### LMI 이론에 의한 삼관성 시스템의 진동억제

# Vibration Suppression Control of 3-mass Inertia System by using LMI Theory

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### 요 약

일반적으로 관성시스템의 제어 문제는 결국, 시스템 자체에서 발생하는 진동을 최대한 억제하면서 빠른 시간에 출력이 기준입력을 추종하는데 있다. 이 경우 문제로 되는 것은 시스템의 모델링 과정에서 발생하는 플랜트의 불확실성과 parameter 변동이다. 여기서는 일반적인 강인한 제어기 설계 이론의 하나인  $H_{\infty}$  이론이 가지는 단점인 제어기의 보수성을 극복하면서, 동시에 출력의 과도응답특성을 개선하기 위한 방법으로  $H_2$  이론을 병용하고 이를 LMI 이론으로 해석하였다. 이 과정에서 3 관성시스템에 LMI 이론을 적용하기 위한 일반화플랜트의 모형을 제시하고 이것의 유효성을, 모델의 불확실성과 parameter 변동을 동시에 고려한 simulation을 통하여 확인하였다.

#### **Abstract**

Generally, it is said that control of the inertia system is to track the reference input quickly while suppressing the vibration due to the system itself. In this case, the difficulty for designing a controller is caused by modeling uncertainty and parameter variation. The purpose of this paper is to propose a design method to suppress the vibration of three-mass inertia system based on the LMI theory. That is, the generalized plant model by which we can cope with conservativeness of the existing  $H_{\infty}$  theory is proposed and analyzed in terms of LMI. The results of simulation for the three-mass inertia system show that the proposed design approach is quite effective under the given situations.

Keywords: 3-mass inertia system, robust servo, LMI theory

### I. Introduction

In the past several years, the  $H_2$  and/or  $H_\infty$  control have attracted many researchers' attentions. Also, its effectiveness has been reported in various application fields in these years. The  $H_\infty$  control theory provides us a quite powerful tool for shaping the loop gain in the frequency domain or obtaining the robust stability property. Therefore, it is quite useful to obtain satisfactory feedback properties such as low sensitivity. On the other hand, the  $H_2$  optimal control theory has been heavily

studied since 1960's as the LQG optimal control problem. The  $\rm H_2$  norm performance measure seems to be suitable for obtaining good command response.

For a servo system design, the following three specifications are of practical interests: (1) internal stability of the closed-loop system which must be guaranteed; (2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance attenuation; (3) desired transient and steady-state properties such as robust tracking to reference inputs.

The  $H_x$  control is a suitable technique to achieve the first two specifications, because they can be naturally expressed as  $H_\infty$  norm constraints. However, since the  $H_\infty$  control is based on the maximum singular value of the transfer function matrix from disturbance to evaluation signals, it is inevitable that the response should be rather conservative. Therefore, it is required to alleviate this phenomenon in order to meet the third specification. Recently, it has been proved that, by introducing  $H_2$  specification into the  $H_\infty$  design, we could simultaneously benefit from the  $H_2$  and  $H_\infty$  control design method [1]. This approach can be achieved by using the so- called LMI (Linear Matrix Inequalities) theory, and is generally called a mixed  $H_2/H_\infty$  control. In consequence, a designer can arbitrarily determine the trade-off between  $H_2$  (e.g. noise rejection) and  $H_\infty$  (e.g. robust stability) performance of the closed loop system.

The formulation of the mixed  $H_2/H_\infty$  control problem for servo system is presented in Section II. The main results are given in Section III, where the structure of generalized plant for robust tracking is proposed and, we also show that there exists a solution to the problem by virtue of an LMI approach. In Section IV, after designing a robust controller based on the proposed method for the three-mass inertia problem, we analyze the results of simulation and check the validity of the proposed structure.

### II. Mixed H<sub>2</sub>/H<sub>∞</sub> Optimal Design Problem by LMI

The basic block diagram used in this paper is given in Fig.1, in which the generalized plant  $\,P\,$  is given by the state-space equations

$$P: \begin{cases} \dot{x} = Ax + B_{1}w + B_{2}u \\ z_{\infty} = C_{\infty}x + d_{\infty 1}w + d_{\infty 2}u \\ z_{2} = C_{2}x + d_{21}w + d_{22}u \\ y = C_{y}x + d_{y1}w \end{cases}$$
(1)

where  $\mathbf{x} \in R^n$  is state vector,  $\mathbf{u}$  is the control input,  $\mathbf{w}$  is an exogenous input (such as a disturbance input, sensor noise etc.),  $\mathbf{y}$  is the measured output and  $\mathbf{z} = [z_{\infty} \ z_2]^T$  is a vector of output signal related to the performance of the control system  $(z_{\infty}$  is related to the  $H_{\infty}$  performance and  $z_2$  is related to the  $H_2$  performance).

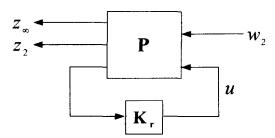


Fig. 1 Block diagram of Mixed H<sub>2</sub>/H<sub>∞</sub> control

Let  $T_{zw}$  be the closed transfer function from w to z for the system P closed with the output-feedback control law  $u=K_ry$ . Our goal is to compute a dynamical output feedback controller  $K_r$ 

$$K_r : \begin{cases} \dot{\mathbf{x}}_K = \mathbf{A}_K \mathbf{x}_K + \mathbf{B}_K y \\ u = \mathbf{C}_K \mathbf{x}_K + d_K y \end{cases}$$
 (2)

that simultaneously meets  $H_2$  and  $H_\infty$  performance on the closed-loop behavior. The closed-loop system  $T_{zw}$  has the following description

$$T_{zw} : \begin{cases} \dot{\mathbf{x}}_{cl} = \mathbf{A}_{cl} \mathbf{x}_{cl} + \mathbf{B}_{cl} w \\ z_{\infty} = \mathbf{C}_{cl1} \mathbf{x}_{cl} + d_{cl1} w \\ z_{2} = \mathbf{C}_{cl2} \mathbf{x}_{cl} + d_{cl2} w \end{cases}$$
(3)

The problem we concerned with can be summarized as minimizing the  $H_2$  norm of the channel  $w \to z_2(:T_2)$ , while keeping the bound  $\gamma$  on the  $H_{\infty}$  norm of the channel  $w \to z_{\infty}(:T_{\infty})$ , i.e.

$$\min \|T_2\|_2$$
 subject to :  $\|T_\infty\|_\infty < \gamma$ 

Since this problem can be reformulated as a convex optimization problem<sup>[1]</sup>, the optimal solution under the given value of  $\gamma$  can be obtained through LMI. As efficient interior-point algorithms are now available to solve the generic LMI problems, the mixed  $H_2/H_\infty$  problem can be solved without much difficulty in order to find the best trade-off between the  $H_2$  and  $H_\infty$  minimization.

## III. The Proposed Structure for LMI Design

In order to find a robust servo controller  $K_r$  that satisfies

desirable transient response ( $H_2$  performance) as well as desired feedback properties ( $H_\infty$  performance), we adopt the mixed  $H_2/H_\infty$  control system rather than the conventional  $H_\infty$  control theory. For the purpose of this, we, first, divide the control objectives into each  $H_\infty$  and  $H_2$  performance criterion, and then describe the two criteria as one formation. In other words, a new structure for the mixed  $H_2/H_\infty$  control is required to deal with two criteria simultaneously. Here, we introduce the following interconnection for robust control system, on which a controller satisfying two criteria is designed.

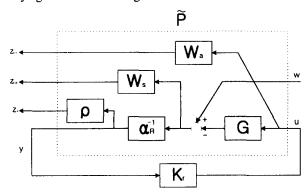


Fig. 2 The proposed generalized plant for Mixed  $H_2/H_{\infty}$  control

A.  $H_{\infty}$  control Problem: In Fig.2,  $\alpha_R^{-1}$  is added in the loop in order to meet the internal model principle [2].  $W_a(s)$  denotes a weighting function related to the plant uncertainty (in this case, additive uncertainty is used) and  $W_s(s)$  is a sensitivity weighting function. We can summarize the robust tracking  $H_{\infty}$  control problem as follows:

(S1)  $K_r(s)$  stabilizes  $\widetilde{P}(s)$ .

(S2) 
$$\|T_{\infty}(s)\|_{\infty} = \left\|\frac{T_{z_{\infty},w}(s)}{T_{z_{\infty},w}(s)}\right\|_{\infty} < \gamma$$

where  $T_{z_{\infty 1}w}(s)$  denotes the transfer function from w to  $z_{\infty 1}$  and is related to robust stability requirement (for additive uncertainty) [2]

$$\|T_{z_{\infty 1}w}\|_{\infty} = \|(1 + GK)^{-1}KW_{a}\|_{\infty} < \gamma$$

$$K = \alpha_{B}^{-1}K_{c}$$
(4)

The nominal performance condition is reflected by  $\left\|T_{z_{\infty 2^w}}(s)\right\|_{\infty}<\gamma \,, \text{ where } T_{z_{2_\infty w}}(s) \text{ denotes the transfer function}$  from w to  $Z_{\infty 2}$ . In this case, if (S2) is satisfied under the

condition of y=0.5, then robust performance can be guaranteed outright, since (S2) will satisfy the SISO robust performance test for additive uncertainty given by Zhou, et al., [3]

$$||T_{z_{\infty 1}}(s)| + |T_{z_{\infty 2}}(s)||_{\infty} < 1$$
 (5)

The robust performance condition that was given in (5) is necessary and sufficient, and the left-hand side is actually the peak value of  $\mu$  [4]. In section 4, we will investigate this value as an index to robust performance when designing a control system for the three-mass inertia problem.

By virtue of the Bound Real Lemma the  $H_{\infty}$  norm of  $T_{\infty}(s)$  is smaller than  $\gamma$  if and only if there exists a symmetric positive definite matrix  $\mathbf{X}_{\infty}$  with

$$\begin{pmatrix} \mathbf{A}_{cl}^{T} \mathbf{X}_{\infty} + \mathbf{X}_{\infty} \mathbf{A}_{cl} & \mathbf{X}_{\infty} \mathbf{B}_{cl} & \mathbf{C}_{clx}^{T} \\ \mathbf{B}_{cl}^{T} \mathbf{X}_{\infty} & -\gamma & d_{cl\infty} \\ \mathbf{C}_{clx} & d_{clx} & -\gamma \end{pmatrix} < 0$$
 (6-a)

$$\mathbf{X}_{\infty} > 0 \tag{6-b}$$

where all the matrices  $\mathbf{A}_{cl}, \mathbf{B}_{cl}, \mathbf{C}_{cl\infty}$  and  $d_{cl\infty}$  are defined in (3).

**B.**  $H_2$  control **Problem**: The traditional  $H_2$  optimization attempts to minimize the energy of the system output when the system is faced with white Gaussian noise input. So, in order to design a controller adept at handling noises,  $H_2$  optimization should be considered. That is, the  $H_2$  norm minimization of the transfer function  $T_{z_2w}(s)$  from w to  $z_2$  in Fig.2 is to be taken as a controller design problem, where  $\rho$  is a varying parameter.

It is well known that the upper bound of  $\|T_{z_2w}\|_2^2$  is defined by  $tr(C_{cl}, W_a C_{cl}^T)$ , where  $W_a$  solves the Lyapunov equation

$$\boldsymbol{A}_{cl}\boldsymbol{W}_{o} + \boldsymbol{W}_{o}\boldsymbol{A}_{cl}^{T} + \boldsymbol{B}_{cl}\boldsymbol{B}_{cl}^{T} = \boldsymbol{0}$$
 (7)

Since  $\mathbf{W}_o < \mathbf{W}$  for any  $\mathbf{W}$  satisfying

$$\boldsymbol{A}_{cl}\boldsymbol{W} + \boldsymbol{W}\boldsymbol{A}_{cl}^T + \boldsymbol{B}_{cl}\boldsymbol{B}_{cl}^T < \boldsymbol{0}$$
 (8)

It is readily verified that  $\|T_{z_2w}\|_2^2 < v$  if and only if there exists W >0 satisfying (8) and  $tr(\mathbf{C}_{cl2}\mathbf{W}\mathbf{C}_{cl2}^T) < v$  [1]. With auxiliary parameter Q, the following analysis result has been known:

[Theorem 1]  $\mathbf{A}_{cl}$  is stable and  $\left\|T_{z_2w}\right\|_2^2 < v$  if and only if there exist symmetric  $X_2 = W^{-1}$  and Q such that

$$\begin{pmatrix} \boldsymbol{A}_{cl}^T \boldsymbol{X}_2 + \boldsymbol{X}_2 \boldsymbol{A}_{cl} & \boldsymbol{X}_2 \boldsymbol{B}_{cl} \\ \boldsymbol{B}_{cl}^T \boldsymbol{X}_2 & -\boldsymbol{I} \end{pmatrix} < 0$$
 (9-a)

$$\begin{pmatrix} \boldsymbol{X}_{2} & \boldsymbol{C}_{cl2}^{T} \\ \boldsymbol{C}_{cl2} & \boldsymbol{Q} \end{pmatrix} > 0$$
 (9-b)

$$tr(\mathbf{Q}) < v$$
 (9-c)

C. Mixed  $H_2/H_\infty$  Control: The mixed  $H_2/H_\infty$  controller  $K_r$  must satisfy both of the following criteria simultaneously

$$\left\| T_{z_{x} \nu} \right\|_{\infty} < \gamma \tag{10}$$

$$\left\|T_{z,\mathbf{w}}\right\|_{2} < \mathbf{v} \tag{11}$$

In order to keep the tractability of the constrained optimization problem, the following assumption is considered.

$$\boldsymbol{X}_{x} = \boldsymbol{X}_{2} \equiv \boldsymbol{P} \tag{12}$$

Therefore, notice that X can be written as

$$\boldsymbol{P} = \begin{bmatrix} X & * \\ * & * \end{bmatrix} = \begin{bmatrix} Y & * \\ * & * \end{bmatrix}^{-1} \tag{13}$$

where  $\dim(X) = \dim(Y) = \dim(A_{cl})$  and  $\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0$  is coupling

LMI. The solution X, Y, and Q under the constraints of (10), (11) is dependent on the value of  $\gamma$  and v, and can be obtained using any available software such as Matlab LMI toolbox [4].

### IV. Controller Design for three-mass Inertia System

We consider the problem given in [6]. The model treated in the problem is a coupled three-mass inertia system that reflects the dynamics of mechanical vibrations. A controller, by which robust performance (both in time and frequency-domain) condition must be satisfied, is required in order to solve the problem.

The three-mass inertia problem is shown in Fig.3, where the meanings of each symbol are as follows:

$$\theta_i$$
 ( $i = 1 \sim 3$ ) [radian]= angular displacement

 $\tau$ = control torque [Nm]

$$\tau_d$$
 ( $i = 1 \sim 3$ ) = torque disturbance

$$j_i (i = 1 \sim 3) [\text{kgm}^2] = \text{moment of inertia}$$

$$d_i$$
 ( $i = 1 \sim 3, a, b$ )[Nms/rad] = viscous-friction coefficient of motors,

 $k_i$  (i = a, b)[Nm/rad] = torsional coefficient of connection part

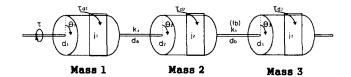


Fig.3 Coupled three-mass inertia system

By using these parameters, the equations of motion can be described as

$$\begin{split} j_{1}\ddot{\theta}_{1} &= -d_{1}\dot{\theta}_{1} - k_{a}(\theta_{1} - \theta_{2}) - d_{a}(\dot{\theta}_{1} - \dot{\theta}_{2}) + \tau + \tau_{d_{1}} \\ j_{2}\ddot{\theta}_{2} &= k_{a}(\theta_{1} - \theta_{2}) + d_{a}(\dot{\theta}_{1} - \dot{\theta}_{2}) - d_{2}\dot{\theta}_{2} - f_{b}(\theta_{2}, \theta_{3}) - d_{b}(\dot{\theta}_{2} - \dot{\theta}_{3}) + \tau_{d} \\ j_{3}\ddot{\theta}_{3} &= f_{b}(\theta_{2}, \theta_{3}) + d_{b}(\dot{\theta}_{2} - \dot{\theta}_{3}) - d_{3}\dot{\theta}_{3} + \tau_{d_{3}} \\ f_{b}(\theta_{2}, \theta_{3}) &= k_{b}(\theta_{2} - \theta_{3}) \end{split}$$

$$(14-a)$$

It is assumed that the control torque  $\tau$  is generated by voltage e [V] through a current amplifier, the equation of which is shown below [6].

$$\dot{\tau} = -a_a \tau + a_a e \tag{14-b}$$

We want to design a controller by which several design specifications are to be satisfied on condition that all of the 11 parameters are subject to change within the range of variation and there must exist hardware constraints.

### 4.1 Feedback Controller Design by Mixed H2/H Control

In this problem, there are 11 physical parameters that are assumed to be changing within the given range of variation. If all the variations are reflected in the controller design, the obtained controller may be considerably conservative as well as complex. Therefore, we first find out principal parameters ( $j_3$  and  $k_a$ ) that strongly affect the resonant frequency of the plant by plotting the frequency response curve, and then a robust controller dependent on these parameters is carried out by making use of the proposed structure for mixed  $H_2/H_\infty$  control. The robustness on variations of the other parameters is evaluated through simulation.

The parameter variations of  $j_3$  and  $k_a$  can be described by additive uncertainty such as

$$j_3 = j_{3.} + W_{i_3} \delta_{i_3} \tag{15.a}$$

$$k_a = k_{a_a} + W_{k_a} \delta_{k_a} \tag{15.b}$$

where  $j_3$ ,  $k_a$  are nominal values,  $W_i$ ,  $W_k$  are constant

values representing the range of variation, and

$$\left|\delta_{i,i}\right| \le 1, \ \left|\delta_{ka}\right| \le 1 \tag{16}$$

Furthermore, if we express the reciprocal of  $j_3$  as

$$\frac{1}{j_3} = \frac{1}{j_{3_o} [1 + (W_{j_3}/j_{3_o})\delta_{j_3}]}$$
 (17)

the conservativeness related to the additive uncertainty may be reduced up to a certain point by adopting (17) in place of (15-a) in the plant dynamics.

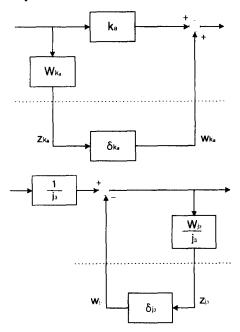


Fig. 4 Representations for additive parameter variation

If we define the input and output of variations as

$$z_{\mu} = [z_{j_1} \quad z_{k_n}]^T, w_{\mu} = [w_{j_1} \quad w_{k_n}]^T$$
 (18-a,b)

and then, parameter variation from  $W_{\mu}$  to  $Z_{\mu}$  can be described as a structured perturbation using

$$\Delta = \operatorname{diag}[\delta_{i}, \quad \delta_{ka}] \tag{19}$$

The purpose of a controller is, if possible, to make

$$\mathbf{z}_{m_2} = [\theta_1 - \theta_2 \quad \theta_2 - \theta_3 \quad \boldsymbol{u}]^T \tag{20}$$

small in the presence of parameter variations and disturbance. This can be achieved by letting the closed-loop transfer function  $T_{\infty}$ , from

$$\boldsymbol{w}_{\infty} = \begin{bmatrix} \boldsymbol{\tau}_{d_1} & \boldsymbol{\tau}_{d_3} \end{bmatrix}^T := \begin{bmatrix} \boldsymbol{w}_1 & \boldsymbol{w}_2 \end{bmatrix}^T \tag{21}$$

to  $\mathbf{Z}_{\infty}$  have robust performance. Therefore, if it is possible to

design a controller by which we keep the  $H_{\infty}$ -norm of the closed-loop transfer function  $T_m$  from  $\mathbf{w}_m = \begin{bmatrix} w_{\mu} & w_{\chi} \end{bmatrix}^T$  to  $z_m = \begin{bmatrix} z_{\mu} & z_{\infty} \end{bmatrix}^T$  low, the design specifications can be satisfied. In other words, the output will track the reference signal and torsional vibrations between  $\theta_I$  and  $\theta_2$ ,  $\theta_2$  and  $\theta_3$  will be also suppressed under the condition of torque disturbance and parameter variations.

Next step, we introduce the generalized plant for this type of problem by the application of Fig.2. That is, a controller, by which  $H_2$ -norm of the transfer function  $T_2$  from  $\mathbf{w}$  (refer to Fig. 5) to  $z_2$  is to be minimized under the condition of  $H_{\infty}$  norm, can be designed based on the structure shown in Fig.5.

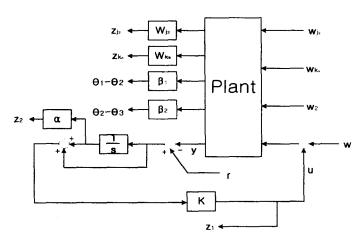


Fig. 5 Structure for Mixed H<sub>2</sub>/H<sub>∞</sub> controller design

In Fig.5,  $\beta_1, \beta_2$  are design variables which can be adjusted to improve the values of  $\theta_1 - \theta_2$  and  $\theta_2 - \theta_3$ , respectively.

### 4.2 Feedforward Controller Design

Since it is impossible to meet all the design specifications related to the output transient response only by a feedback controller, we adopt the structure of two-degree-of-freedom system to cope with this problem. That is, by adding a feedforward path, we try to improve the output time response. The feedforward controller is designed by using the model matching method [2].

Fig.6 shows the basic structure for the two-degree-of-freedom system used, where G denotes the real transfer function from the control input  $\tau$  to output y (= $\theta_I$ ) and  $G_m$  denotes an ideal model for design, and F can be arbitrarily determined if only  $G_m^{-1}F$  is

stable and proper. If G and  $G_m$  are identical, we can prescribe the output response by making use of feedforward controller F independent of the feedback controller  $K_r$ , because the transfer function from r to y becomes F regardless of  $K_r$ . And in case of G not coinciding with  $G_m$ , the feedback controller  $K_r$  will act as a compensator for the tracking error.

It is assumed that  $G_m$  is an ideal model that has no viscous friction and torsion, that is,  $\theta_1 = \theta_2 = \theta_3$ . Therefore, we define

$$G_m^{-1} = (j_1 + j_2 + j_3)s^2 (22)$$

$$F = \frac{1}{(T_1^2 s^2 + 2\zeta T_1 s + 1)(T_2 s + 1)}$$
 (23)

where  $T_1 = 0.023$ ,  $T_2 = 0.03$ ,  $\zeta = 0.9$ , by which the output response can sufficiently satisfy the design specifications when a step reference is added.

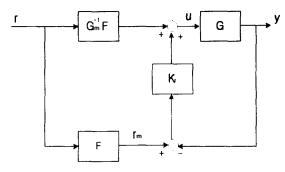


Fig.6 Two-Degree-of-Freedom System

#### 4.3 Simulation Results

After designing a robust controller based on Fig.5, we check the six design specifications through simulation. Matlab is used for computation.

In Table 1, the parameter values used in the simulation are shown, where 'nominal' means an ideal case without parameter variations, 'Case1' represents that the moments of inertia have their minimum values and the torsional and viscous-friction coefficients are varied maximally within the range of variation, 'Case2', on the contrary, represents that the moments of inertia have their maximum values and the torsional and viscous-friction coefficients are varied minimally, and the case that all of the parameters have their minimal value is represented by 'Case3'.

The values of  $W_{j_3}$ ,  $W_{k_a}$  which express the magnitude of variations on  $j_3$ ,  $k_a$  are given as 0.004 (20% variation) and 8 (10% variation), respectively. And constants  $\alpha$ ,  $\beta_1$ ,  $\beta_2$  are determined

as 10, 0.08, 0.05, respectively through several trial and errors. Although six specifications were originally given in [6], we will show two representative simulation results on account of space considerations.

Table 1. Nominal and varied parameters

Para- meters	Nominal	Case 1	Case 2	Case 3
j <sub>1</sub>	0.001	0.0009	0.0011	0.0009
j <sub>2</sub>	0.001	0.0009	0.0011	0.0011
j <sub>3</sub>	0.002	0.001	0.003	0.003
k <sub>a</sub>	920	1012	828	828
$k_b$	80	88	72	72
d <sub>1</sub>	0.005	0.055	0.045	0.045
$d_2$	0.001	0.0011	0.0009	0.0009
$\mathbf{d}_3$	0.007	0.035	0.0014	0.0014
$d_a$	0.001	0.01	0.0002	0.0002
$\mathbf{d}_{\mathbf{b}}$	0.001	0.0011	0.0009	0.0009
a <sub>e</sub>	5000	5000	5000	4500

### (1) Tracking ability (vibration suppression) - specification 1

The reference tracking ability is shown in Fig.7, where  $\theta_3$  and  $\tau$  represent plant output and control input, respectively. The results of simulation are arranged in the Table 2, from which we know that the design specification were sufficiently satisfied in the presence of the parameter variations and disturbance.

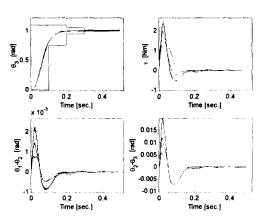


Fig. 7 Step Responses to the reference input

### (2) Complementary Sensitivity Function - Specification 4

The gain plots of the complementary sensitivity function are shown in Fig.8. We know that, although the condition - the gain

must be under 20[dB] over all the frequencies considered – is satisfied despite of variations, the other one – the gain must be under –20[dB] above 300[rad/sec] frequency – cannot be met in any cases. Actually, since it was already known that specification no. 4 and the others had a reciprocal relationship each other [6], it is impossible to meet all the specifications simultaneously. Numerical results are arranged in Table 3.

Table 2. Results for specification 1

	Require- ments	Nominal	Worst Case
max  τ  t≥0	≤3	1.9637	2.4508 (Case 2)
$\max_{t \ge 0} \frac{ \theta_1 - \theta_2 }{t}$	≤0.02	0.0015	0.0023 (Case 3)
$\max_{t \ge 0} \frac{ \theta_2 - \theta_3 }{t}$	≤0.02	0.0121	0.0197 (Case 2)

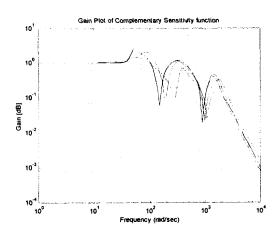


Fig.8 Gain plots of complementary sensitivity function

Table 3. Results for specification 4

	Require- ments	Nominal	Worst Case
$\max  G(j\omega)  [dB]$ $\omega$	≤20	6.3577	8.7150 (Case 2)
max G(jω)  [dB] ω≥300 [rad/sec]	≤-20	0.7484	1.4813 (Case 2)

### (3) Robust stability by $\mu$ -analysis

After closing the plant with the controller  $K_r$ , we take the additive uncertainty of the parameters  $j_3, k_a$  as input and output of the closed-loop system (see Fig.4). Then the system is

robustly stable for all structured  $\Delta(s)$  satisfying  $\|\Delta\|_{\infty} < 1$  if and only if the interconnected system in Fig.9 is stable. This can be done by checking

$$\mu_{m} := \sup_{\omega} \mu_{\Delta}(\widetilde{P}(j\omega)) < 1 \tag{24}$$

That is, we can check robust stability of Fig.9 by evaluating (24). The  $\mu$ -value is shown in Fig.10, when we let the class of model error as  $\Delta \in diag[C\ C]$ , where C denotes the set of complex numbers. Since the maximum  $\mu$ -value is about 0.28 at  $\omega$ =300[rad/sec], we can confirm that the robust stability is satisfied.

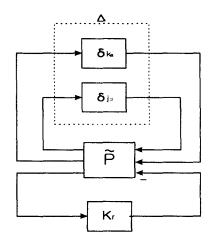


Fig.9 Representation for the structured singular value

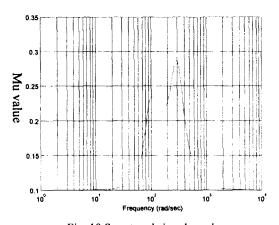


Fig. 10 Structured singular value

### V. Conclusion

In this paper, we have proposed a generalized plant structure for the application of LMI theory to cope with some difficulties in the three-mass inertia system. And the effectiveness of the proposed structure was confirmed through simulations. For the purpose of designing a robust controller, the design objectives such as sufficient vibration suppression and robust tracking are first defined in terms of  $H_2$  and  $H_{\infty}$  optimization theory, then the generalized plant for the mixed  $H_2/H_{\infty}$  control is determined and solved by using a LMI algorithm. Practical computation to get a controller is now quite easy thanks to some excellent software such as Matlab.

It is thought that the difficulty in selecting weighting functions, which is essential to the general  $H_{\infty}$  control theory, can be avoided to some degree if we use the proposed structure for controller design. While the LMI-based approach is computationally more involved for large problems, it has the merit of eliminating the regularity restrictions attached to the Riccati-based solution. For example, the problems caused by adding an integrator in the loop for servo system design can be easily handled through the generalized plant proposed here.

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### References

- [1] C.D. Scherer, D. Gahinet and M. Chilali, "Multiobjective Output-Feedback Control via LMI Optimization," *IEEE Trans. Auto. Contr.*, vol. AC-42, no.2 pp. 896-911, 1997
- [2] T. Sugie and M. Vidyasagor, "Further Results on the robust servo tracking problems in the two-degree-of-freedom control systems," System and Control Letters, 13, pp.100-108, 1989
- [3] K. Zhou, J.C. Doyle and K. Glover, *Robust and Optimal Control*, New Jersey, U.S.A., Prentice-Hall, 1996
- [4] A. Packard, and J.C. Doyle, "The Complex Structured Singular Value," *Automatica*, 29-1, pp.71-109, 1993
- [5] P. Gahinet, A. Nemirovski, A.J. Laub and M. Chilali, LMI Control Toolbox, The MATHWORKS Inc., 1995
- [6] S. Hara, etc, "A coupled three-inertia benchmark problem," Measurement and Control, vol.34, no.5, pp. 403-409, 1995 (In Japanese)



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