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L-fuzzy topologies on complete MV-algebras

Yong-chan Kim and Jung-mi Ko

Department of Mathematics, Kangnung National University
123, Kangnung, Kangwondo, 210-702, Korea

Abstract

In this paper, we introduce neighborhood systems in an L-fuzzy topology using complete MV-algebras. We investigate the relationship between L-fuzzy topologies and the neighborhood systems. We study the properties of neighborhood systems.

Key Words : Complete MV-algebra, Neighborhood systems, Adherent points

1. Introduction

Ward and Dilworth [12] introduced residuated lattices as the foundation of the algebraic structures of fuzzy logics. Hajeck [2] introduced a BL-algebra which is a general tool of a fuzzy logic. Recently, Hohle [3,4] extended the fuzzy set $f:X\to L$ where L is a complete MV-algebra in stead of an unit interval I or a lattice L.

It is a remarkable work to apply fuzzy topologies to fuzzy logics. Ying[15] introduced the neighborhood systems as a new method.

In this paper, we introduce neighborhood systems in an L-fuzzy topology in a view of [15] using complete MV-algebras. We investigate the relationship between L-fuzzy topologies and the neighborhood systems. We study the properties of neighborhood systems.

2. Preliminaries

Definition 2.1 [4.10] A lattice $(L,\le,\wedge,\vee,\odot,\to,0,1)$ is called a residuated lattice if it satisfies the following conditions: for each $x,y,z\in L$,

(R1) $(L,\odot,1)$ is a commutative monoid,

(R2) if $x\le y$, then $x\odot z\le y\odot z$ ($\odot$ is an isotone operation),

(R3) (Galois correspondence): $(x\odot y)\le z$ iff $x\le y\to z$.

In a residuated lattice L, $x^*=(x\to 0)$ is called *complement* of $x\in L$.

Lemma 2.2 [10] In a residuated lattice $(L,\le,\wedge,\vee,\odot,\to,0,1)$, we have the following properties: for $x,y,z\in L$,

(1) $x=1\to x$,

(2) $1=x\to x$,

(3) $x\odot y\le x,y$,

(4) $x\odot y\le x\wedge y$,

(5) $y\le x\to y$,

(6) $x\odot y\le x\to y$,

(7) $x\le y$ iff $1=x\to y$,

(8) $x=y$ iff $1=x\to y=y\to x$,

(9) if $y\le z$, $(x\to y)\ge(x\to z)$.

Definition 2.3 [4,10] A residuated lattice $(L,\le,\wedge,\vee,\odot,\to,0,1)$ is called a BL-algebra if it satisfies the following conditions:for each $x,y\in L$,

(B1) $x\wedge y=x\odot(x\to y)$,

(B2) $x\vee y=[(x\to y)\to y]\wedge[(y\to x)\to x]$,

(B3) $(x\to y)\vee(y\to x)=1$.

Definition 2.4 [4,10] A BL-algebra L is called an MV-algebra if $x=x^{**}$ for each $x\in L$.

Definition 2.5 [4,10] An MV-algebra L is called complete if $\bigwedge_{i\in\Gamma}x_i\in L$ and $\bigvee_{i\in\Gamma}x_i\in L$ for any $x_i\in L$.

Theorem 2.6 [4,10] Let L be a complete MV-algebra. For each $x\in L$, $\{y_i\mid i\in\Gamma\}\subset L$, we have the following properties.

(1) $\bigwedge_{i\in\Gamma}y_i^*=(\bigvee_{i\in\Gamma}y_i)^*$.

(2) $\bigvee_{i\in\Gamma}y_i^*=(\bigwedge_{i\in\Gamma}y_i)^*$.

(3) $\bigwedge_{i\in\Gamma}(x\vee y_i)=x\vee(\bigwedge_{i\in\Gamma}y_i)$.

(4) $\bigvee_{i\in\Gamma}(x\wedge y_i)=x\wedge(\bigvee_{i\in\Gamma}y_i)$.

(5) $x\to\bigvee_{i\in\Gamma}y_i=\bigvee_{i\in\Gamma}(x\to y_i)$.

(6) $x\to\bigwedge_{i\in\Gamma}y_i=\bigwedge_{i\in\Gamma}(x\to y_i)$.

(7) $(\bigvee_{i\in\Gamma}y_i)\to x=\bigwedge_{i\in\Gamma}(y_i\to x)$.

(8) $\bigwedge_{i\in\Gamma}y_i\to x=\bigvee_{i\in\Gamma}(y_i\to x)$.

(9) $x\odot y=(x\to y^*)^*$.

(10) $x\le y$ iff $x^*\ge y^*$.

Throughout this paper, let L be a complete MV-algebra and $L_0=L-\{0\}$. The class of all fuzzy

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sets on a set X will be denoted by L^X and the fuzzy sets by the Greek symbols λ, μ, ν, etc.

Definition 2.7 [4] All algebraic operations on L can be extended pointwise to the set L^X as follows:

$$\mu \to \rho \text{ iff } \mu(x) \to \rho(x), \text{ for all } x \in X,$$
$$(\mu \odot \rho)(x) = \mu(x) \odot \rho(x), \text{ for all } x \in X.$$

The set of all fuzzy points in X is denoted by $Pt(X)$. For $x_t \in Pt(X)$, $x_t \in \lambda$ iff $t \le \lambda(x)$.

All the other notations and the other definitions are standard in fuzzy set theory.

Definition 2.8 [1,6] A subset T of L^X is called an L-fuzzy topology on X if it satisfies the following conditions:

(O1) $\overline{0}, \overline{1} \in T$, where $\overline{0}(x) = 0$ and $\overline{1}(x) = 1$ for all $x \in X$.

(O2) If $\mu_1, \mu_2 \in T$, $\mu_1 \wedge \mu_2 \in T$.

(O3) If $\mu_i \in T$ for each $i \in \Gamma$, $\bigvee_{i \in \Gamma} \mu_i \in T$.

The pair (X, T) is called an L-fuzzy topological space.

3. Fuzzy neighborhood systems

Definition 3.1 Let $\lambda \in L^X$ and $x_p \in Pt(X)$.

Then the degree to which x_p belongs to λ is

$$[x_p \to \lambda] = p \to \lambda(x).$$

Lemma 3.2 For $\lambda, \mu_i \in L^X$ and $x_p \in Pt(X)$, we have the following properties:

(1) $[x_1 \to \lambda] = \lambda(x)$.

(2) $[x_p \to \lambda] = 1$ iff $x_p \in \lambda$.

(3) $[x_p \to \lambda] = 0$ iff $p = 1$ and $\lambda(x) = 0$.

(4) $[x_p \to \bigvee_{i \in \Gamma} \mu_i] = \bigvee_{i \in \Gamma} [x_p \to \mu_i]$, for any $\{\mu_i\}_{i \in \Gamma} \subset L^X$.

(5) $[x_p \to \bigwedge_{i \in \Gamma} \mu_i] = \bigwedge_{i \in \Gamma} [x_p \to \mu_i]$, for any $\{\mu_i\}_{i \in \Gamma} \subset L^X$.

Proof. (1) From Lemma 2.2(1),

$$[x_1 \to \lambda] = 1 \to \lambda(x) = \lambda(x).$$

(2) From Lemma 2.2(7),

$$[x_p \to \lambda] = p \to \lambda(x) = 1 \text{ iff } p \le \lambda(x) \text{ iff } x_p \in \lambda.$$

(3) Let $[x_p \to \lambda] = 0$. Since $p \to \lambda(x) = 0$, by Lemma 2.2(5), $\lambda(x) \le (p \to \lambda(x)) = 0$. Thus, $\lambda(x) = 0$. Since $p^* = (p \to 0) = 0$ and L is a MV-algebra, by Lemma 2.2(2), $1 = (0 \to 0) = 0^*$ implies $p = (p^*)^* = 0^* = 1$. Conversely, let $p = 1$ and $\lambda(x) = 0$. From Lemma 2.2(1),

$$[x_p \to \lambda] = 0.$$

(4) and (5) are easily proved from Theorem 2.6(5,6).

Definition 3.3 Let (X, T) be an L-fuzzy topological space, $\mu \in L^X$ and $e \in Pt(X)$. Then the degree to which λ is a neighborhood of e is defined by

$$N_e(\lambda) = \bigvee \{ [e \to \mu] \mid \mu \le \lambda,\ \mu \in T \}.$$

A mapping $N_e: L^X \to I$ is called the fuzzy neighborhood system of e.

Theorem 3.4 Let (X, T) be an L-fuzzy topological space and N_e the fuzzy neighborhood system of e.

For $\lambda, \mu \in L^X$, it satisfies the following properties:

(1) $N_e(\overline{0}) = [e \to \overline{0}]$ and $N_e(\overline{1}) = 1$.

(2) $N_e(\lambda) \le [e \to \lambda]$.

(3) $N_e(\lambda) \le N_e(\mu)$, if $\lambda \le \mu$.

(4) $N_e(\lambda) \wedge N_e(\mu) \le N_e(\lambda \wedge \mu)$.

(5) $N_e(\lambda) \le \bigvee \{ N_e(\mu) \mid \mu \le \lambda, [d \to \mu] \le N_d(\mu, r)\ \forall d \in Pt(X) \}$.

(6) $N_{x_p}(\lambda) = p \to N_{x_1}(\lambda)$, for each $x_p \in Pt(X)$.

Proof. (1) Since $\overline{0}, \overline{1} \in T$, $N_e(\overline{0}) = [e \to \overline{0}]$ and $N_e(\overline{1}) = [e \to \overline{1}] = 1$ because $e \in \overline{1}$ from Lemma 3.2(2).

(2) It is proved from the following:

$$\begin{aligned} N_e(\lambda) &= \bigvee \{ [e \to \mu_i] \mid \mu_i \le \lambda,\ \mu_i \in T \} \\ &= \{ [e \to \bigvee \mu_i] \mid \bigvee \mu_i \le \lambda,\ \mu_i \in T \} \\ &\quad \text{(by Lemma 3.2(4))} \\ &\le [e \to \lambda]. \end{aligned}$$

(3) It is trivial from the definition of N_e.

(4) It is proved from the following:

$$\begin{aligned} & N_e(\lambda) \wedge N_e(\mu) \\ &= \bigvee \{ [e \to \rho] \mid \rho \le \lambda,\ \rho \in T \} \wedge N_e(\mu) \\ &= \bigvee \{ [e \to \rho] \wedge N_e(\mu) \mid \rho \le \lambda,\ \rho \in T \} \\ &\quad \text{(by Theorem 2.6(4))} \\ &= \bigvee \{ \bigvee \{ [e \to \rho] \wedge [e \to \omega] \mid \rho \le \lambda,\ \omega \le \mu,\ \rho \in T,\ \omega \in T \} \} \\ &= \bigvee \{ \bigvee \{ [e \to \rho \wedge \omega] \mid \rho \le \lambda,\ \omega \le \mu,\ \rho \in T,\ \omega \in T \} \\ &\quad \text{(by Lemma 3.2(5))} \\ &\le \bigvee \{ [e \to \rho \wedge \omega] \mid \rho \wedge \omega \le \lambda \wedge \mu,\ \rho \wedge \omega \in T \} \\ &= N_e(\lambda \wedge \mu). \end{aligned}$$

(5) If $\mu \in T$, then $N_d(\mu) = [d \to \mu]$, for each $d \in Pt(X)$. It implies

$$\begin{aligned} & N_e(\lambda) \\ &= \bigvee \{ [e \to \mu] \mid \mu \le \lambda, \mu \in T \} \\ &= \bigvee \{ N_e(\mu) \mid \mu \le \lambda, N_e(\mu) = [d \to \mu], \forall d \in Pt(X) \} \\ &\le \bigvee \{ N_e(\mu) \mid \mu \le \lambda, [d \to \mu] \le N_e(\mu), \forall d \in Pt(X) \}. \end{aligned}$$

(6) For each $x_p \in Pt(X)$, we have

$$\begin{aligned} & N_{x_p}(\lambda) \\ &= \bigvee \{ p \to \mu(x) \mid \mu \le \lambda, \mu \in T \} \\ &= p \to \bigvee \{ \mu(x) \mid \mu \le \lambda, \mu \in T \} \\ &= p \to \bigvee \{ [x_1 \to \mu] \mid \mu \le \lambda, \mu \in T \} \\ &\quad \text{(by Lemma 3.2(1))} \\ &= p \to N_{x_1}(\lambda). \end{aligned}$$

Theorem 3.5 Let N_e be a fuzzy neighborhood system of e satisfying the above conditions (1)-(4), for each $e \in Pt(X)$. We define

$$T_N = \{\lambda \in L^X \mid [e \to \lambda] \le N_e(\lambda),\ \forall e \in Pt(X)\}.$$

(1) T_N is an L-fuzzy topology on X.

(2) If N_e is the fuzzy neighborhood system of e induced by (X, T), then $T_N = T$.

(3) If N_e's satisfy the conditions (1)-(6), then

$$T_N = \bigvee\{\lambda \in L^X \mid [x_1 \to \lambda] \le N_{x_1}(\lambda),\ \forall x \in X\}.$$

Proof.

(1) (O1) It is easily proved from Theorem 3.4(1).

(O2) Let $\mu_1, \mu_2 \in T_N$. For each $i \in \{1,2\}$, we have

$$[e \to \mu_i] \le N_e(\mu_i),\ \forall e \in Pt(X).$$

It implies

$$\begin{aligned}[e \to \mu_1 \wedge \mu_2] &= [e \to \mu_1] \wedge [e \to \mu_2] \\ &\quad (\text{by Lemma } 3.2(5)) \\ &\le N_e(\mu_1) \wedge N_e(\mu_2). \\ &\le N_e(\mu_1 \wedge \mu_2). \\ &\quad (\text{by Theorem } 3.4(4))\end{aligned}$$

Hence $\mu_1 \wedge \mu_2 \in T_N$.

(O3) Let $\mu_i \in T$ for each $i \in \Gamma$. Since for each $i \in \Gamma$,

$$[e \to \mu_i] \le N_e(\mu_i),\ \forall e \in Pt(X),$$

we have

$$\begin{aligned}[e \to \bigvee_{i \in \Gamma} \mu_i] &= \bigvee_{i \in \Gamma} [e \to \mu_i] \quad (\text{by Lemma } 3.2(4)) \\ &\le \bigvee_{i \in \Gamma} N_e(\mu_i) \\ &\le N_e(\bigvee_{i \in \Gamma} \mu_i) \quad (\text{by Theorem } 3.4(3))\end{aligned}$$

Thus, $\bigvee_{i \in \Gamma} \mu_i \in T_N$.

Hence T_N is an L-fuzzy topology on X.

(2) Let $\lambda \in T_N$. From the definition of T_N and Theorem 3.4(2), we have $[e \to \lambda] = N_e(\lambda)$. Since, for each $e \in Pt(X)$,

$$\begin{aligned}[e \to \lambda] &= N_e(\lambda) \\ &= \bigvee\{[e \to \mu_i] \mid \mu_i \le \lambda, \mu_i \in T\},\end{aligned}$$

then, for each $x_1 \in Pt(X)$,

$$\begin{aligned}\lambda(x) &= [x_1 \to \lambda] \quad (\text{by Lemma } 3.2(1)) \\ &= \bigvee\{[x_1 \to \mu_i] \mid \mu_i \le \lambda, \mu_i \in T\} \\ &= [x_1 \to \bigvee \mu_i] \quad (\mu_i \in T) \\ &= \bigvee \mu_i(x).\end{aligned}$$

Thus, $\lambda = \bigvee \mu_i$ with $\mu_i \in T$. So, $\lambda \in T$.

Hence $T_N \subset T$.

Let $\mu \in T$. Then

$$\begin{aligned}N_e(\mu) &= \bigvee\{[e \to \lambda] \mid \lambda \le \mu, \lambda \in T\} \\ &= [e \to \mu].\end{aligned}$$

So, $\mu \in T_N(\lambda)$. Hence $T \subset T_N$.

(3) We only show that

$$\begin{aligned}&[x_t \to \lambda] \le N_{x_t}(\lambda),\ \forall x_t \in Pt(X) \\ \Leftrightarrow &[x_1 \to \lambda] = \lambda(x) \le N_{x_1}(\lambda),\ \forall x \in X.\end{aligned}$$

($\Rightarrow$) It is trivial.

($\Leftarrow$) From the condition (6),

$$\begin{aligned}N_{x_t}(\lambda) &= t \to N_{x_1}(\lambda) \\ &\ge t \to \lambda(x) \quad (\text{by Lemma } 2.2(9)) \\ &= [x_t \to \lambda].\end{aligned}$$

Definition 3.6 Let (X, T) be an L-fuzzy topological space, $\lambda \in L^X$ and $e \in Pt(X)$. Then the degree to which e is an adherent point of λ is defined by

$$Ad_e(\lambda) = N_e(\lambda^*)^*.$$

Theorem 3.7 Let (X, T) be an L-fuzzy topological space .For each $\lambda \in L^X$, we define operators $C_T, I_T : L^X \to L^X$ as follows:

$$C_T(\lambda) = \bigwedge\{\rho \in L^X \mid \lambda \le \rho, \rho^* \in T\},$$
$$I_T(\lambda) = \bigvee\{\nu \in L^X \mid \nu \le \lambda, \nu \in T\}.$$

For each $\lambda \in L^X, e, x_t \in Pt(X)$, we have the following properties.

(1) $I_T(\lambda^*) = C_T(\lambda)^*$.

(2) $[e \to I_T(\lambda)] = N_e(\lambda)$.

(3) $[e \to C_T(\lambda)^*] = Ad_e(\lambda)^*$.

(4) $Ad_{x_t}(\lambda) = [x_t \odot Ad_{x_1}(\lambda)]$.

Proof. (1) Since $(\lambda^*)^* = \lambda$, we have

$$\begin{aligned}I_T(\lambda^*) &= \bigvee\{\nu \in L^X \mid \nu \le \lambda^*, \nu \in T\} \\ &= \bigvee\{\nu \in L^X \mid \nu^* \ge \lambda, \nu \in T\} \\ &\quad (\text{by Theorem } 2.6(10)) \\ &= \bigvee\{(\nu^*)^* \in L^X \mid \lambda \le \nu^*, \nu \in T\} \\ &= (\bigwedge\{\nu^* \in L^X \mid \lambda \le \nu^*, (\nu^*)^* = \nu \in T\})^* \\ &\quad (\text{by Theorem } 2.6(2)) \\ &= C_T(\lambda)^*.\end{aligned}$$

(2)
$$\begin{aligned}[e \to I_T(\lambda)] &= [e \to \bigvee\{\mu_i \mid \mu_i \le \lambda, \mu_i \in T\}] \\ &= \bigvee\{[e \to \mu_i] \mid \mu_i \le \lambda, \mu_i \in T\} \\ &\quad (\text{by Lemma } 3.2(4)) \\ &= N_e(\lambda).\end{aligned}$$

(3)
$$\begin{aligned}[e \to C_T(\lambda)^*] &= [e \to I_T(\lambda^*)] \quad (\text{by } (1)) \\ &= N_e(\lambda^*) \quad (\text{by } (2)) \\ &= Ad_e(\lambda)^*.\end{aligned}$$

(4)
$$\begin{aligned}Ad_{x_t}(\lambda) &= N_{x_t}(\lambda^*)^* \\ &= (t \to N_{x_1}(\lambda^*))^* \\ &\quad (\text{by Theorem } 3.4(6)) \\ &= [x_t \odot N_{x_1}(\lambda^*)^*] \\ &\quad (\text{by Theorem } 2.6(9)) \\ &= [x_t \odot Ad_{x_1}(\lambda)]\end{aligned}$$

Example 3.8 Let $L = ([0,1], \le, \wedge, \vee, \odot, \to, 0, 1, ^*)$ be a complete MV-algebra defined by (called generalized Lukasiewicz logic, ref.[5,10])

$$a \to b = \min\{1, (1 - a^p + b^p)^{\frac{1}{p}}\}$$
$$a \odot b = \max\{0, (a^p + b^p - 1)^{\frac{1}{p}}\}$$

where p is a natural number.

Let $X = \{x, y\}$ be a set and $\mu \in L^X$ as follows:

$$\mu(x)=0.3,\quad \mu(y)=0.4.$$

We define an L-fuzzy topology

$$T=\{\overline{0},\overline{1},\mu\}.$$

From Definition 3.3, we can obtain $N_{x_{0.6}}, N_{x_{0.2}}: L^X \to L$ as follows:

$$N_{x_{0.6}}(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{1},\\ (1-0.6^p+0.3^p)^{\frac{1}{p}}, & \text{if } \overline{1}\neq\lambda\geq\mu,\\ (1-0.6^p)^{\frac{1}{p}}, & \text{otherwise},\end{cases}$$

$$N_{x_{0.2}}(\lambda)=\begin{cases}1, & \text{if } \lambda\geq\mu,\\ (1-0.2^p)^{\frac{1}{p}}, & \text{otherwise}.\end{cases}$$

Moreover, $N_{x_1}, N_{y_1}: L^X \to L$ as follows:

$$N_{x_1}(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{1},\\ 0.3, & \text{if } \overline{1}\neq\lambda\geq\mu,\\ 0, & \text{otherwise},\end{cases}$$

$$N_{y_1}(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{1},\\ 0.4, & \text{if } \overline{1}\neq\lambda\geq\mu,\\ 0, & \text{otherwise}.\end{cases}$$

From Theorem 3.4 and Theorem 3.5 (2-3), we have

$$T_N=\{\overline{1},\overline{0},\mu\}=T.$$

From Definition 3.6, we can obtain $Ad_{x_{0.6}}, Ad_{x_1}: L^X \to L$ as follows:

$$Ad_{x_{0.6}}(\lambda)=\begin{cases}0, & \text{if } \lambda=\overline{0},\\ (0.6^p-0.3^p)^{\frac{1}{p}}, & \text{if } \overline{0}\neq\lambda\leq\mu^*,\\ 0.6, & \text{otherwise},\end{cases}$$

$$Ad_{x_1}(\lambda)=\begin{cases}0, & \text{if } \lambda=\overline{0},\\ (1-0.3^p)^{\frac{1}{p}}, & \text{if } \overline{0}\neq\lambda\leq\mu^*,\\ 1, & \text{otherwise}.\end{cases}$$

Thus, $Ad_{x_{0.6}}(\lambda)=[x_{0.6}\odot Ad_{x_1}(\lambda)]$.

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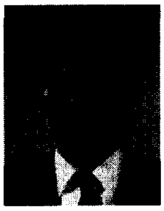
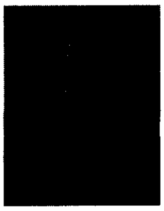


Yong-Chan Kim

Yong-Chan Kim was born August 6, 1957. He received the B.S. , M.S. , and Ph.D. degree in Department of Mathematics from Yonsei University, Seoul, Korea, in 1991. Since 1991, he has been a faculty member of the Department of Mathematics in Kangnung National University, where he is currently an Associate Professor. His research interests are Fuzzy topology, Fuzzy logic . He is a member of KMS, KKMS, and KFIS.

Phone : +82-33-640-2270
Fax : +82-33-647-3256
E-mail : yck@kangnung.ac.kr



Jung-Mi Ko

Jung-Mi Ko was born March 16, 1958. She received the B.S. , M.S. , and Ph.D. degree in Department of Mathematics from Yonsei University, Seoul, Korea, in 1988. Since 1988, she has been a faculty member of the Department of Mathematics in Kangnung National University, where she is currently a Professor. Her research interests are Fuzzy topology, Fuzzy logic, and Differential Geometry. She is a member of KMS, KKMS, and KFIS.

Phone : +82-33-640-2268
Fax : +82-33-647-3256
E-mail : jmko@kangnung.ac.kr