Yield and Compression Behavior of Semi-Solid Material by Upper-Bound Method

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ABSTRACT

The compression behavior of semi-solid materials is studied from a viewpoint of yield criteria and analysis methods. To describe the behavior of materials in semi-solid state, several theories have been proposed by extending the concept of plasticity of porous compressible materials. In the present work, the upper-bound method and the finite element method are used to model the simple compression process using yield criteria of Kuhn and Doraivelu. Segregation between solid and liquid which cause defect of product is analysed for Sn-15%Pb and A356 alloys during deformation in semi-solid state. The comparison of analyses is made according to yield criteria and analysis methods. In addition, the analysis result for semi-solid dendritic Sn-15%Pb alloy is compared with the experimental result of Charreyron et al..

Keywords : Semi-solid material, Finite element method, Upper-bound method, Yield criterion, Porous material

1. Introduction

Semi-solid forming has been developed to fabricate near-net-shape products and lots of the relating researches have been in progress from an economical and environmental viewpoint.

Mechanical property of a product by semi-solid forming process is closely related to microstructure of the final product [1]. Especially, separation between liquid and solid by friction condition and deformation of the material causes the main defect of the product. Therefore, prediction of the liquid segregation is necessary to prevent the defect during the semi-solid forming process.

Many investigations have been made since Spencer et al. [2] investigated the behavior of semi-solid alloy at MIT in 1972 and plasticity theory of porous material has been applied to behavior of semi-solid material. Choi et al. [3] has made use of the yield criterion of Kuhn and the finite element analysis taking induction heating into account has been made for the closed-die compression process. In the meanwhile, Charreyron et al. [4] has predicted the distribution of liquid fraction depending on the degree of deformation and friction condition by upper-bound method for Sn-15%Pb alloy using the yield criterion of Shima & Oyane. In addition, Choi et al. [5] predict distribution of liquid fraction for Sn-15%Pb alloy by applying Doraivelu's yield criterion to upper-bound method according to friction condition and degree of deformation. Nguyen et al. [6] has proposed new analysis model for semi-solid behavior of A356 alloy and performed simple compression test and filtering experiment for the A356 alloy.

In this study, finite element analysis and upper-bound analysis is performed by using Kuhn and Doraivelu's criterions. The results of analyses will be compared with those of Charreyron et al. [4] in which Shima & Oyane's yield criterion. In addition, liquid distribution is obtained for simple compression process of A356 alloy by performing analysis of semi-solid behavior according to yield criterions and analysis methods.

2. Yield and constitutive equations of semi-solid material

2.1 Yield of semi-solid material by Kuhn's yield criterion

The yield criterion for compressible porous material proposed by Kuhn is expressed as equation (1).

$$AJ'_2 + BJ_1^2 = \delta Y_0^2 = Y_R^2 \tag{1}$$

$$A = 2(1+\nu),\ B = \frac{1}{3}(1-2\upsilon) = 1 - \frac{A}{3}$$

$$\nu = 0.5(R)^2 = 0.5(g_s)^2$$

$$\sqrt{\delta} = R^3$$

Here, Y_R is yield stress of material with relative density R, Y_0 is yield stress of fully dense material, J_1 is the first invariant of stress tensor, J_2' is the second invariant of deviatoric stress, and g_s is volume fraction of solid.

2.2 Yield of semi-solid material by Doraivelu's yield criterion

The yield criterion for porous material as equation (2) was suggested by Doraivelu et al., which is used as the yield criterion for deformation of solid skeleton in semi-solid material.

$$AJ'_2 + BJ_1^2 = \delta Y_0^2 = Y_R^2 \tag{2}$$

$$A = 2(1+\nu),\ B = \frac{1-2\nu}{3}$$

$$Y_0 = \left[\frac{1}{\delta}\left[\frac{1+\upsilon}{3}\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\} + \frac{1-2\upsilon}{3} 9\sigma_m^2\right]\right]^{1/2}$$

Here, ν is Poisson's ratio.

Using equation (2), the stress-strain rate equation yields

$$\dot{\varepsilon}_i = \frac{1}{2}\frac{\dot{\varepsilon}_{eq}}{Y_0}\frac{1-g_L}{\delta}[2(1+\nu)\sigma_i - 6\nu\sigma_m],\quad i = 1,2,3 \tag{3}$$

Here, g_L signifies liquid fraction.

In order to calculate internal deformation energy, equivalent strain rate is represented as follows.

$$\dot{\varepsilon}_i = \frac{\sqrt{2\delta}}{1-g_L}\left[\frac{1}{6(1+\upsilon)}\left\{(\dot{\varepsilon}_1-\dot{\varepsilon}_2)^2 + (\dot{\varepsilon}_2-\dot{\varepsilon}_3)^2 + (\dot{\varepsilon}_3-\dot{\varepsilon}_1)^2\right\} + \frac{\dot{\varepsilon}_v^2}{6(1-2\upsilon)}\right]^{1/2} \tag{4}$$

Let $(\sigma_{1,c})_{max}$ the maximum stress in compression experiment by Pinsky [7]. The yield criterion is given by

$$\delta Y_0^2 = (\sigma_{1,c})_{max}^2 \tag{5}$$

Supposing that there is no friction on the wall of the container in filtration experiment by Charreyron et al., which results in $\dot{\varepsilon}_2 = \dot{\varepsilon}_3 = 0$, equations (6) and (7) can be obtained from the constitutive equations.

$$\sigma_{2,pf} = \sigma_{3,pf} = \frac{3\nu}{1+\nu}\cdot\sigma_m \tag{6}$$

$$\sigma_{1,pf} = \frac{3(1-\nu)}{1+\nu}\cdot\sigma_m \tag{7}$$

Using equations (6), (7) and (2), δ yields

$$\delta = \left(\frac{\sigma_{1,pf}}{Y_0}\right)^2 (1-2\nu)\left(\frac{1+\nu}{1-\nu}\right) \tag{8}$$

3. Finite element analysis for simple compression process

Semi-solid material is composed of solid with rigid visco-plastic deformation and liquid. Total stress applied on the material is expressed as summation of stresses on solid and liquid. Semi-solid model to obtain equilibrium equation is shown in Fig. 1.

As shown in equation (9), the stress applied on semi-solid material, $\sigma_{T_{ij}}$ is summation of the stress on solid, $\sigma_{S_{ij}}$ and the stress on liquid, p.

$$\sigma_{T_{ij}} = \sigma_{S_{ij}} + \delta_{ij} p A_L \tag{9}$$

Here, A_L is liquid area.

The equation (9) is expressed as equation (10) and equilibrium equation is shown in equation (11).

$$\sigma_{T_{ij}} = \sigma_{S_{ij}} \frac{A_S}{A_T} + \delta_{ij} p \frac{A_L}{A_T} = \sigma_{ij} + \delta_{ij} p g_L \qquad (10)$$

Here, A_s is solid area, A_T is total area, and g_L is liquid fraction.

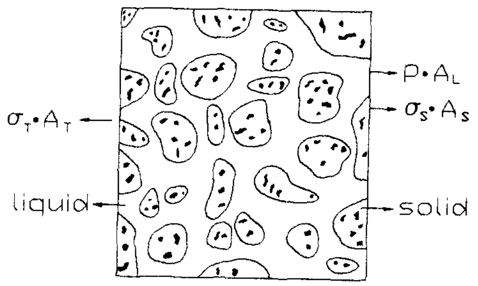


Fig. 1 Schematic illustration of microstructure at semi-solid state

Kuhn's yield criterion is used for the criterion for deformation of solid, and Darcy's law is adopted as governing equation for liquid as shown in equation (12). Equation (13) shows continuity equation.

$$u_{L,i} \cdot g_L = \frac{\chi}{\mu_L} \frac{\partial p}{\partial x_i} \qquad (12)$$

$$\vec{\nabla} \cdot \vec{U}_S + \vec{\nabla} \cdot (g_L \vec{U}_L) = 0 \qquad (13)$$

$$\vec{\nabla} = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y}$$

Here, $\vec{U}_S$ is velocity vector of solid and $\vec{U}_L$ is velocity vector of liquid.

Finally, finite element equation is obtained by equation (11)-(13).

$$\begin{bmatrix} K_S & K_L \\ L_S & L_L \end{bmatrix} \begin{pmatrix} \Delta V_S \\ \Delta V_P \end{pmatrix} = \begin{bmatrix} F_S \\ Q_L \end{bmatrix} \qquad (14)$$

Here, F_S and Q_L are residual of nodal force vector.

Flow equation and mechanical property applied by Toyoshima [8] are used for finite element analysis of Sn-15%Pb alloy. Radius of the material is 6.35mm, height is 6.35mm, temperature is 184°C, and strain rate is $1.33\times10^{-2}s^{-1}$.

Flow equation and mechanical property applied by Choi [3] are used for finite element analysis of A356 alloy. Radius of the material is 10mm, height is 22mm, temperature is 584°C, and strain rate is $0.8\times10^{-2}s^{-1}$.

4. Upper-bound analysis for simple compression process

Upper-bound analysis for simple compression process is performed for deformation analysis in solid skeleton of semi-solid material by applying yield criterion of Doraivelu et al. and kinematically admissible velocity field proposed by Charreyron is used.

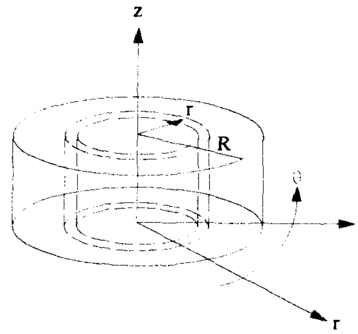


Fig. 2 Schematic diagram of a compression sample

For a compression model shown in Fig. 2, continuity equation is obtained from equation (15).

$$\frac{\partial}{\partial t}(1-g_L) \cdot \rho_S + \nabla[(1-g_L)\rho_S V_S] = 0 \qquad (15)$$

Assuming that density of solid is constant, equation (16) is obtained.

$$\frac{\partial}{\partial t}(1-g_L) + \nabla[(1-g_L)V_S] = 0 \qquad (16)$$

Equation (16) is adjusted and then equation (17) is obtained.

$$\frac{\partial g_L}{\partial t} = \nabla[(1-g_L)]V_S \qquad (17)$$

Equation (17) is applied to a compression model as shown in Fig. 2 and yields equation (18).

$$\frac{1}{(1-g_L)}\frac{\partial g_L}{\partial t} = \frac{V_r}{r}+\frac{\partial V_r}{\partial r}+\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{\partial V_Z}{\partial Z} = \dot{\varepsilon}_v \quad (18)$$

The kinematically admissible velocity field, which satisfies the continuity equation and boundary condition as equation (19), is expressed as equation (20).

$$z=0\ ;\ V_{S,z}=0$$
$$z=H\ ;\ V_{S,z}=-U \quad (19)$$

$$V_{S,r}=\frac{U}{2}\frac{R}{H}\left[a\left(\frac{r}{R}\right)+b\left(\frac{r}{R}\right)^3\right] \quad (20)$$
$$V_{S,\theta}=0$$
$$V_{S,z}=-\frac{U}{H}z$$

Power of plastic deformation is given by equation (21) and power due to friction dissipation is in equation (22), which result in total energy dissipation, J^* in equation (23).

$$\dot{W}_i=\int_V(1-g_L)\sqrt{\delta}Y_0\dot{\varepsilon}_{eq}dV \quad (21)$$

$$\dot{W}_f=\int_{S_f}\overline{m}\frac{\sqrt{\delta}Y_0}{\sqrt{3}}|V_{S_r}|dS_f \quad (22)$$

$$J^*=\sum\dot{W}_i+\sum\dot{W}_j \quad (23)$$

The parameters a and b, which control densification intensity and distribution and are used in determining the velocity field, can be obtained by minimizing equation (23), total energy dissipation, using FPS (Flexible Polyhedron Search)[9] which is the kind of direct search method.

The linear interpolation equation is obtained as follows by using equations (5) and (8), and $(\sigma_{1,c})_{max}$ and $\sigma_{1,pf}$ on liquid fraction which result from the experiment of Pinsky et al. as follows:

$$\nu=3.88\cdot g_L-0.98 \quad (24)$$

From equation (8), the relationship between flow stress and liquid fraction is obtained by use of equation (24) and $\sigma_{1,pf}$ which results from the experiment of Pinsky et al.

$$\sqrt{\delta}\cdot Y_0=-17.47\cdot g_L+10.89 \quad (25)$$

In order to describe the behavior of solid skeleton, the flow stress is expressed as

$$\sqrt{\delta}\cdot Y_0=k\dot{\varepsilon}_{ave}^n \quad (26)$$

Here, supposing that n=0.23 which was obtained in the experiment of Suery et al. [10], $Y_0=10.89\ MPa$ at a liquid fraction of zero from equation (9), and $\dot{\varepsilon}_{ave}=1.33\times10^{-2}\ s^{-1}$, then k is to be $26.97\ MPa\cdot s^{-n}$.

In addition, the linear interpolation equation is obtained as follows by using equations (5) and (8), and $(\sigma_{1,c})_{max}$ and $\sigma_{1,pf}$ on liquid fraction which result from the experiment of Nguyen et al. as follows:

$$\nu=3.4221\cdot g_L-0.84902 \quad (27)$$

From equation (8), the relationship between flow stress and liquid fraction is obtained by use of equation (27) and $\sigma_{1,pf}$ which results from the experiment of Nguyen et al.

$$\sqrt{\delta}\cdot Y_0=-5.83363\cdot g_L+3.25387 \quad (28)$$

In order to describe the behavior of solid skeleton, the flow stress is expressed as

$$\sqrt{\delta}\cdot Y_0=k\dot{\varepsilon}_{ave}^n \quad (29)$$

Here, supposing that n=0.23 which was obtained in the experiment of Choi et al., $Y_0=3.25387\ MPa$ at a liquid fraction of zero from equation (28), and $\dot{\varepsilon}_{ave}=0.8\times10^{-2}\ s^{-1}$ which is the same value as

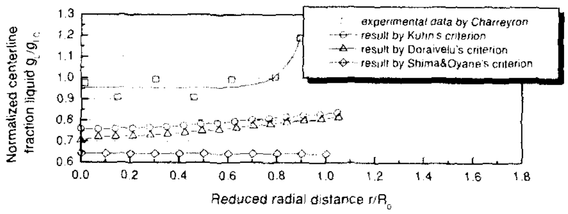
Nguyen's experiment, then k is to be 9.87847 $MPa \cdot s^{-n}$.

5. Results and discussions

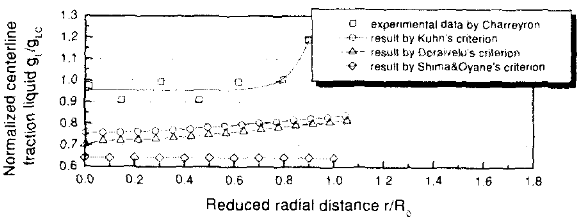
5.1 Comparison of yield criteria for Sn-15%Pb alloy

Finite element analysis for simple compression process is performed for cylindrical material of Sn-15%Pb applying Kuhn and Doraivelu's criteria. In order to verify the FE analysis, the result of the FE analysis is compared with that of the experiment performed by Charreyron. Additionally, the result of this study is compared with that of analysis performed by Charreyron in which Shima & Oyane's yield criterion is used.

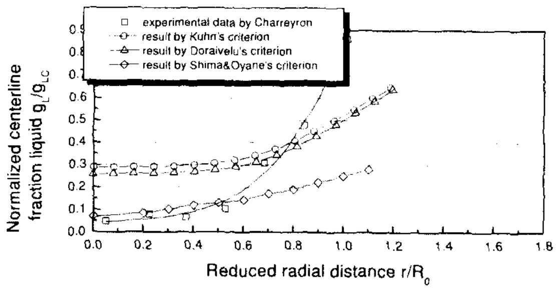
As a result, Fig. 3 shows radial distribution of liquid fraction for Sn-15%Pb at various strains according to the above mentioned yield criteria. It is shown that liquid distribution of the material increases with an increase of the radius and decreases with an increase of strain.



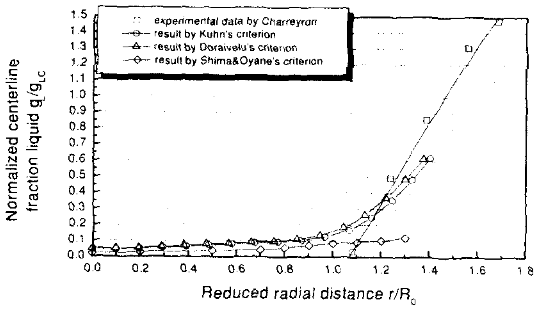
(a) compressed at ε=0.20 (m=1.0)



(b) compressed at ε=0.58 (m=1.0)



(c) compressed at ε=0.58 (m=1.0)



(d) compressed at ε=0.97 (m=1.0)

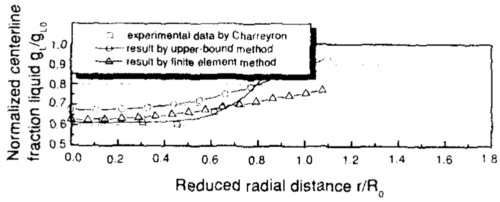
Fig. 3 Radial distribution of liquid fraction for Sn-15%Pb according to yield criterion

The analysis result in this study which Kuhn and Doraivelu's yield criterion is used is on better agreement with expremental result than that of Charreyron which Shima & Oyane's yield criterion is used. Although the result by using Kuhn's criterion is nearly similar to that of Doraivelu's, the result by using Kuhn's criterion has better similarity to the experimental result at the early deformation and the result by using Doraivelu's has better at the later deformation.

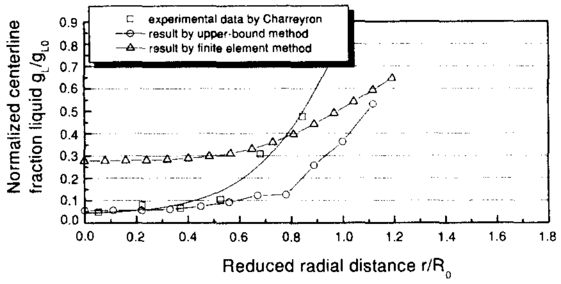
5.2 Comparison of analysis method for Sn-15%Pb alloy

Upper-bound analysis and Finite Element analysis by using Doraivelu's yield criterion are performed for simple compression process of cylindrical billet of Sn-15%Pb. And its results are compared with those of experiment of Charreyron, so that feasibility according to analysis methods is proposed. Fig. 4 shows radial distribution of liquid fraction for Sn-15%Pb according to analysis methods.

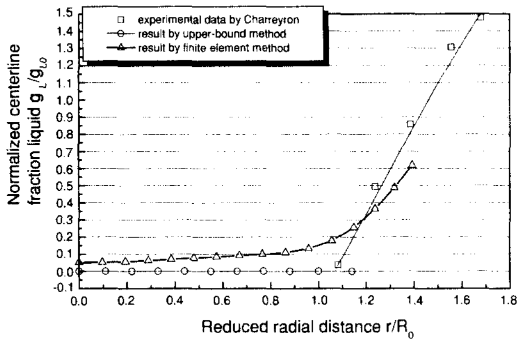
As shown in Fig. 4, radial distribution of liquid fraction increases with an increase of strain and the result of finite element analysis is on better agreement with experimental results by Charreyron than that of upper-bound analysis.



(a) compressed at ε=0.27 (m=1.0)



(b) compressed at ε=0.58 (m=1.0)



(c) compressed at ε=0.97 (m=1.0)

Fig. 4 Radial distribution of liquid fraction for Sn-15%Pb according to yield criteria analysis methods

5.3 Comparison of analysis method and yield criterion for A356 alloy

Radial distribution of liquid fraction for A356 alloy according to yield criteria and analysis methods is proposed.

Fig. 5 shows radial distribution of liquid fraction for A356 alloy according to yield criteria by using finite element analysis. Fig. 6 shows radial distribution of liquid fraction for A356 according to analysis methods.

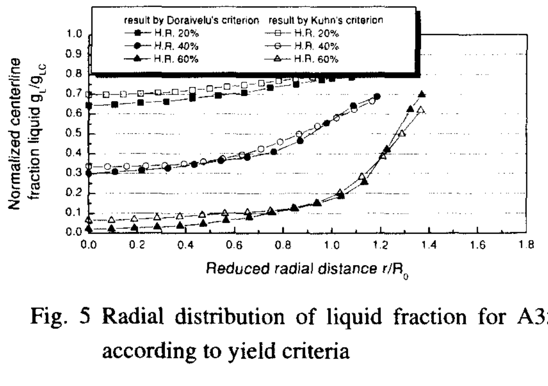


Fig. 5 Radial distribution of liquid fraction for A356 according to yield criteria

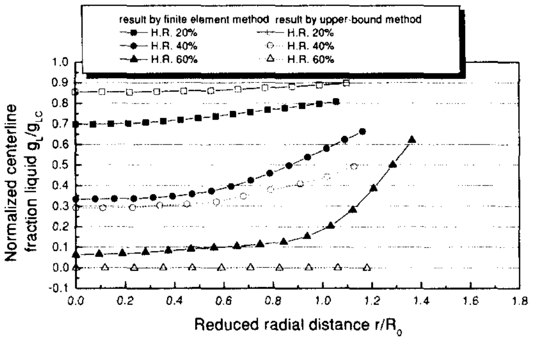


Fig. 6 Radial distribution of liquid fraction for A356 according to analysis methods

As a result in Fig. 5, the result of Doraivelu's yield criterion has better similarity to the experimental result at the initial state of deformation. As shown in Fig. 6, big difference is found between upper-bound method and finite element method and finite element analysis shows better result than upper-bound method both in appearance of deformation and in liquid fraction.

6. Conclusions

In this study, Kuhn and Doraivelu's yield criteria among many yield criteria for porous material is adopted to semi-solid material such as Sn-15%Pb and A356. In addition, upper-bound method and finite element method are compared for simple compression process of semi-solid material according to various yield criteria. From the results of this study, we come to the following conclusion.

(1) Theoretical analysis model for semi-solid material such as Sn-15%Pb and A356 is proposed by using Kuhn and Doraivelu's yield criteria.

(2) The improved result is obtained by using Kuhn and Doraivelu's criteria than Shima & Oyane's criterion in aspect of distribution of liquid fraction.

(3) In analysis of simple compression process in semi-solid material, the increase of deformation results in the increase of liquid fraction in radial direction, which indicates segregation between solid and liquid.

(4) There is big difference between upper-bound analysis and finite element analysis for semi-solid alloy and finite element analysis shows better results in a viewpoint of deformation and liquid fraction.

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