A Method to Calculate Charge for Reactive Power Service under Competition of Electric Power Utilities

Kyoung-Soo Ro and Sung-Chul Park

Abstract - As electric power systems have been moving from vertically integrated utilities to a deregulated environment, the charging of reactive power management is a new challenging theme for market operators. This paper proposes a new methodology to compute the costs of providing reactive power management service in a competitive electrical power market. The proposed formulation, which is basically different from those shown in the literature, consists of two parts. One is to recover investment capital costs of reactive power supporting equipment based on a reactive power flow tracing algorithm. The other is to recover operational costs based on variable spot prices using the optimal power flow algorithm. The charging shapes resulted from the proposed approach exhibit a quite good meaning viewed from a practical sense. It turns out that reactive power charges are mostly due to recovery of capital costs and slightly due to recovery of operational costs. The method can be useful in providing additional insight into power system operation and can be used to determine tariffs of a reactive power management service.

Keywords - reactive power charging, optimal power flow(OPF), reactive power tracing, power utility deregulation.

1. Introduction

In recent years, some developed countries such as England, Australia, etc, opened their electricity industries to private enterprises and tried to improve the efficiency of the total power system through a competitive market. These things became possible due to development of energy-related technologies and information communication through internet. Deregulation and unbundling of the services provided by electric power utilities promotes competition by enabling access to transmission services for all wholesale buyers and sellers of electric power. Transmission utilities must provide non-discriminatory transmission service to third parties at cost-based rates. Under the deregulated circumstance, there is a necessity for separately charging the component parts of electricity production and delivery such as power generation, transmission, distribution, and ancillary services.

Transmission service providers need to know the precise operating costs of providing ancillary services to their customers since the costs vary as a function of time, location, and system status. Ancillary services are defined, by Federal Energy Regulatory Commission (FERC), as "*those services necessary to support the transmission of electric power from seller to purchaser given the obligations of control areas and transmitting utilities within those control areas to maintain reliable operations of the interconnected transmission system.*"[1]

The major eight ancillary services may be thought of as (i) scheduling and dispatch, (ii) reactive power management (voltage control), (iii) load following (automatic generation control), (iv) energy imbalance, (v) spinning reserve, (vi) supplemental reserve, (vii) black start, (viii) real power losses. Cost related to each unbundled ancillary service is to be calculated separately, and then is to be added together to get the total ancillary service costs.

Calculating costs of these ancillary services became, recently, one of the most active areas to research. Analyzing the costs is the first step of determining the price. This paper deals with the ancillary service of reactive power management, which is essential for the system security and voltage control. An approach of a reactive power market for reactive power charging has been presented.[2, 3] Gil et al. proposes two different reactive power markets such as a reactive energy market based on losses spot prices and a reactive capacity market based on a reactive power regulating capacity payment.[2] Ahmed and Strbac present an arrangement for reactive power market based on combined reactive power capacity and energy payments.[3]

Other several papers have been published for calculating reactive power charges using real-time pricing methods, which use a modification of optimal power flow(OPF) program to determine nodal marginal costs. [4, 5, 6, 7] These methods of real-time pricing of reactive power can be regarded as the extension of real-time

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This work was sponsored by Next-Generation Power Technology Center supported by Ministry of Science and Technology and Korea Science and Engineering Foundation, and by Electrical Engineering & Science Research Institute (EESRI) that is supported by Korea Electric Power Corporation (KEPCO). (EESRI-00-018)

Manuscript received: Sep. 26, 2001, accepted: Nov. 20, 2001.

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pricing of active power, which is developed in [8]. In all of the real-time pricing methods the objective is to minimize the total generation production cost of active power or its variations. Choi et al. attempts a different concept to evaluate the reactive power charge, which uses the objective function of maximizing social benefit instead of minimizing the generation production cost.[9] Meanwhile, there exist assertions that the reactive power charge should contain the capital cost portion as well as the nodal marginal cost.[3, 10]

Thus, a proper charging structure for reactive power support should not only recover the investment capital costs of reactive power supplying equipment, but also presents an economic signal for real-time operations. The marginal cost of reactive power at each bus, computed from the OPF program, represents the sensitivity of generation production cost with respect to reactive power demand. That marginal cost is really small and irrelavant to the fuel cost of generators, so it cannot charge correctly the actual reactive power costs. The marginal cost can be viewed to account for the system's operation costs that consist of energy losses, operation and maintenance, etc. Therefore, the capital costs of reactive power supplying equipment should be included in the reactive power charging process.

This paper proposes a new method to calculate the costs for providing a reactive power management service. It consists of two components- recovering investment capital costs and recovering operational costs. The capital costs associated with reactive power supplying equipment are distributed using a reactive power tracing algorithm, which can compute what portion of the reactive power generation is used by a certain customer. This paper calculates the contribution of individual reactive power generations to loads using a graph theory that is presented in [14]. And the operational costs are distributed based on variable spot prices derived from the real-time pricing of reactive power.

2. Real-time Pricing of Reactive Power

Real-time pricing of reactive power is made of using the OPF model. The OPF model has the goal of determining active power outputs of generating units, which solutions minimize the total operating cost subject to a set of the operational constraints. The goal is achieved by letting reactive power charge at each bus equal to the short-run marginal cost of supplying reactive power at that bus. In this description we assume the customer responses to reactive power charges do not vary.

The descriptions here for real-time pricing of reactive power mainly refer to [4]. The objective function to be minimized, illustrated in eq. (1), consists of active power production costs of generators, and the costs of system operation and maintenance are assumed to be constant so that they are not included in the objective function.

$$C = \sum_{i=1}^{N_G} C_i(P_{Gi}) \tag{1}$$

where, N_G is the number of all generators. $C_i(P_{Gi})$ is the active power cost function of the generator at bus i.

Since reactive power generating costs are much less than those of real power and are not available under the current utility operation, the operating costs of providing a generator's reactive power is assumed to be neglected in the above equation (1). The function of active power costs is generally represented as a quadratic function.

$$C_i(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i \tag{2}$$

The equality constraints of the OPF problem are power flow equations at all buses which are written as follows.

$$P_i - \sum_{j=1}^{N_B} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) = 0 \tag{3}$$

$$Q_i - \sum_{j=1}^{N_B} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) = 0 \tag{4}$$

where, P_i is active bus power at bus i, $P_i = P_{Gi} - P_{Di}$. Q_i is reactive bus power at bus i, $Q_i = Q_{Gi} - Q_{Di}$. P_{Gi} and Q_{Gi} are active and reactive power generation at bus i. P_{Di} and Q_{Di} are active and reactive power demand at bus i. $|Y_{ij}|\angle\theta_{ij}$ is an element of Y_{BUS} matrix. $|V_i|\angle\delta_i$ is the bus voltage at bus i. N_B is the number of all buses.

The inequality constraints of the OPF problem are listed next.

(1) Limits on active and reactive power generation.

$$P_{Gi,\min} \le P_{Gi} \le P_{Gi,\max} \tag{5}$$

$$Q_{Gi,\min} \le Q_{Gi} \le Q_{Gi,\max} \tag{6}$$

(2) Transmission limits.

$$P_{ij,\min} \le P_{ij} \le P_{ij,\max} \tag{7}$$

$$P_{ij} = |V_i|^2 |Y_{ij}| \cos(\theta_{ij}) - |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) \tag{8}$$

where, P_{ij} is the active power flow from bus i to bus j. Equation (8) is obtained when assuming that the shunt admittance is neglected.

(3) Voltage limits.

$$|V_i|_{\min} \le |V_i| \le |V_i|_{\max} \tag{9}$$

In order to solve the nonlinear programming problem mentioned above, the Lagrangian function is formulated. The Lagrangian multipliers corresponding to the equality constraint of equation (4) indicate the short-run marginal cost of supplying reactive power at each bus, so the real-time reactive power charge at bus i can be written as the following equation.

$$\lambda_{Q,i} = \frac{\partial L}{\partial Q_i} \tag{10}$$

3. Proposed Method using Tracing of Reactive Power Flow

Real-time pricing of reactive power mentioned in the previous section is a well-developed method to find operational costs of reactive power, but it is not enough to recover all costs relevant to the provision of reactive power. In the real-time pricing method the objective is to minimize the total generation production cost of active power. The production cost of reactive power is neglected in the objective function since the cost of reactive power production is relatively very small. Even if a trial is made to include reactive power production cost into the objective function, its acceptable functional form has not been known yet. Therefore, real-time pricing method can be suited well to calculate active power charges, but not sufficient for reactive power charges.

A proper charging structure for reactive power support should not only recover the capital costs of reactive power supplying equipment, but also present an economic signal for real-time operations. So, the costs for providing a reactive power service should consist of two components- recovering investment capital costs and recovering operational costs. This section provides a method to distribute the capital costs associated with reactive power supplying equipment. The important idea is to distribute the capital costs according to the customer's usage of reactive power, which can be calculated from the tracing method of reactive power flow. The contributions of individual reactive power generations to loads are, in this paper, found using the graph theory.[14] Next is the explanation of the graph theory to evaluate the reactive power tracing.

First, here are some assumptions to simplify the problem. Reactive power required by transmission line resistance, reactance and charging capacitance has been moved to the line terminal buses and modeled as equivalent loads according to ac power flow solution. The buses of a network can be classified as generator buses, load buses and network buses based on their net injection to the system. The flows of electricity obey the proportional-sharing rule.

The downstream tracing is used for calculating the contribution factors of individual generators to loads. To solve the problem we build up two matrices. One is an extraction factor matrix of loads from bus total passing power. The other is a contribution factor matrix of generators to bus total passing power. The product of these two matrices constitutes the contribution factor matrix of generators to loads.

An extraction factor matrix (A_l) of lines from total passing reactive power of their upstream buses is built up, which matrix satisfies the equation of $Q_l = A_l Q$, where Q_l is the vector of line reactive power, Q is the vector of bus total passing reactive power calculated from ac power flow solution. The nonzero element in A_l is calculated by the following equation.

$$(A_l)_{line\ j,bus\ i} = \frac{line\ j\text{'}s\ power\ flow}{bus\ i\text{'}s\ total\ passing\ power\ Q_i} \tag{11}$$

where, bus i is the upstream bus of line j.

Similarly an extraction factor matrix (A_L) of loads is formed from total passing reactive power of their upstream buses, which matrix satisfies the equation of $Q_L = A_L Q$, where Q_L is the vector of load reactive power. And the matrix A_L will be a diagonal matrix.

$$A_{Lii} = \begin{cases} 0 & i \notin net\ load\ buses \\ \dfrac{net\ load\ power\ on\ bus\ i}{Q_i} & i \in net\ load\ buses \end{cases} \tag{12}$$

A contribution factor matrix (B) of generators to bus total passing reactive power is defined as

$$Q = BQ_G \tag{13}$$

where Q_G is the vector of generator reactive power. The matrix B is formed row by row. The elements in matrix B can be calculated by the following equation.

$$B_{bus-i,bus-k} = \begin{cases} 1 & k = i,\ k \in net\ gen.buses \\ 0 & k = i,\ k \notin net\ gen.buses \\ 0 & k > i \\ 0 & k < i,\ k \notin net\ gen.buses \\ \sum_{l \in i} A_{lj-m} \cdot B_{m-k} & k < i,\ k \in net\ gen.buses \end{cases} \tag{14}$$

where, $k<i$ implies k is an upstream bus of bus i. $k>i$ implies k is a downstream bus of bus i. $l_j \in i$ implies line j is an inflow line of bus i. A_{lj-m} is the unique non-zero element corresponding to line j in matrix A_l with bus m. B_{m-k} represents the contribution of generator k to the total injection power of bus m. ($A_{lj-m} \cdot B_{m-k}$) represents the contribution of generator k to the total injection power of bus i through line j from bus m to bus i.

Next we determine the contribution factors of individual generators to loads. Since $Q_L = A_L Q$ and $Q = BQ_G$, we have

$$Q_L = A_L Q = A_L B Q_G = K Q_G \tag{15}$$

where, K is the contribution factor matrix of generators to loads, which represents what portion of the reactive power generation is used by loads.

Then the reactive power charge($\beta_{Q,i}$) at bus i contributed from capital costs of reactive power supporting equipment is calculated by

$$\beta_{Q,i} = \sum_{j=1}^{N_G} K_{ij} CC_{Q,j} \tag{16}$$

where, K_{ij} is an element of the K matrix, and $CC_{Q,i}$ is the capital cost for reactive power at bus i, which calculation is shown in detail in the next section. Finally, the total cost of reactive power($TOT_{Q,i}$) at bus i can be computed by summing up the real-time reactive power charge and the charge from a capital cost allocation method using reactive power flow tracing.

$$TOT_{Q,i} = \lambda_{Q,i} + \beta_{Q,i} \tag{17}$$

4. Case Study

The proposed algorithm is tested on a 6-bus sample system shown in Fig. 1. There are three generators on buses 1, 2, and 3 respectively. The sample system data including line data, and generation and load at all buses are given in Appendix.

Investment capital costs of individual generators are assumed to be 200$/MW-h, 140$/MW-h and 170$/MW-h, respectively. The lower and upper limits of active and reactive power at each generator are given as follows.

$50MW < P_{G1} < 200MW$, $-100MVAR < Q_{G1} < 150MVAR$
$37.5MW < P_{G2} < 150MW$, $-100MVAR < Q_{G2} < 150MVAR$
$45MW < P_{G3} < 180MW$, $-100MVAR < Q_{G3} < 150MVAR$

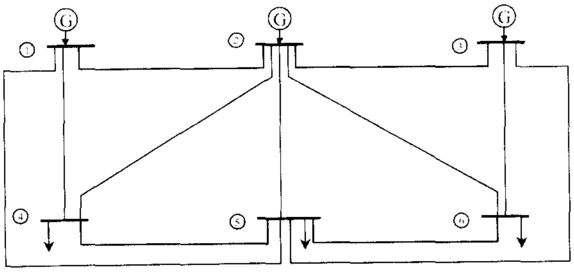

Fig. 1 6-bus sample system

The cost functions of individual generators for active power production are as follows.

$$C_1(P_{G1}) = 0.00533P_{G1}^2 + 11.669P_{G1} + 213.1$$
$$C_2(P_{G2}) = 0.00889P_{G2}^2 + 10.333P_{G2} + 200.0$$
$$C_3(P_{G3}) = 0.00741P_{G3}^2 + 10.833P_{G3} + 240.0$$

4.1 Result for the real-time pricing method

The OPF program is used to evaluate real-time pricing of reactive power. The OPF formulation was solved using a simulation package on MATLAB. Table 1 is the result of running the OPF program.

Thus, the charge of reactive power service for bus 4 is 73.3$/h, which is calculated from 100MVAR* 0.733$/MVAR-h. Similarly the charges for buses 5 and 6 are 80.1$/h and 49.7$/h, respectively.

Table 1 Result of real-time pricing method.

bus	Voltage Mag(pu) Ang(pu)	Generation P(MW) Q(MVAR)	Load P(MW) Q(MVAR)	λ ($/MVAR-hr) Q
1	1.100 1.050	79.26 61.20	- -	-
2	1.099 0.074	122.04 122.34	- -	-
3	1.100 0.516	112.35 100.40	- -	-
4	1.008 -1.880	- -	100.00 100.00	0.733
5	0.992 -2.675	- -	100.00 100.00	0.801
6	1.016 -2.354	- -	100.00 100.00	0.497

4.2 Result for the proposed method

Reactive power flow tracing method starts from power flow solutions. Table 2, as a result of equation (15), shows the contributions of generators to reactive power demands at load buses. Reactive power demand at bus 4 is 100MVAR, 34.9% of which is contributed from the generator 1.

Next, we need calculations of capital costs of generators for reactive power production. The capital costs of generators are usually given in terms of the active power in $/MW, but it does not give us the capital costs for reactive power. Here is adopted one of the joint cost allocation approaches presented in [10] to find the portion for reactive power. The triangular relationship like $S = \sqrt{P^2 + Q^2}$ between active, reactive and apparent power of a generator is used. The capital cost for reactive power(CC_Q) can be computed from the following equation.

Table 2 Contributions of generators to reactive power demands.

Bus	Load	Supplied by Gen.1	Supplied by Gen.2	Supplied by Gen.3
1	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000
4	100.0000	34.8534	57.8301	0.0000
5	100.0000	26.9096	27.5999	25.9032
6	100.0000	0.0000	30.6462	65.8624
Total	300.0000	61.7630	116.0762	91.7656

$$CC_Q = CC_P * \tan(\cos^{-1} 0.9) \tag{18}$$

where CC_P is the capital cost of a generator for active power, and 0.9 is the value of nominal power factor of the generator.

Therefore, the capital cost of generator 1 for reactive power is 96.9$/MVAR-h (=200*tan(cos^{-1}0.9)). And the capital costs of generators 2 and 3 for reactive power are 67.8$/MVAR-h and 82.3$/MVAR-h, respectively.

Then, the reactive power flow tracing method gives us the sharing portions of reactive loads for the capital costs of reactive power. The sharing portion of bus 4 can be calculated as follows.

$$\cdot\ bus\ 4 \rightarrow \frac{34.8534}{61.7630} \times 96.9 + \frac{57.8301}{116.0762} \times 67.8 \fallingdotseq 88.46(\$/h)$$

The sharing portions of buses 5 and 6 for the reactive power capital costs are 81.57$/h and 76.97$/h, respectively.

Finally, the reactive power charges for individual loads can be found by summing up the charges of real-time pricing method and the charges of the reactive power flow tracing method. So, the reactive power charge for bus 4 is 161.76$/h (=73.3$/h + 88.46$/h).

Table 3 illustrates the results of the two methods together. There exists a large difference between the two results since the real-time pricing method ignores the costs contributed from reactive power supplying equipment. These fixed costs are much larger than operational costs in reactive power management.

Table 3 Comparison of the charges calculated by each method ($/h).

bus \ method	real-time pricing	proposed method	Difference
4	73.3000	161.76	88.46
5	80.1000	161.67	81.57
6	49.7000	126.67	76.97

5. Conclusion

As electric power industries have been moving from regional monopolies to competition modes, it becomes necessary to calculate accurate costs of power generation, transmission, distribution and ancillary services.

In this paper, we proposed a new charging method of reactive power management, which is required for system security and voltage control. The traditional real-time pricing method turns out not to recover fully the embedded costs of reactive power. The proposed algorithm, composed of two parts, attempted to recover investment capital costs of reactive power supporting equipment by a reactive power flow tracing method, as well as to recover operational costs by a real-time pricing method of OPF algorithm. A charge of active power at each bus is generally calculated by the real-time pricing method, but the real-time pricing method is not enough to calculate reactive power charges. It turns out that reactive power charges are mostly due to recovery of capital costs and slightly due to recovery of operational costs.

The proposed method contributes power companies to calculate an ancillary service cost of reactive power management and makes power markets active in offering an adequate signal of reactive power charge under power system deregulation.

Acknowlegements

This work was sponsored by Next-Generation Power Technology Center supported by Ministry of Science and Technology and Korea Science and Engineering Foundation, and by Electrical Engineering & Science Research Institute (EESRI) that is supported by Korea Electric Power Corporation (KEPCO). (EESRI-00-018)

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Appendix. Data for the 6-bus sample system.

1. Bus data (rating : 345kV, 100MVA)

Bus	Bus type	Voltage schedule (pu V)	P_{gen} (pu MW)	P_{load} (pu MW)	Q_{load} (pu MVAR)
1	Swing	1.05			
2	Gen.	1.05	1.0	0.0	0.0
3	Gen.	1.07	1.0	0.0	0.0
4	Load		0.0	1.0	1.0
5	Load		0.0	1.0	1.0
6	Load		0.0	1.0	1.0

2. Line data

From bus	To bus	R(pu)	X(pu)	B(pu)
1	2	0.10	0.20	0.04
1	4	0.05	0.20	0.04
1	5	0.08	0.30	0.06
2	3	0.05	0.25	0.06
2	4	0.05	0.10	0.02
2	5	0.10	0.30	0.04
2	6	0.07	0.20	0.05
3	5	0.12	0.26	0.05
3	6	0.02	0.10	0.02
4	5	0.20	0.40	0.08
5	6	0.10	0.30	0.06

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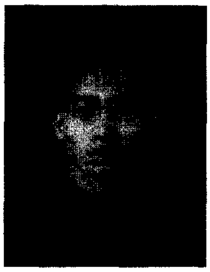

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