퍼지 및 지능시스템학회 논문지 2001, Vol. 11, No. 7, pp. 628-632

Generalization of the Spreading Function and Weyl Symbol for Time-Frequency Analysis of Linear Time-Varying Systems

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Abstract

We propose time-frequency (TF) tools for analyzing linear time-varying (LTV) systems and nonstationary random processes. Obtained warping the narrowband Weyl symbol (WS) and spreading function (SF), the new TF tools are useful for analyzing LTV systems and random processes characterized by generalized frequency shifts. This new Weyl symbol (WS) is useful in wideband signal analysis. We also propose WS as tools for analyzing systems which produce dispersive frequency shifts on the signal. We obtain these generalized, frequency-shift covariant WS by warping conventional, narrowband WS. Using the new, generalized WS, we provide a formulation for the Weyl correspondence for linear systems with instantaneous frequency characteristics matched to user specified characteristics. We also propose a new interpretation of linear signal transformations as weighted superpositions of non-linear frequency shifts on the signal. Application examples in signal analysis and detection demonstrate the advantages of our new results.

Key Words: Spreading functions, Weyl symbol, Time-frequency symbols

1. Introduction

Time-frequency (TF) formulations of the conventional Weyl symbol (WS) and its 2-D Fourier transform (FT), the spreading function (SF), have been successfully used in the analysis of linear time-varying systems and nonstationary random processes [5,6,12]. The conventional WS and SF are defined, respectively, as [5]

$$WS_L(t,f)=\int_{-\infty}^{\infty}K_L(t+\frac{\tau}{2},t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau \qquad (1)$$

$$SF_L(\tau,\nu)=\int_{-\infty}^{\infty}K_L(t+\frac{\tau}{2},t-\frac{\tau}{2})e^{-j2\pi t\nu}dt \qquad (2)$$

for an operator L on $L_2(\mathbf{R})$ with operator kernel $K_L(t,\tau)$ [2]. The WS can be interpreted as the transfer function of a linear time-varying (LTV) system or as the time-varying spectrum of a nonstationary random process. The 1-D inner product of the operator input x(t) and output (L x)(t) can be expressed as the 2-D inner product of the Wigner distribution (WD) [3] of the operator input and the WS of the operator,

$$\int(Lx)(t)x^*(t)dt=\int\int WS_L(t,f)\,WD_x(t,f)\,dtdf \qquad (3)$$

and the relationship in (3) is called the quadratic form of x(t) [12]. Here, $WD_x(t,f)=\int x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\,e^{-j2\pi\tau f}d\tau$ is the Wigner distribution [3] of the process x(t). The quadratic form provides a definition of a TF concentration measure [12], and is useful in TF detection [7,11] and analysis [8] applications. In the sense of the quadratic form (3), we say that the WS is associated with the WD.

The WS in (1) preserves constant time shifts, constants frequency shifts, and scale changes on a random process [5,6]

$$Y(f)=(S_\tau X)(f)=x(f)e^{-j2\pi\tau f}\Rightarrow WS_{R_Y}(t,f)=WS_{R_X}(t-\tau,f)$$

$$Y(f)=(M_\nu X)(f)=X(f-\nu)\Rightarrow WS_{R_Y}(t,f)=WS_{R_X}(t,f-\nu)$$

$$Y(f)=\frac{1}{\sqrt{|a|}}X(\frac{f}{a})\Rightarrow WS_{R_Y}(t,f)=WS_{R_X}(at,\frac{f}{a})$$

where S_τ and M_ν are the constant time-shift and constant frequency-shift operators, respectively. Here, R_X is the autocorrelation operator of X(f) with the operator kernel $K_{RX}(f,\nu)=E\{X(f)X^*(\nu)\}$ and the expectation operator E{ • }. The WS also satisfies the unitarity property defined a

$$\int_{t,f}WS_L(t,f)WS_V(t,f)dtdf=\sum_{m,n}\varepsilon_m\gamma_n\left|\int Hn(f)Gm(f)df\right|^2 \qquad (4)$$

where ε_m and $G_m(f)$ are eigenvalues and eigenfunctions, respectively, of the kernel of the operator L, and γ_n and $H_n(f)$ are similarly defined for the operator V on $L_2(\mathbf{R})$. Using (1) and the kernel expansion $\Gamma_L(f,\nu)=\sum\varepsilon_n G_n(f)G_n^*(\nu)$, one can express the WS of L as a weighted summation of the Wigner distribution of the eigenfunctions of L, i.e. $WS_L(t,f)=\sum\varepsilon_n WD_{G_n}(t,f)$.

The SF provides an important interpretation of a time varying system output as a weighted superposition of time-shifted and frequency-shifted versions of the input signal x(t), where the weight is the SF [12], i.e.

Maniscript received Month Day, 2001; revised Month Day, 2001.

$(Lx)(t)=\int\int SF_L(\tau,\nu)e^{-j\pi\tau\nu}(M_\nu S_\tau x)(t)d\tau d\nu$. Here, $(M_\nu S_\tau x)(t)=x(t-\tau)e^{j2\pi\nu t}$ is the TF shifted version of x(t). Thus, the SF provides the amount of time shifts and frequency lags produced by the LTV system. This is comparable to the conventional interpretation of the (convolution) output of a linear time-invariant (LTI) system as a weighted superposition of time-shifted versions of the input signal. The weight is the impulse response of the LTI system and shows the amount of time shifts produced by the LTI system. The support region of the SF has been used to define underspread random processes [5], a useful concept in detection applications [7].

When an LTV system produces dispersive (non-constant) frequency shifts, the conventional WS and SF are no longer adequate to characterize linear systems whose nonstationary process is not matched to simple time and frequency shifts. Thus, in this paper, we propose new TF symbols and spreading functions as tools for analyzing systems which produce dispersive frequency shifts on the signal. These new TF symbols are important since they can be interpreted as time-varying transfer functions for such systems. We derive such generalized TF symbol and spreading function by warping the conventional narrowband WS and SF, respectively. We provide a generalized TF formulation of the quadratic form in (3) for linear systems with instantaneous frequency characteristics matched to a specified warping. Special examples will be given to demonstrate how the generalized TF symbol and spreading function greatly simplify when matched to the system. Analysis and detection application examples demonstrate the importance of these new TF techniques.

2. Generalization of Narrowband Weyl correspondence

2.1 Hyperbolic Weyl Symbol and Spreading Function

If a system imposes hyperbolic frequency shifts and scale changes on the input signal, new WS and new SF are needed for analysis. The TF geometry of these new WS and SF should reflect the hyperbolic system changes on the input signal. Thus, for an operator $\boldsymbol{Y}$ on $L_2(\mathbf{R}^+)$ with kernel $K_Y(t,\tau)$, we define the hyperbolic WS (HWS) and SF (HSF), respectively, as

$$HWS_Y(t,f)=t\int K_Y(te^{\zeta/2},te^{-\zeta/2})e^{-j2\pi t f\zeta}d\zeta,\ t>0 \qquad (5)$$

$$HSF_Y(\zeta,\beta)=\int K_Y(te^{\zeta/2},te^{-\zeta/2})e^{-j2\pi\beta\ln(t)}dt.$$

The relation between the HWS and the HSF is given as

$$HSF_Y(\zeta,\beta)=F_{\gamma\to\zeta}^{-1}[P_{t\to\beta}[HWS_Y(t,\gamma/t)]]$$

where $\boldsymbol{F}^{-1}$ is the inverse Fourier transform operator and $\boldsymbol{P}_t\to\beta\{x(t)\}=\int x(t)e^{-j2\pi\beta\ln(t)}/t\ dt=\rho x(\xi\ln)(\beta)$, $t>0$, is a version of the Mellin transform [1]. Note the similarities between the conventional WS (and SF) and the HWS (and HSF) summarized in Table 1 and 2. Row 4 shows that the hyperbolic WS in (5) can be obtained from the conventional WS in (1) by first unitarily warping the operator $\boldsymbol{Y}$ and then transforming the TF axes. For the HSF, the axes are simply scaled since they show only relative TF lags, not absolute TF locations.

The HWS preserves hyperbolic frequency shifts and scale changes on a random process x(t), i.e

$$y(t)=x(t)e^{j2\pi\beta\ln(t)}\Rightarrow HWS_{Ry}(t,f)=HWS_{Rx}(t,f-\frac{\beta}{t}),$$

$$y(t)=\frac{1}{\sqrt{|a|}}x(\frac{t}{a})\Rightarrow HWS_{Ry}(t,f)=HWS_{Rx}(\frac{t}{a},af),$$

where **Ry** and **Rx** are the autocorrelation operators of y(t) and x(t), respectively. The HWS also satisfies the unitarity property in (4). The quadratic form in (3) can now be written in terms of the HWS and Qx(t,f), the Altes-Marinovic Q-distribution [9],

$$\int(Yx)(t)x^*(t)dt=\int\int HWS_Y(t,f)Q_x(t,f)dtdf.$$

This new form of the quadratic form may be useful in detection applications of nonstationary processes and systems with hyperbolic instantaneous frequency characteristics. These formulations are important as they provide a new interpretation of these system outputs as weighted superpositions of hyperbolic frequency-shifted and scale changed versions of the input signal, i.e.

$$(Yx)(t)=\int\int HSF_Y(\zeta,\beta)e^{-j\pi\zeta\beta}(H_\beta C_{e^\zeta}x)(t)d\zeta d\beta$$

where $(\boldsymbol{H}_\beta x)(t)=e^{j2\pi\beta\ln(t)}x(t)$ is the hyperbolic frequency shift operator and $(\boldsymbol{C}_a x)(t)=x(t/a)/|a|1/2$ is the scaling operator. Thus, HSFY weighs the relative importance of hyperbolic frequency shifts and scale changes caused by a linear system. In Section 3, we provide applications to demonstrate the importance of the HWS.

2.2 Power Weyl Symbol and Spreading Function

We obtain the κ-th power WS ($PWS^{(\kappa)}$) and the κ-th power SF ($PSF^{(\kappa)}$), for an operator $\boldsymbol{Y}$ on $L_2(\mathbf{R})$, by warping the conventional WS and SF as shown in row 5 of Table 1 and 2. The relation between $PWS^{(\kappa)}$ and $PSF^{(\kappa)}$ is given by

$$PSF^{(\kappa)}(\zeta,\beta)=F^{-1}{}_{\gamma\to\zeta}\{P^{(\kappa)}{}_{t\to\beta}\{PWS^{(\kappa)}{}_Y(t,\gamma\varphi_\kappa(t))\}\}(6)$$

where $\boldsymbol{P}^{(\kappa)}{}_{t\to beta}\{x(t)\}=\int x(t)\ e^{-j2\pi\beta|t|^\kappa}\kappa|t|^{\kappa-1}dt$ is the κ-th power transform. The quadratic form can now be expressed [4] in terms of PWS(κ) and a power warped version of the WD [10].

The operator output can be interpreted as a weighted superposition of κ-th power frequency shifts on the input signal with weights $PSF^{(\kappa)}{}_Y$, i.e.

$$(Yx)(t)=\int\int PSF^{(\kappa)}{}_Y(\zeta,\beta)e^{-j\pi\zeta\beta}e^{j2\pi\beta sgn(t)|t|^\kappa}x_{\zeta,\kappa}(t)d\zeta d\beta$$

where $x\zeta,\kappa(t)=|1-sgn(t)\zeta|t|-\kappa|(1-\kappa)/2\kappa\ x(t|1-sgn(t)\zeta|t|-\kappa|1/\kappa)$. An important fact is that when κ=1, the PWS(κ) and PSF(κ) simplify to the conventional WS and SF in

(1) and (2), respectively. Hence, the relationship between the PWS(κ) and PSF(κ) in (6) simplifies to the 2-D Fourier transform relationship between the WS and SF.

Table 1. Various Weyl symbols for a given warping function ξ(b). Here, Y is defined based on the domain of ξ(b). For example, for the HWS, Y is defined on L2(R+). The warping operator is (Wξx)(t)=x(ξ-1(t))/|φ(ξ-1(t))|0.5 and (WξWξ-1x)(t)=x(t). Here, ξln(b)=ln(b), ξκ(b)=sgn(b) |b|κ, and ξexp(b)=eb.

$\xi(b)$	Weyl Symbol (WS) time-frequency representation
1 to 1	$GWS_Y(t,f)=WS_{W_\xi Y W_\xi^{-1}}(\xi(t), f/\varphi(t))$
b	$WS_L(t,f)=\int K_L(t+\frac{\tau}{2}, t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau$
$\xi_{\ln}(b)$	$HWS_Y(t,f)=WS_{W_{\xi_{\ln}} Y W_{\xi_{\ln}}^{-1}}(\ln(t), tf)$
$\xi_\kappa(b)$	$PWS_Y^{(\kappa)}(t,f)=WS_{W_{\xi_\kappa} Y W_{\xi_\kappa}^{-1}}(\xi_\kappa(t), f/\varphi_\kappa(t))$
$\xi_{\exp}(b)$	$EWS_Y(t,f)=WS_{W_{\xi_{\exp}} Y W_{\xi_{\exp}}^{-1}}(e^t, fe^{-t})$

Table 2. Various spreading functions for a given warping function ξ(b). Here, Y is defined based on the domain of ξ(b).

$\xi(b)$	Spreading Function (SF)
1 to 1	$GSF_Y(\zeta,\beta)=SF_{W_\xi Y W_\xi^{-1}}(\zeta,\beta)$
b	$SF_L(\tau,\nu)=\int K_L(t+\frac{\tau}{2}, t-\frac{\tau}{2})e^{-j2\pi t\nu}dt$
$\xi_{\ln}(b)$	$HSF_Y(\zeta,\beta)=SF_{W_{\xi_{\ln}} Y W_{\xi_{\ln}}^{-1}}(\zeta,\beta)$
$\xi_\kappa(b)$	$PSF_Y^{a}(\kappa)(\zeta,\beta)=SF_{W_{\xi_\kappa} Y W_{\xi_\kappa}^{-1}}(\zeta,\beta)$
$\xi_{\exp}(b)$	$ESF_Y(t,f)=SF_{W_{\xi_{\exp}} Y W_{\xi_{\exp}}^{-1}}(\zeta,\beta)$

2.3 Exponential Weyl Symbol and Spreading Function

Using the warping ξexp(b)=eb in row 6 of Table 1 and 2, we obtain the exponential WS (EWS) and the exponential SF (ESF) for Y on L2(R). Two transforms link EWSY(t,f) and ESFY(ζ,β),

$$ESF_Y(\zeta,\beta)=F^{-1}_{\gamma\to\zeta}\{E_{t\to\beta}\{EWS_Y(t,\gamma e^{t})\}\}.$$

Here, Et→β {x(t)}=∫ x(t) e-j2πβe^t et dt. The exponential version of the quadratic form uses the EWS and an exponential warped WD [4,10]. Operator output can be interpreted as a weighted superposition of exponential frequency shifts on the input signal, i.e.

$$(Yx)(t)=\int\int ESF_Y(\zeta,\beta)e^{-j\pi\zeta\beta}e^{j2\pi\beta e^t}x_\zeta(t)d\zeta d\beta$$

where xζ(t)=[et/(et-ζ)]1/2 x(ln(et-ζ)).

2.4 Generalized Weyl Symbol and Spreading Function

If a system imposes TF operators different from simple time or frequency shifts on the input signal, then new WS and SF are needed for analysis to reflect the dispersive changes on the input signal. We obtain the new generalized WS (GWS) and the new generalized SF (GSF) of an operator ***Y*** representing a system whose input signal is shifted in frequency in a non-linear manner related to a one-to-one warping function ξ(b). The new GWS is defined as

$$GWS_Y(t,f)=\int K_Y(\Xi(\xi(t),\zeta),\Xi(\xi(t),-\zeta))\cdot |\varphi(\Xi(\xi(t),\zeta))\varphi(\Xi(\xi(t),-\zeta))|^{-\frac{1}{2}}e^{-j2\pi f\zeta/\varphi(t)}d\zeta, \quad (7)$$

where Ξ(c,ζ)=ξ-1(c+ζ/2), ξ-1(ξ(b))=b, and φ(t)=ξ'(t). Here, KY(t,τ) is the kernel of the operator ***Y*** defined on L2([α,β]) and [α,β] depends on the domain of ξ(b). The GWS preserves generalized frequency shifts on a random process x(t), i.e.

$$y(t)=x(t)e^{j2\pi c\xi(t)}\Rightarrow GWS_{Ry}(t,f)=GWS_{Rx}(t,f-c\varphi(t)),$$

where **R**y and **R**x are the correlation operators of y(t) and x(t), respectively.

The new GSF is

$$GSF_Y(\zeta,\beta)=\int K_Y(\Xi(c,\zeta),\Xi(c,-\zeta))\cdot |\varphi(\Xi(c,\zeta))\varphi(\Xi(c,-\zeta))|^{-1/2}e^{-j2\pi c\beta}dc. \quad (8)$$

The integration limits in (7) and (8) are determined by the range of ξ(b). The relation between GWSY and GSFY is

$$GSF_Y(\zeta,\beta)=F^{-1}_{\gamma\to\zeta}\{G_{t\to\beta}\{GWS_Y(t,\gamma\varphi(t))\}\}$$

where ***G***t → c{x(t)}=∫ x(t) e-j2πcξ(t) |φ(t)| dt is a generalized transform dependent on the warping function ξ(b). The quadratic form in (3) can now be expressed in terms of the generalized WS, GWSY,

$$\int (Yx)(t)x^*(t)dt=\int\int GWDx(t,f)\ GWS_Y(t,f)dtdf.$$

Here, GWDx(t,f) is the generalized warped version of the WD [10] that depends on ξ(b). This generalized form of the quadratic form may be useful in detection applications of systems with arbitrary instantaneous frequency characteristics.

The operator output can now be interpreted as a weighted superposition of dispersive frequency shifts on the input

$$(Yx)(t)=\int\int GSF_Y(\zeta,\beta)e^{-j\pi\zeta\beta}(D_\beta\mathfrak{T}_\tau x)(t)d\zeta d\beta$$

where (***D***βx)(t)=ej2πβξ(t) x(t) is the generalized frequency shifted signal, and $\mathfrak{T}_\tau=W^{-1}_\xi S_\tau W_\xi$ is a generalized warped time-shift operator which can be further simplified depending on the specific warping function, ξ(b).

Depending on the choice of ξ(b), all the WS and SF in Section 2 and Tables 1 and 2 are special cases of the GWS in (7) and GSF in (8). For example, the GWS and GSF in rows 2 in Table 1 and 2 simplify to the conventional WS examples in rows 2 when $\xi(b)=b$.

3. Application Examples

3.1 Analysis problem

In order to demonstrate the importance of the new generalized WS, we analyze a hyperbolic random process $x(t)=\sum \alpha_i x_i(t)$. Here, α_i are uncorrelated, zero-mean random weights and $x_i(t)=e^{j2\pi c_i \ln(t)}$, $t>0$, $i=1,2,3$, are hyperbolic FM, deterministic signals. Note that each signal term $x_i(t)$ has hyperbolic instantaneous frequency, c_i/t. One can show that the hyperbolic WS in (5) of the correlation operator **Rx** with kernel $K_{Rx}(t, \tau)=\mathbf{E}[x(t)x^*(\tau)]$ simplifies to

$$HWS_{Rx}(t,f)=\sum E[|\alpha_i|^2]\delta(f-c_i/t), t>0 \qquad (9)$$

where E[•] is the expectation operator. Figure 1 shows the contour plots of (a) the conventional WS versus (b) the HWS of $\mathbf{R}_x$ of a windowed x(t). Both show time-varying transfer functions with hyperbolic TF characteristics. The advantage of the HWS in (9), is that it is ideally localized along the three instantaneous frequency curves $f=c_i/t$ in the TF plane. The disadvantage of the conventional WS is that it produces spurious components along hyperbolae since it does not match the intrinsic hyperbolic TF characteristics.

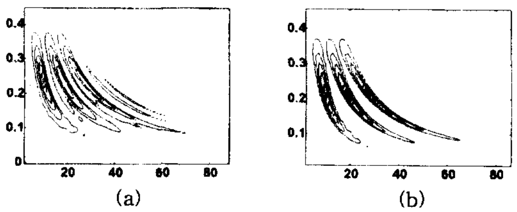

Fig 1. (a) Weyl symbol, WSRx(t, f), and (b) hyperbolic Weyl symbol, HWS_{Rx}(t, f), of a windowed hyperbolic process x(t). The horizontal axis for time and the vertical for normalized frequency.

3.2 Detection problem

Next, we consider the detection of a known deterministic signal s(t) with hyperbolic TF characteristics in nonstationary Gaussian random noise n(t). Assume that the noise has the correlation function $\mathbf{R}n(t,\tau)$ whose support region area is less than unity in the hyperbolic SF domain. Here, the support region of a hyperbolic SF, $HSF_{Rn}(\zeta,\beta)$, of the noise process n(t) is the region in (ζ, β) where $HSFRn(\zeta,\beta)\neq 0$. The test statistic of the optimal likelihood ratio detector is $\mathbf{Re}\{<\mathbf{R}_n^{-1}x, s>\}$ where $\mathbf{R}_n$ is the correlation operator and x(t) is the received signal. The inner product is defined as $<x,y>=\int x(t)y^*(t)dt$ and $\mathbf{Re}\{a\}$ is the real part of a.

Using the hyperbolic version of the quadratic form, one obtains

$$Re[<R_n^{-1}x, s>]=\int\int HWS_{R_n^{-1}}(t,f)\, Re[\, Q_{xs}(t,f)]\,dtdf$$

where $Q_{xs}(t,f)$ is the cross Q-distribution of x(t) and s(t). Similar to the conventional underspread operator approximations in [6,7], we show that if the hyperbolic SFs of two operators $\boldsymbol{Y}$ and $\boldsymbol{S}$ are confined in a small area (jointly underspread), then the hyperbolic WS of the composite operator $\boldsymbol{YS}$ can be approximated as the product of the hyperbolic WS of each operator [4], i.e.

$$HWS_{YS}(t,f)\approx HWS_Y(t,f)\; HWS_S(t,f).$$

For the two correlation operators $\boldsymbol{R}_n$ and $R_{n^{-1}}$, we show that

$$HWS_{R_nR_n^{-1}}(t,f)\approx HWS_{R_n}(t,f)\; HWS_{R_n^{-1}}(t,f)\approx 1.$$

This simplifies the TF test statistic for detecting a deterministic signal [7]

$$Re[<R_n^{-1}x, s>]\approx \int\int Re[\, Q_{xs}(t,f)]\,/\, HWS_{R_n}(t,f)\,dtdf.$$

4. Conclusions

The conventional WS and SF are most useful for systems producing constant time shifts and frequency shifts on the signal. The WS are time-frequency representations that can be interpreted as time-varying spectra for random processes. In this paper, using warping techniques, we generalized the conventional narrowband WS and SF to new WS and SF better matched to dispersive systems. For example, we defined the hyperbolic WS and SF matched to hyperbolic frequency shifts and scale changes, the power WS and SF matched to power law frequency shifts, and the exponential WS and SF matched to exponential frequency shifts. We presented specialized forms of the new WS, SF, and corresponding quadratic forms. We also provided application examples in analysis and detection to demonstrate the advantages of our new results.

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