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Absolute Stability of the Simple Fuzzy Logic Controller

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Abstract

The stability analysis for the fuzzy logic controller (FLC) has widely been reported. Furthermore many research in the FLC has been introduced to decrease the number of parameters representing the antecedent part of the fuzzy control rule. In this paper we briefly explain a single-input fuzzy logic controller (SFLC) or simple-structured FLC which uses only a single input variable. And then we analyze that it is absolutely stable based on the sector bounded condition. We also show the feasibility of the proposed stability analysis through a numerical example of a mass-damper-spring system.

Key Words : Fuzzy Logic Control, Stability Analysis, Absolute Stability, Mass-damper-spring System

1. Introduction

According as the controlled plants become more complex and large-scaled, the development of more intelligent control schemes is required in the control field. A fuzzy logic control is one of proper schemes for this tendency.

Recently, fuzzy control has been applied successfully to many industrial applications due to a number of advantages. But it still has some disadvantages: excessive number of tuning factors, difficulty to stability analysis, and etc..

The conventional FLC has many tuning parameters: membership functions, scaling factors, and so forth. In order to improve this problem, most fuzzy logic controls use the error and the change-of-error as fuzzy input variables regardless of the complexity of controlled plants. But these FLC's are typically suitable for the case of some simple lower order plants. In case of general complex higher order plants, all process states are commonly required as fuzzy input variables for a desirable performance. Unfortunately, it needs a huge number of control rules, membership functions, and scaling factors. The single-input fuzzy logic control which greatly simplifies the design process of the conventional FLC was proposed in some papers[1-4]. It used only a single input variable for the FLC of an arbitrary controlled plant with the minimum phase property. Then the control performance was quite a good. Therefore the SFLC had many advantages besides the simplification of the design procedure of the conventional FLC.

Many research has been proposed to develop the stability analysis of the FLC. Kickert and Mamdani[5] used describing function techniques to study the stability analysis. Ray et al.[6] applied circle criteria scheme for linear SISO and MIMO systems associated with FLC. Tanaka and Sugeno[7] studied the stability and design technique for a new type of the FLC. Fuh and Tung[8] analyzed the robust stability of a conventional FLC.

In this paper we analyze the absolute stability of the SFLC and represent a numerical example. We first expand a nonlinear controlled plant into a Taylor series about a nominal operating point. And a fuzzy control system is transformed into a Lure system with nonlinearities. We also prove that the closed-loop system with the SFLC satisfies the sector condition globally.

This paper is organized as follows. We simply describe the SFLC in the following section. In Section 3, we analyze the absolute stability of the SFLC. We present a numerical example in order to show the feasibility of the proposed stability analysis in Section 4. We finally discuss a concluding remarks in Section 5.

2. Single-input Fuzzy Logic Control

The SFLC was designed for FLC's with skew-symmetric property in a fuzzy control rule table[1,2].

Let the controlled process be a system with n-th order (linear or nonlinear) state equation:

$$\begin{aligned} \dot{\mathrm{x}} &= \mathrm{F}(\mathrm{x}, u), \\ y &= x \end{aligned} \tag{1}$$

with

$$\begin{aligned} \mathrm{x} &= [x_1, x_2, \cdots, x_n]^T \\ &= [x, \dot{x}, \cdots, x^{(n-1)}]^T, \end{aligned} \tag{2}$$

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where $F(x, u)$ are partially known continuous functions representing both system dynamics and unknown external disturbances, $x(t) \in R^n$ is the process state vector, and $u(t) \in R$ and $y(t) \in R$ are the input and output of the system, respectively.

The control problem is to force $y(t)$ to track a given bounded reference input signal $x_d(t)$. Let $e(t)$ be the tracking error vector as follows

$$\begin{aligned} e(t) &= x(t) - x_d(t) \\ &= [e(t), \dot{e}(t), \cdots, e^{(n-1)}(t)]^T. \end{aligned} \tag{3}$$

The rule form for the conventional (PD-type) FLC using two fuzzy input variables of the error and the change-of-error is as follows:

$$R^{ij}_{old}: \text{ If } e \text{ is } LE_i \text{ and } \dot{e} \text{ is } LDE_j, \text{ then } u \text{ is } LU_{ij}$$
$$i = 1, 2, \cdots, M, \quad j = 1, 2, \cdots, N,$$

where LE, LDE, and LU are the linguistic values taken by the process state variables e, $\dot{e}$, and u, respectively. If the controlled plant has minimum phase property, then the fuzzy control rule table is skew symmetric. That is, the absolute magnitude of the control input, $|u|$, is approximately proportional to the distance from a straight line called the switching line. Then a signed distance d_s is defined as follows:

$$\begin{aligned} d_s &= sgn(s_l) \cdot \frac{|\dot{e} + \lambda e|}{\sqrt{1+\lambda^2}} \\ &= \frac{\dot{e} + \lambda e}{\sqrt{1+\lambda^2}}, \end{aligned} \tag{4}$$

where

$$sgn(s_l) = \begin{cases} 1 & \text{for } s_l > 0 \\ -1 & \text{for } s_l < 0 \end{cases}. \tag{5}$$

It is a distance between an operating point and the switching line. Thus, we conclude that:

$$u \propto -d_s. \tag{6}$$

Then, a fuzzy rule table can be established on an one-dimensional space of d_s instead of a two-dimensional space of the phase plane. That is, the control action can be determined by d_s only. So, we call it SFLC. The rule form for the SFLC is given as follows:

$$R^k_{new}: \text{ If } d_s \text{ is } LDL_k \text{ then } u \text{ is } LU_k,$$

where LDL_k is the linguistic value of the signed distance in the k-th rule. Then the rule table can be established on an one-dimensional space like Table 1.

Table 1. Rule table for the SFLC.

d_s	LDL_{-2}	LDL_{-1}	LDL_0	LDL_1	LDL_2
u	LU_2	LU_1	LU_0	LU_{-1}	LU_{-2}

In Table 1, subscripts -2, -1, 0, 1, and 2 denote fuzzy linguistic values of Negative Big (NB), Negative Small (NS), ZeRo (ZR), Positive Small (PS), and Positive Big (PB), respectively. Hence, the number of rules is greatly reduced compared to the case of the conventional FLC's. Also, we can easily add or modify rules for fine control.

The general n-input FLC has rules of the following form:

$$R^k_{GO}: \text{ If } e_1 \text{ is } LE^1_k, \ e_2 \text{ is } LE^2_k, \cdots, \text{ and } e_n \text{ is } LE^n_k \text{ then } u \text{ is } LU_k,$$
$$k = 1, 2, \cdots, m^n,$$

where m is the number of fuzzy sets for each fuzzy input variable and LE^i_k ($i = 1, 2, \cdots, n$) is the linguistic value taken by the process state variable e_i $(= x^{(i-1)} - x_d^{(i-1)})$ in the k-th rule.

Similar to the 2-dimensional rule table for R^{ij}_{old}, the n-dimensional one for R^k_{GO} also satisfies skew-symmetry property and the absolute magnitude of the control input is proportional to the distance from its main diagonal hyperplane (instead of the diagonal line in the two-dimensional table). Then d_s of Eq. (4) is changed to a general signed distance D_s as follows:

$$D_s = \frac{e^{(n-1)} + \lambda_{n-1} e^{(n-2)} + \cdots \lambda_2 \dot{e} + \lambda_1 e}{\sqrt{1 + \lambda^2_{n-1} + \cdots \lambda^2_2 + \lambda^2_1}}. \tag{7}$$

That is, D_s represents the signed distance from the operating point to the switching hyperplane. Then the rule table is still equivalent to Table 1 except D_s instead of d_s. From Eq. (7) we can see that the general signed distance, D_s, contains knowledge of all process states as well as the error and the change-of-error.

3. Absolute Stability of the SFLC

We analyze the stability in the case that the SFLC operates as a nonlinear controller. That is, we assume that the relationship between input and output of the SFLC is nonlinear.

We first expand a nonlinear controlled plant (1) into a Taylor series about (x_0, u_0):

$$\begin{aligned} \dot{x} &= A_0 x + B_0 u + g(x, u), \\ y &= C_0^T x, \end{aligned} \tag{8}$$

where

$$A_0 = \frac{\partial f}{\partial x}\bigg|_{(x_0, u_0)}, \tag{9}$$

$$B_0^T = [\,0 \quad b(x_0)\,], \tag{10}$$

and

$$C_0^T = [1 \quad 0]. \tag{11}$$

Here g(x,u) includes modeling error, aging, uncertainties, and disturbances. And x_0 and u_0 denote the nominal operating point and the nominal input, respectively. Then Eq. (8) is called by the perturbed Lure system.

We consider the SFLC. Similar to R^k_{new}, its generalized rule form is as follows:

$$R^{(k)}_{GSI}\ :\ \text{If } D_s \text{ is } LGDL^{(k)} \text{ Then } u \text{ is } LU^{(k)}.$$

From Table 1, we can see that the output of the SFLC is symmetric with respect to zero action and bounded by a linear gain. That is, the control input u is expressed as follows:

$$u = -\psi(D_s) \tag{12}$$

That is, $\psi(\cdot)$ is a nonlinear function that belongs to a sector [0, τ], where τ is a positive constant.

The block diagram of the closed-loop system with the SFLC can be represented as Fig. 1.

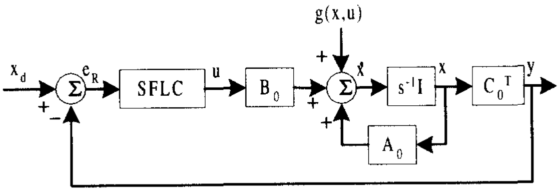

Fig. 1. Block diagram of the closed-loop system with the SFLC.

As $x_d = 0$, D_s is expressed by the following equation.

$$\begin{aligned} D_s &= \frac{e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \cdots + \lambda_2\dot{e} + \lambda_1 e}{\sqrt{1+\lambda_{n-1}^2 + \cdots \lambda_2^2 + \lambda_1^2}} \\ &= \frac{1}{\sqrt{\sum_{i=1}^{n}\lambda_i^2}}\left(x^{(n-1)} + \lambda_{n-1}x^{(n-2)} + \cdots + \lambda_2\dot{x} + \lambda_1 x\right) \\ &= C_{d0}^T x, \end{aligned} \tag{13}$$

where

$$C_{d0}^T = \frac{1}{\sqrt{\sum_{i=1}^{n}\lambda_i^2}}[\lambda_1, \lambda_2, \cdots, \lambda_{n-1}, 1], \quad \lambda_n = 1. \tag{14}$$

Then the closed-loop system with the SFLC is summarized as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu + g(x,u), \\ u &= -\psi(D_s), \\ D_s^T &= C_d^T x, \end{aligned} \tag{15}$$

where $A = A_0$, $B = K_u B_0$, and $C_d^T = K_d C_{d0}^T$. Now the block diagram of Fig. 1 is changed to Fig. 2.

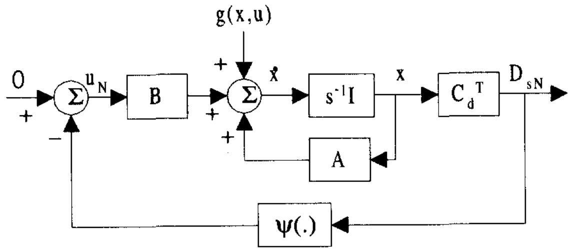

Fig. 2. Block diagram of the system with the nonlinear SFLC.

As explained in Eq. (12), $\psi(\cdot)$ is a time-invariant nonlinearity that satisfies the following sector condition globally.

$$\psi(D_s)[\psi(D_s) - \tau D_s] \le 0 \tag{16}$$

Then we can guarantee that the proposed nonlinear SFLC is absolutely stable.

Consider the system (15), where A is Hurwitz, (A,B,Cd) is a minimal realization of $G(s) = C_d(sI - A)^{-1}B$, and the nonlinearity g(x,u) is bounded as follows:

$$\|g(x,u)\|_2 \le \nu\|x\|_2 \le \frac{\varepsilon_g}{2\|P\|_{i2} + 2\eta\tau^2\|C_d\|_2^2}\|x\|_2, \tag{17}$$

$$\|P\|_{i2} = [\lambda_{\max}(P^*P)]^{1/2}, \ \nu > 0, \ \varepsilon_g > 0, \tag{18}$$

and $\psi(\cdot)$ is a time-invariant nonlinearity that satisfies the sector condition (16) globally. Then the system is absolutely stable if there is $\eta \ge 0$ with $-\frac{1}{\eta}$ not an eigenvalue of A such that

$$\mathrm{Re}[1 + (1 + j\eta\omega)\tau G(j\omega)] > 0, \ \forall\omega \in R, \tag{19}$$

where

$$G(j\omega) = \mathrm{Re}[G(j\omega)] + j\,\mathrm{Im}[G(j\omega)]. \tag{20}$$

The above proposition is proved as follows: Consider the following Lure-type Lyapunov function[9].

$$V_s = x^T P x + 2\eta\int_0^{D_s}\psi(\sigma)\tau d\sigma \tag{21}$$

The derivative of V_s along the trajectories of the system is given by

$$\begin{aligned} \dot{V}_s &= x^T(PA + A^TP)x + 2x^TPBu + 2x^TPg + 2\eta\tau\dot{D}_s\psi \\ &= x^T(PA + A^TP)x - 2x^TPB\psi + 2x^TPg + 2\eta\tau\psi C_d(Ax + Bu + g). \end{aligned} \tag{22}$$

From the sector condition, we can see that

$$-2\psi(D_s)[\psi(D_s) - \tau D_s] \ge 0. \tag{23}$$

Thus,

$$\begin{aligned} \dot{V}_s &\le x^T(PA + A^TP)x - 2x^TPB\psi + 2\eta\tau\psi C_d(Ax + Bu) + 2x^TPg + 2\eta\tau\psi C_d g - 2\psi(\psi - \tau C_d x) \\ &\le x^T(PA + A^TP)x - 2x^T(PB - \eta\tau A^TC_d^T - \tau C_d^T)\psi - (2 + 2\eta\tau C_d B)\psi^2 + 2x^TPg + 2\eta\tau\psi C_d g. \end{aligned} \tag{24}$$

Choose η such that

$$2(1+\eta\tau C_d B) = w^2. \tag{25}$$

From the given condition (19) we see that there are symmetric positive-definite matrices P and P_0, a vector L, and a positive constant ε_p such that[9]

$$\begin{aligned} PA + A^T P &= -L^T L - \varepsilon_p P \\ PB &= \tau C_d^T + \eta\tau A^T C_d^T - wL^T \\ \varepsilon_p P &= \varepsilon_p P_0 + \varepsilon_g I. \end{aligned} \tag{26}$$

Therefore,

$$\begin{aligned} \dot{V}_s \le & -\varepsilon_p x^T P x + 2x^T P g + 2\eta\tau\psi C_d g \\ & - [x^T L^T L x - 2w\psi x^T L^T + w^2\psi^2] \\ \le & -\varepsilon_p x^T P x - [Lx - w\psi]^T [Lx - w\psi] \\ & + 2\|x\|_2 \|P\|_{i2} \|g\|_2 + 2\eta\tau^2 \|C_d\|_2^2 \|x\|_2 \|g\|_2. \end{aligned} \tag{27}$$

From Eqs. (17) and (18), the inequality (27) can be summarized as follows:

$$\begin{aligned} \dot{V}_s &\le -x^T(\varepsilon_p P - \varepsilon_g I)x \\ &\le -\varepsilon_p x^T P_0 x, \end{aligned} \tag{28}$$

which is negative definite.

4. Numerical Example

Let's consider a mass-damper-spring system in order to show the feasibility of the proposed stability analysis. The controlled plant is illustrated in Fig. 3, in which the spring is a soft spring, $m = c = 1$, and $k(x_1) = 3 - x_1^2$. Then the nominal dynamic equations of the system are as follows[8]:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -3 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ x_1^3 \end{bmatrix}, \\ y &= [1 \quad 0] x, \end{aligned} \tag{29}$$

where x_1 and x_2 represent the position and the velocity, respectively.

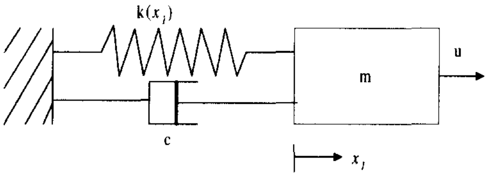

Fig. 3. Mass-damper-spring system.

In this example, A_0, B_0, C_0, and $g(x, u)$ are known as Eq. (29). In the SFLC, we let the scaling factors of K_d and K_u be equal to $2\sqrt{1+\lambda^2}$ and 1, respectively. And 0.5 is used as the value of the design parameter λ. Like Table 1, five fuzzy sets are applied to both fuzzy variables. The max-product inference and the center of gravity defuzzification are selected for a general fuzzy logic system. Then we can obtain the value of τ of 1.1.

Now we can get the closed-loop system of Eq. (15). That is,

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_d = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Therefore,

$$\begin{aligned} G(s) &= C_d^T (sI - A)^{-1} B \\ &= \frac{2s+1}{s^2+s+3}. \end{aligned} \tag{30}$$

From the Popov plot of Eq. (30), we can obtain the value of η of 0.4. The value of w^2 is given by Eq. (25). From Eq. (26),

$$\begin{aligned} L^T L = & \frac{\tau^2}{w^2}[(I+\eta A^T) C_d C_d^T (I+\eta A)] \\ & - \frac{\tau}{w^2}[PBC_d^T (I+\eta A) \\ & + (I+\eta A^T) C_d B^T P] + \frac{1}{w^2} PBB^T P. \end{aligned} \tag{31}$$

From Eqs. (26) and (31), the following Riccati equation is derived.

$$\bar{A}^T P + P\bar{A} - PRP + Q = 0, \tag{32}$$

where

$$\begin{aligned} \bar{A} &= A + \frac{\varepsilon_p}{2} I - \frac{\tau}{w^2} BC_d^T (I+\eta A), \\ R &= -\frac{1}{w^2} BB^T, \\ Q &= \frac{\tau^2}{w^2} (I+\eta A^T) C_d^T C_d (I+\eta A). \end{aligned}$$

Let ε_p=0.83, solve Eq. (32) using MATLAB, we have

$$P = \begin{bmatrix} 5.79 & 1.11 \\ 1.11 & 1.90 \end{bmatrix}.$$

Then, $\|P\|_2 = 6.08$. From Eq. (26), we get $\varepsilon_g = 1.33$. Therefore, ν of the condition (17) is given by
$\nu \le 0.078$.

5. Concluding Remarks

We briefly explained a simple FLC called SFLC. The SFLC had many advantages: The number of fuzzy rules was greatly reduced compared to conventional FLC's, and thus computational complexity was mitigated. Also, generation, modification, and tuning of control rules were very easy. Furthermore, the control performance of the proposed SFLC was nearly the same as that of the conventional skew-symmetric FLC's.

We showed that the SFLC is absolutely stable from the sector bounded condition and also proved the feasibility of the proposed stability analysis through a numerical example of a mass-damper-spring system.

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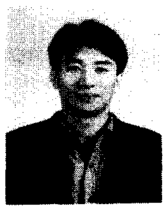

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