Optimal Siting of UPFC for Reducing Congestion Cost by using Shadow Prices

Kwang-Ho Lee and Jun-Mo Moon

Abstract - As competition is introduced in the electricity supply industry, congestion becomes a more important issue. Congestion in a transmission network occurs due to an operating condition that causes limit violations on the transmission capacities. Congestion leads to inefficient use of the system, or causes additional costs (Congestion cost). One way to reduce this inefficiency or congestion cost is to control the transmission flow through the installation of UPFC (Unified Power Flow Controller). This paper also deals with an optimal siting of the UPFC for reducing congestion cost by using shadow prices. A performance index for an optimal siting is defined as a combination of line flow sensitivities and shadow prices. The proposed algorithm is applied to the sample system with a condition, which is concerning the quadratic cost functions. Test results show that the siting of the UPFC is optimal to minimize the congestion cost by the proposed algorithm.

Keywords - Congestion Cost, UPFC, Optimal Power Flow, Shadow Price

1. Introduction

Electric power utilities in many countries have focused considerable interest on the institutional structure of open transmission access (OTA). Competition in OTA allows the market participants easy access to the transmission system in a non-discriminatory and equitable manner.

Transmission congestion occurs when transmission line power flows reach the finite network capacities, and precludes the simultaneous delivery of power from an associated set of power transactions [1-3]. Congestion can result in an overall increase in the cost of power delivery. Such a congestion cost can cause large differences in spot prices in a system that is under severe stress and possibly in need of transmission expansion [4]. Also the congestion cost can be much greater than the cost of transmission losses. Therefore, congestion is quite an important factor in the total cost of operating the power system, and has been at the center of the extensive debate on OTA.

Flexible AC transmission system (FACTS) devices, which were first defined by Hingorani [5] in 1988, have a large potential ability to make power systems operate in a flexible, secure and economic way. Presently, the studies on FACTS are concerned with FACTS device developments and their impacts on the power systems. It is also significant to study the impact of the FACTS devices on improving the performance of power systems such as optimization (OPF) [6-8].

An optimal power flow (OPF) is a procedure to determine the optimal steady state operation of a power system so as to minimize a given objective function while satisfying a set of physical and operating constraints [9]. OPF plays a key role in the pricing mechanism that is important for providing services based on the standards of quality, reliability, and security in the OTA environment. The need for OPF has been increased to solve the problems of today's power system of OTA such as calculating spot prices and performing the necessary decomposition of power prices into components reflecting the generation, losses and congestion [10].

This paper presents a method of utilizing unified power flow controller (UPFC) by adjusting the power flows on the congested lines to reduce the congestion cost. UPFC is one of the main types of FACTS, that is adequate for controlling line power flows. Some previous papers [6,7] presented methods to incorporate the power flow control needs of FACTS in studying the optimal active power flow However, those considered only the FACTS that are installed at a pre-defined position. This paper focuses on the optimal siting for UPFC to be installed to reduce the congestion cost.

In order to determine the optimal siting, not only are the sensitivities of UPFC parameters to the power flows on the congested lines, but also shadow prices on the congested lines are introduced in the new performance index in this paper. The shadow price is a dual variable represented as a Lagrange multiplier corresponding to a constraint, and indicates a sensitivity that means a marginal change in the cost function due to a change in the finite transmission capacity [11,12]. The new performance index is proposed to each candidate siting where a UPFC is to be installed in the form of a combination of power flow sensitivity and shadow price to the congested lines. Therefore the index gives a measure of the optimal siting of a UPFC to minimize the congestion cost. The simulation studies on the

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IEEE 14-bus system are presented and discussed to show the effectiveness of the proposed method.

2. Modeling of UPFC

The UPFC is composed of a shunt transformer, a series transformer and two switching converters; these converters are operated from a common dc link provided by a dc storage capacitor(V_{dc}). In Fig.1, m and δ refer to amplitude modulation index and phase-angle of the control signal of each converter respectively. And then Converter II injects an ac voltage with controllable magnitude and phase angle in series with the transmission line via a series transformer. Converter I supply or absorb the real power demand by converter II at the common dc link. It can also supply or absorb controllable reactive power [13].

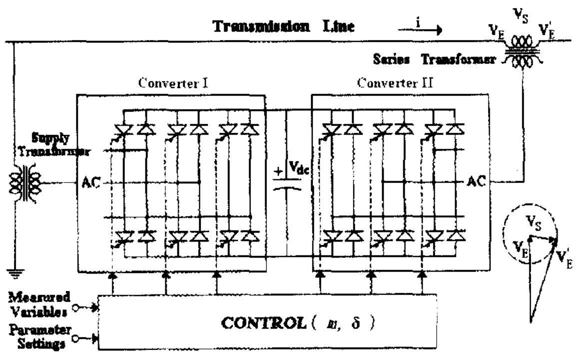

Fig. 1 Common structure for UPFC

Neglecting UPFC losses, in steady-state the UPFC is equivalent to a voltage source($\overline{V}_S$) inserted in series with line and a current source($\overline{I}_S$) connected in shunt with line as Fig.2. Where $\overline{V}_E\,'$ is a fictitious voltage behind the series reactance and X_S is the effective reactance seen from the line side of the series transformer.

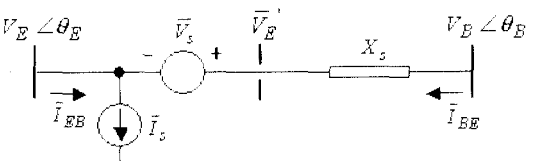

Fig. 2 Steady-state equivalent circuit diagram of UPFC

It is derived the UPFC model as Fig.3 for power flow. Let's consider the Fig.3 (a) that is used to maintain a prespecified power flow from E-bus to B-bus, and to regulate the B-bus voltage at specific value. Using power flow terminology, B-bus is a P-V bus and E-bus is a P-Q bus. A power flow analysis is performed where the UPFC is derived the Decoupled Model as given in Fig.3 (b). The prespecified value (P_E, Q_E, P_B and V_B) instead of m and δ can be used UPFC control variable [14].

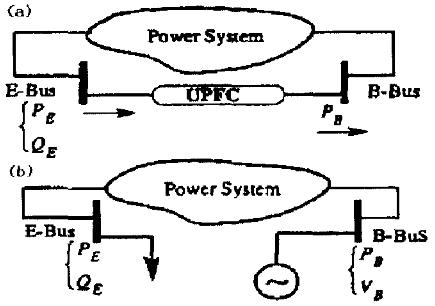

Fig. 3 Decoupled Model of UPFC for Power Flow

Where the pre-specified value is:
P_E , $P_{B,}$: Active Power of B-Bus and E-Bus
Q_E : Reactive Power of E-bus
V_B : Voltage Amplitude of B-bus

3. Optimal Power Flow

3.1 Formulation of OPF

The objective of active power optimization is to minimize production cost while observing the transmission line and the generation active power limits. The problem can be stated as follows:

$$\textit{minimize} \quad F_T = \sum_{i=1}^{m} C_i(P_{Gi}) \tag{1}$$

$$\textit{subject to} \quad \sum_{i=1}^{m} P_{Gi} - \sum_{k=1}^{n} P_{Dk} - P_L = 0 \tag{2}$$

$$P_l \le P_l^{\max} \tag{3}$$

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \tag{4}$$

Where n and m are the number of system buses and number of generating units respectively, and
$C_i(P_{Gi})$:Production cost of the unit at bus i,
F_T :Total production cost of m generators,
$P_{Gi}^{\min}$, $P_{Gi}^{\max}$:Active power limits of the unit at bus i,
P_{Dk} :Active power load at bus k,
P_L :Network active power loss,
P_l, $P_l^{\max}$:Active power flow and its limit on line l.

The augmented Lagrangian is:

$$L(P_{Gi}) = F_T(P_{Gi}) + \lambda(\sum_{k=1}^{n} P_{Dk} + P_L - \sum_{i=1}^{m} P_{Gi}) + \sum_{l=1}^{N_l} \mu_l(P_l - P_l^{\max})$$
$$+ \sum_{i=1}^{m} [\mu_i^{\min}(P_{Gi}^{\min} - P_{Gi}) + \mu_i^{\max}(P_{Gi} - P_{Gi}^{\max})] \tag{5}$$

where the Lagrangian multipliers are:
λ : for the power balance equation, referred to system lambda,
$\mu_i^{\min}(\mu_i^{\max})$:for lower(upper) active power limits of the unit at bus *i*,
μ_l : for active power flow limit on line *l*,
and N_l is the number of transmission line flow violations.

In this paper the main objective is to determine the optimal siting of UPFC for reducing the congestion cost. This problem is not solved simultaneously with the OPF problem, but solved by using the results of the OPF calculation. We propose that the Lagrangian multipliers for power flow limit on lines can be effective to determine the optimal siting of UPFC.

3.2 Shadow Prices

In order to examine the meaning of μ_l, we consider a small change in $P_l^{\max}$ on the congested line *l* at the optimal solution satisfying the KKT (Karush-Khun-Tucker) condition. Thus the different optimal solution, P_G', can be obtained which corresponds to the changed $P_l^{\max\prime}$. The change in Lagrangian resulting from the small change in line flow limit satisfies

$$\Delta L(P_G) = \Delta F_T + \mu_l \Delta P_l \tag{6}$$

In (6), it is assumed that the loss does not change and the binding situation on $\mu_i^{\min}(\mu_i^{\max})$ does not change either. If the change in line flow limit is sufficiently small, $\Delta L(P_G)$ in (6) goes to zero. Thus we have

$$\mu_l = -\frac{\Delta F_T}{\Delta P_l} \tag{7}$$

Equation (7) means that the Lagrangian multiplier μ_l is the sensitivity of the optimal cost with respect to small change in the line flow constraint. Thus, μ_l may be equivalently considered as the marginal price of the power flow control on the congested line. So it can be referred to a shadow price of the power flow control. This meaning of μ_l is applied to the new performance index for determining the optimal siting of UPFC. With the larger value of μ_l acting as a power flow control on the line, the more effective is the reduction of the congestion cost.

4. Optimal Siting of UPFC

4.1 Power Flow Control of UPFC

This paper regards UPFC as a means to control the line power flow. The control parameter is the variable real power of a UPFC (difference between i-j line power and the injection power of UPFC) that is denoted by u, while x is the vector of state variables (voltage angle of the buses). Thus, the power flow equation of UPFC can be formulated as follows:

$$P_l = G_l(u, x) \tag{8}$$

$$h(u, x) = 0 \tag{9}$$

$$u^{\min} \le u \le u^{\max} \tag{10}$$

Where (8) represents the line flow equation on line *l* based on the model of Fig. 3, while (9) represents the power flow equations. The relation of the changes in the line flow and control parameter is derived as follows:

$$\Delta P_{line} = \frac{dG}{du} \cdot \Delta u \tag{11}$$

$$= \left\{ \frac{\partial G}{\partial u} - \frac{\partial G}{\partial x} \cdot \left[\frac{\partial h}{\partial x} \right]^{-1} \cdot \frac{\partial h}{\partial u} \right\} \cdot \Delta u \tag{12}$$

$$= \left[G_u - G_x h_x^{-1} h_u \right] \cdot \Delta u \tag{13}$$

$$= S_{Gu} \cdot \Delta u \tag{14}$$

Where ΔP_{line} is a vector of line flow changes, *G* is a vector of functions for line power flows, and S_{Gu} $(= G_u - G_x h_x^{-1} h_u)$ is the matrix of the line flows sensitivities with respect to control variables *u*, while G_u, G_x, h_u and h_x are Jacobian matrices.

The (*i,j*) element of the sensitivity matrix, $S_{Gu}(i, j)$ indicates the degree of power flow reduction on line *i* from increasing control of u by a unit at line *j*. So the sensitivities can be utilized to find an effective siting of UPFC for power flow control on a certain transmission line. However, an amount of the congestion cost change following such a control cannot be given directly. That is, the UPFC that is installed at the optimal siting for controlling the power flow of congested lines is not always optimal for reducing the congested cost.

On the other hand, the shadow prices as (7) mean the marginal relation between a congestion cost and the power flow controls at an optimal condition. Thus, a combination of line flow sensitivities and shadow prices is proposed in this paper as a performance index for an optimal siting of UPFC.

4.2 Performance Index for Siting of UPFC

The short-run marginal cost (SRMC) of power production is important for appropriate price signals in an OTA environment. The congestion cost is one of the components of the SRMC. According to the marginal cost theory [15], the SRMC for active power at bus i (MC_i) is

$$MC_i = \lambda - \lambda \frac{\partial P_L}{\partial P_i} - \sum_{l=1}^{N_l} \mu_l \frac{\partial P_l}{\partial P_i} \tag{15}$$

$$= \frac{\partial C_i}{\partial P_i} - \mu_i^{\min} + \mu_i^{\max} \tag{16}$$

where λ, μ_l, $\mu^{\min}$, $\mu^{\max}$ are equivalent to those of (5).

Equation (15) corresponds to the overall system buses, while (16) corresponds to only the bus with a generating unit. From (15), the SRMC has three components, the first is the system lambda. The second has relation to the incremental loss caused by transmitting active power to bus i. And, the third is associated with the network thermal loading constraints, which corresponds to the congestion cost in an OTA environment. In the term of congestion cost, the shadow price of the constraint is multiplied by the marginal flow along the line caused by an extra MW of demand or generation at the node.

Therefore the congestion cost (CC) from the overall small changes of injection powers is derived by the summation with respect to overall buses. Thus we have

$$\Delta CC = \sum_{l=1}^{N_l} \mu_l \left\{ \sum_{i=1}^{n} \left(\frac{\partial P_l}{\partial P_i} \Delta P_i \right) \right\} \tag{17}$$

$$= \sum_{l=1}^{N_l} \mu_l \Delta P_l \tag{18}$$

Equation (18) means that the congestion cost can be reduced by the line flow control in the congested line at an optimal condition. In order to relate the UPFC control to the congestion cost, some part of the sensitivity equation (14) for UPFC is substituted for ΔP_l. Thus we have

$$\Delta CC = \left(\sum_{l=1}^{N_l} \mu_l S_{Gu}^l \right) \cdot \Delta u \tag{19}$$

$$= S_{cu} \cdot \Delta u \tag{20}$$

Where S_{Gu}^l is the vector of line flow sensitivities on the congested line l with respect to control variables u, while $S_{cu} (= \sum_{l=1}^{N_l} \mu_l S_{Gu}^l)$ is the vector of the congestion cost sensitivities with respect to control variables u. The vector Δu corresponds to the candidate lines where a UPFC is to be installed. So the line corresponding to the element of the maximum value in sensitivity vector S_{cu} can be selected as an optimal siting of a UPFC for reducing the congestion cost.

5. Test Results and Discussion

The IEEE 14-bus system shown in Fig. 4 is used to demonstrate the application of the proposed algorithm. The generation cost functions are modeled as the conventional quadratic type. For simplicity, the load powers are assumed to have only active powers in Table 1. All lines are assumed to have only reactance as listed in Table 2. The MW flows on the lines are assumed to be constrained to the values as listed in Table 2.

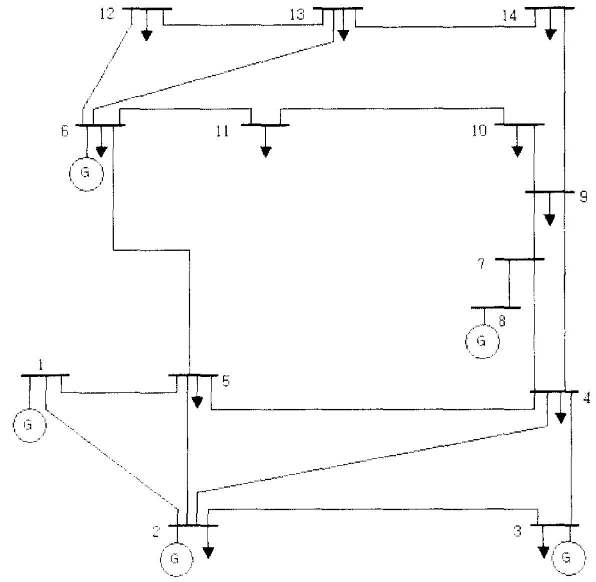

Fig. 4 Network diagram of test system (IEEE 14-Bus)

Table 1 Load data of test system

Buses	1	2	3	4	5	6	7
MW Loads	0	21.7	94.2	47.8	7.6	11.2	0
Buses	8	9	10	11	12	13	14
MW Loads	0	29.5	9.0	3.5	6.1	13.5	14.9

Table 2 Line data of test system

Lines	From -To Bus	Reactance [pu]	MW capacities
1	1-2	0.05917	60
2	1-5	0.17388	60
3	2-3	0.17632	40
4	2-4	0.22304	25
5	2-5	0.17103	40
6	3-4	0.04211	30
7	4-7	0.20912	40
8	4-5	0.04211	40
9	4-9	0.55618	20
11	6-11	0.19890	25
12	6-12	0.25581	20
13	6-13	0.13027	30
14	7-9	0.11001	50
15	7-8	0.17615	60
16	9-14	0.27038	30
17	9-10	0.08450	20
18	10-11	0.19207	20
19	12-13	0.19988	20
20	13-14	0.34802	20

The quadratic functions of the generation costs are given as Table 3 with the minimum and maximum active power outputs of the generators, where the cost functions are

$f_i = a_i + b_i P_{gi} + c_i P_{gi}^2$.

Table 3 Generation data of test system

Gen. No.	$P_G^{\min}$ [MW]	$P_G^{\max}$ [MW]	a	b	c
1	10	100	0	2.00	0.00315
2	20	50	0	1.75	0.01750
3	15	80	0	1.00	0.06250
6	10	45	0	3.25	0.00834
8	10	45	0	3.00	0.02500

Four cases of optimizations have been studied in this condition. Case I is the economic dispatch, case II is the conventional OPF without any UPFC, while case III and case IV are the OPF with a UPFC installed. In case III, only the effectiveness of power flow control of a UPFC which is represented as in (14) is considered as a performance index for an optimal siting. In case IV, the sensitivities in (20) are used as a new performance index proposed in this paper. In the test studies, the most compensated active power of the line is assumed to be ±5MW of its original active power uncompensated.

Table 4 Comparisons of computational results of each case

Case. No.	P_{G1} [MW]	P_{G2} [MW]	P_{G3} [MW]
I	100.000	50.000	21.714
II	100.000	46.805	43.702
III	100.000	50.000	38.664
P_{G6} [MW]	P_{G8} [MW]	F_T^* [\$]	ΔF_T^{**} [\$]
45.000	34.286	743.031	0.0
23.193	45.000	781.395	38.364
25.409	44.927	768.051	25.020
45.000	34.286	743.031	0.0

F_T^* : total generation cost
ΔF_T^{**} : increase in generation cost from economic dispatch

The optimization results of the four cases are listed in Table 4. The optimal dispatch of generation power, total generation cost, and the increase in generation cost of each case are shown for comparison.

In case I, since the line constraints are neglected in economic dispatch, its optimal cost (\$743.031) can be lower than the other three cases. But the power flows on the line 4 and 8 are 26.11 and 49.96MW respectively, which overflow the maximum limits. Thus, such a generation dispatch without line power controls is not permissible.

In case II, the conventional OPF gives the optimal dispatch satisfying the line flow constraints. The generation cost is \$781.395 that is higher by \$38.364 than that of the economic dispatch. This increased cost results from the congestion on the lines. The shadow prices on the congested lines 4 and 8 are 4.1296 and 6.3669 respectively. In order to reduce this congestion cost, we consider the installation of a UPFC at an optimal siting.

Table 5 shows the sensitivities, S_{Gu} of the congested lines with respect to the candidate sitings (line 1 ~ line 20) where a UPFC is to be installed. The sensitivities are computed at the state of the generation dispatch in case II. The values of performance indices in case III and case IV are listed in Table 6. In case III, we select the siting which is the most effective to control the power flow on the congestion lines. The siting of line 3 is selected because the index of line 3 has the largest value in Table 6, which is computed by only the sensitivities in Table 5. The generation cost (\$768.051) becomes lower than that of the OPF results without any UPFC. However that cost can be much lower by using the proposed performance index.

Table 5 Sensitivities of congested lines

Lines / Sitings	Line 4	Line 8	Lines / Sitings	Line 4	Line 8
1	-0.0026	0.0051	11	0.0005	0.0050
2	0.0026	0.0050	12	0.0001	0.0006
3	0.0038	0.0058	13	0.0001	0.0014
4	-0.0100	0.0058	14	-0.0004	-0.0041
5	0.0030	-0.0058	15	0.0000	0.0001
6	-0.0037	-0.0058	16	-0.0004	-0.0041
7	0.0004	0.0041	17	-0.0005	-0.0049
8	0.0039	-0.0100	18	0.0005	0.0050
9	-0.0003	-0.0030	19	0.0001	0.0006
10	0.0008	0.0083	20	0.0004	0.0041

Table 6 Performance index of candidate sitings

Cases / Sitings	Case III	CaseIV	Cases / Sitings	Case III	CaseIV
1	0.0025	0.0217	11	0.0005	0.0050
2	-0.0024	-0.0211	12	0.0001	0.0006
3	**0.0096**	0.0526	13	0.0001	0.0014
4	0.0042	0.0045	14	-0.0004	-0.0041
5	-0.0028	-0.0246	15	0.0000	0.0001
6	-0.0095	-0.0526	16	-0.0004	-0.0041
7	0.0045	0.0278	17	-0.0005	-0.0049
8	-0.0061	-0.0476	18	0.0005	0.0050
9	-0.0033	-0.0203	19	0.0001	0.0006
10	0.0091	**0.0561**	20	0.0004	0.0041

In case IV, a UPFC is installed at the siting that is optimal to decrease the congestion cost. As in Table 6, the largest value of performance index corresponds to line 10, so it is selected. The value is computed by weighted summation of the sensitivities in Table 6, where the weights for the congested lines are the shadow prices correspondingly. The OPF calculation of the system with a UPFC at line 10 results in the generation cost of \$743.031 which is much lower than that of case III.

6. Conclusions

This paper has presented an algorithm for reducing congestion cost which occurs in a competitive electricity market. Since congestion occurs when power flows reach the capacities of transmission lines, the UPFC is used to control the line power flow. The UPFC compensated line is represented as the line with a variable reactance. In order to determine the optimal siting of a UPFC installation, two factors are incorporated into a performance index: one is the sensitivity matrix of a UPFC with respect to the congested lines and the other is the shadow price corresponding to the congested line.

In the test studies, the proposed algorithm is applied to the sample system (IEEE 14-bus) with the quadratic functions of generation cost. Through comparing the optimization results of the four cases, the performance index proposed in this paper is verified to be effective for reducing the congestion cost.

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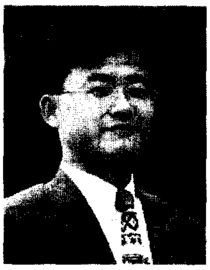

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