 \\ 1 & T & T & \frac{2(T-1)}{T}\tilde{\sigma}_e^2 \\ 0 & \frac{2(T-1)}{T}\tilde{\sigma}_e^2 & \frac{2(T-1)}{T}\tilde{\sigma}_e^2 & \frac{2(T-1)}{T}\tilde{\sigma}_e^4 \end{bmatrix} \tag{19}$$

with

$$\tilde{J}_{11}^{-1} = \frac{2\tilde{\sigma}_e^4}{MNT^2(T-1)(T-2)(N-1)} \cdot$$

$$\begin{bmatrix} (N-1)T(T^2-2(T-1)) & 0 & -T^2(N-1) & \frac{T^2(N-1)}{\tilde{\sigma}_e^2} \\ 0 & (T-1)(T-1) & -(T-1)(T-2) & 0 \\ -T^2(N-1) & -(T-1)(T-2) & (NT^2-3T+2) & -\frac{T^2(N-1)}{\tilde{\sigma}_e^2} \\ \frac{T^2(N-1)}{\tilde{\sigma}_e^2} & 0 & -\frac{T^2(N-1)}{\tilde{\sigma}_e^2} & \frac{T^2(N-1)}{2\tilde{\sigma}_e^4} \end{bmatrix} \tag{20}$$

Therefore the joint LM test statistic for $H_0 : \sigma_\mu^2 = 0$, $\sigma_\nu^2 = 0$ and $\rho = 0$ is given by

$$LM = \frac{MN}{2(N-1)}\Big(A^2 - 2AB + B^2\Big) + \frac{MNT^2}{2(T-1)(T-2)}\Big(B^2 - 4BC + 2TC^2\Big), \tag{21}$$

where $A = \frac{\tilde{u}'(I_M \otimes J_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}} - 1$, $B = \frac{\tilde{u}'(I_M \otimes I_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}} - 1$ and $C = \frac{\tilde{u}'\tilde{u}_{-1}}{\tilde{u}'\tilde{u}}$. It is asymptotically distributed as χ^2_3 when the null hypothesis is true(Engle, 1984). This joint LM test requires only OLS residuals and is easy to compute.

It is noted that, if $\rho = 0$ and one is testing $H_0 : \sigma^2_\mu = 0$ and $\sigma^2_\mu = 0$, then the same derivation is applied except $\theta = (\sigma^2_e, \sigma^2_\mu, \sigma^2_\nu)'$ and we ignore the fourth element of $\tilde{D}_1$ in (18) and the fourth row and column of $\tilde{J}_{11}$ in (19). In this case, the joint LM statistic reverts to

$$LM_1 = \frac{MN}{2(N-1)}(A^2 - 2AB + \frac{NT-1}{T-1}B^2). \tag{22}$$

Similarly, if $\sigma^2_\nu = 0$ and one is testing $H_0 : \sigma^2_\mu = 0$ and $\rho = 0$, then $\theta = (\sigma^2_e, \sigma^2_\mu, \rho)'$ and we ignore the third element of $\tilde{D}_1$ in(18) and the third row and column of $\tilde{J}_{11}$ in (19). In this case, the LM statistic becomes

$$LM_2 = \frac{NT^2}{2(T-1)(T-2)}(A^2 - 4AC + 2TC^2), \tag{23}$$

reported in Baltagi and Li(1991). The LM statistic for the joint test $H_0 : \rho = 0$, $\sigma^2_\mu = 0$ and $\sigma^2_\nu = 0$ given in (21), involves an interaction term ($2AB$, $4BC$) in addition to the familiar A^2, B^2 and C^2 terms.

3.2. An LM test for $\rho = 0$ given $\sigma^2_\mu > 0$ and $\sigma^2_\nu > 0$

Next, we consider the LM test for $\rho = 0$ given the existence of random industry and random nested effects. A similar test is considered by Baltagi and Li(1995) in one-way error component model with serial correlation . The null hypothesis for this model is $H_0 : \rho = 0$ (given $\sigma^2_\mu > 0$ and $\sigma^2_\nu > 0$) vs $H_1 : \rho \neq 0$ (given $\sigma^2_\mu > 0$ and $\sigma^2_\nu > 0$). Under the null hypothesis, we have

$$(\Omega^{-1})_{\rho=0} = \frac{1}{\sigma^2_1}Q^*_1 + \frac{1}{\sigma^2_2}Q^*_2 + \frac{1}{\sigma^2_e}Q^*_3, \tag{24}$$

where $\sigma^2_1 = NT\sigma^2_\mu + T\sigma^2_\nu + \sigma^2_e$, $\sigma^2_2 = T\sigma^2_\nu + \sigma^2_e$ and correspondingly, $Q^*_1 = I_M \otimes \bar{J}_N \otimes \bar{J}_T$, $Q^*_2 = I_M \otimes E_N \otimes \bar{J}_T$ and $Q^*_3 = I_M \otimes I_N \otimes E_T$, respectively. Using the formula of Hemmerle and Hartly(1973),

$$\frac{\partial L}{\partial \sigma^2_e} = \frac{\partial L}{\partial \sigma^2_\mu} = \frac{\partial L}{\partial \sigma^2_\nu} = 0$$

$$\frac{\partial L}{\partial \rho} = D_\rho = \frac{M(T-1)}{T}\Big[N - \frac{\hat\sigma_e^2}{\hat\sigma_1^2} - \frac{(N-1)\hat\sigma_e^2}{\hat\sigma_2^2}\Big]$$
$$+\frac{\hat\sigma_e^2}{2}\hat u'\Big\{I_M \otimes \Big[\Big(\frac{1}{\hat\sigma_1^2}(\bar J_N \otimes \bar J_T) + \frac{1}{\hat\sigma_2^2}(E_N \otimes \bar J_T)$$
$$+\frac{1}{\hat\sigma_e^2}(I_N \otimes E_T)\Big)\Big(I_N \otimes G\Big)\Big(\frac{1}{\hat\sigma_1^2}(\bar J_N \otimes \bar J_T) + \frac{1}{\hat\sigma_2^2}(E_N \otimes \bar J_T) + \frac{1}{\hat\sigma_e^2}(I_N \otimes E_T)\Big)\Big]\Big\}\hat u$$

where $\hat\sigma_e^2 = \hat u'(I_M \otimes I_N \otimes E_T)\hat u/MN(T-1)$, $\hat\sigma_1^2 = \hat u'(I_M \otimes \bar J_N \otimes \bar J_T)\hat u/M$ and $\hat\sigma_2^2 = \hat u'(I_M \otimes E_N \otimes \bar J_T)\hat u/M(N-1)$ are the m.l.e of σ_e^2, σ_1^2 and σ_2^2, respectively, where $\hat u$ is maximum likelihood residuals under the null hypothesis. G is bidiagonal matrix with bidiagonal elements all equal to one. Therefore, we have

$$\hat D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ D_\rho \end{bmatrix}. \tag{25}$$

Using the the formula of Harville(1977), the information matrix under the null hypothesis ($\rho = 0$) is

$$\hat J_\rho = \begin{bmatrix} \frac{MN^2T^2}{2\hat\sigma_1^4} & & & \\ \frac{MNT^2}{2\hat\sigma_1^4} & \frac{MT^2}{2}(\frac{1}{\hat\sigma_1^4} + \frac{N-1}{\hat\sigma_2^4}) & & \\ \frac{MNT}{2\hat\sigma_1^4} & \frac{MT}{2}(\frac{1}{\hat\sigma_1^4} + \frac{N-1}{\hat\sigma_2^4}) & \frac{M}{2}(\frac{1}{\hat\sigma_1^4} + \frac{N-1}{\hat\sigma_2^4} + \frac{N(T-1)}{\hat\sigma_e^4}) & \\ \frac{MN(T-1)\hat\sigma_e^2}{\hat\sigma_1^4} & M(T-1)\hat\sigma_e^2(\frac{1}{\hat\sigma_1^4} + \frac{N-1}{\hat\sigma_2^4}) & \frac{M(T-1)}{T}\hat\sigma_e^2(\frac{1}{\hat\sigma_1^4} + \frac{N-1}{\hat\sigma_2^4} - \frac{N}{\hat\sigma_e^4}) & J_{\rho\rho} \end{bmatrix},$$

where $J_{\rho\rho} = 2M(T-1)^2(Na^2 + 2ab + b^2) + 2M(2T-3)(Na+b) + MN(T-1)$, and $a = -\dfrac{\hat\sigma_2^2 - \sigma_e^2}{T\hat\sigma_2^2}$ and $b = -\dfrac{\hat\sigma_e^2}{T}\Big(\dfrac{1}{\hat\sigma_2^2} - \dfrac{1}{\hat\sigma_1^2}\Big)$, see Baltagi(1995, p. 225). Thus the resulting LM test statistic is

$$LM_3 = \hat D'\hat J_\rho \hat D = \frac{\hat J_\rho^{44}}{det(\hat J_\rho)}(D_\rho)^2 = \frac{M^3N^3T^3(N-1)(T-1)}{8det(\hat J_\rho)\hat\sigma_1^4\hat\sigma_2^4\hat\sigma_e^4}(D_\rho)^2, \tag{26}$$

where *det* denotes the determinants. Under the null hypothesis, LM_3 is asymptotically distributed as χ_1^2. It is also noted that, if $\sigma_\nu^2 = 0$ and one is testing $H_0 : \rho = 0$ (given $\sigma_\mu^2 > 0$), then $\theta = (\sigma_e^2, \sigma_\mu^2, \rho)'$ and we ignore the third row and column of $\hat{J}_\rho$, the LM statistic becomes $((\hat{D}_\rho)^2 \hat{J}^{\rho\rho})$ reported in Baltagi(1995, p. 92). Further, if $\rho = 0$ and one is testing $H_0 : \sigma_\nu^2 = 0$ (given $\sigma_\mu^2 > 0$), then $\theta = (\sigma_e^2, \sigma_\mu^2, \sigma_\nu^2)'$ and we ignore the fourth row and column of $\hat{J}_\rho$. In this case, the LM statistic becomes

$$LM_4 = \frac{M(N-1)(NT-1)}{2N(T-1)} \Big(\frac{NT-1}{N-1} \frac{\hat{u}'(I_M \otimes E_N \otimes \bar{J}_T)\hat{u}}{\hat{u}'(I_M \otimes E_{NT})\hat{u}} - 1 \Big)^2. \tag{27}$$

4. CONCLUSION

In this paper, we derived the LM test which is jointly testing for the presence of serial correlation, random individual effects and nested effects, and the LM test for $\rho = 0$ given the existence of random industry and random nested effects. These joint and conditional LM tests are attractive competitor to the LR and the Wald tests because it requires only the OLS residuals. The resulting tests are useful for specification of the serially correlated nested error component model. Some empirical panel data are given by Montemarquette and Mahseredjian(1989) and Antweiler(2001).

REFERENCES

Antweiler, W. (2001), nested random effects estimation in unbalanced panel data, *Journal of Econometrics,* **101**, 295-314.

Baltagi, B.H. (1990), A Lagrange Multiplier test for the error components model with incomplete panels, *Econometric Review,* **9**, 103-107.

Baltagi, B.H. (1995), *Econometric Analysis of Panel Data,* Wiley, New York.

Baltagi, B.H. and Li, Q. (1991), A joint test for serial correlation and random individual effects, *Statistics and Probability Letters,* **11**, 277-280.

Baltagi, B.H. and Li, Q. (1995), Testing AR(1) against MA(1) disturbances in an error component model, *Journal of Econometrics,* **68**, 133-151.

Bera, A.K. and Jarque, C.M. (1982), Model specification tests: a simultaneous aproach, *Journal of Econometrics,* **20**, 59-82.

Breusch, T.S. and Pagan, A.R. (1980), The Lagrange Multiplier test and its application to model specification in econometrics, *Review of Economic Studies*, **47**, 239-254.

Engle, R. (1984), Wald, Likelihood–Ratio, Lagrange Multiplier tests in econometrics, in: Griliches, Z., M.D. Intriligator, eds., *Handbook of Econometrics,* North Holland, Amsterdam.

Godfrey, L.G. (1989), *Misspecification Tests in Econometrics,* Cambridge University Press, Cambridge.

Harville, D.A. (1977), Maximum likelihood approaches to variance component estimation and to related problems, *Journal of the American Statistical Association*, **72**, 320-338.

Hemmerle, W.J. and Hartley, H.O. (1973), Computing maximum likelihood estimates for the mixed A.O.V. model using the W-transformation, *Technometrics*, **15**, 819-831.

Lillard, L.A. and Willis, R.J. (1978), Dynamic aspects of earning mobility, *Econometrica*, **46,** 985-1012.

Montmarquette, C. and Mahseredjian, S. (1989), Does school matter for educational achievement? A two-way nested error components analysis, *Journal of Applied Econometrics*, **4**, 181-193.

Pantula, S.G. and Pollock, K.H. (1985), Nested analysis of variance with autocorrelated error, *Biometrics*, **41,** 909-920.

Wansbeek, T. and Kapteyn, A. (1982), A simple way to obtain the spectral decomposition of variance components models for balanced data, *Communications in Statistics - Theory and Method*, **11**, 2105-2111.

Wansbeek, T. and Kapteyn, A. (1983), A note on spectral decomposition and maximum likelihood estimation in ANOVA models with balanced data, *Statistics and Probability Letters*, **1**, 213-215.