Control Algorithm for Stabilization of Tilt Angle of Unmanned Electric Bicycle

Sangchul Han, Jongkil Han, and Woonchul Ham

Abstract: In this papers, we derive a simple kinematic and dynamic formulation of an unmanned electric bicycle. We also check the controllability of the stabilization problem of bicycle. We propose a new control algorithm for the self stabilization of unmanned bicycle with bounded wheel speed and steering angle by using nonlinear control based on the sliding patch and stuck phenomena which was introduced by W. Ham. We also propose a sort of optimal control strategy for steering angle and driving wheel speed that make the length of bicycle's path be the shortest. From the computer simulation results, we prove the validity of the proposed control algorithm.

Keywords: electric bicycle, sliding mode control, sliding patch, lyapunov function, stabilization

I. Introduction

One of the hot research topic in mobile robot is the automatic motion planning [1]. P. Ferbach[2] dealt with tractor-trailer robots evolving in a plane between obstacles by using a progressive constraint approach. There has been many automatic planning problems with car-like robots. In 1988, I.J.Cox made experimental cart, "Blanche" that was a tricycle configuration with a single front wheel which serves both for steering and driving the cart and two passive load-bearing rear wheels [5] [6]. In those paper, he introduced the cart coordinates and kinematics and solved path tracking problem. We adopted the same approach from those papers to derive the kinematics of coordinate and an unmanned Electric bicycle [5]. Recently, there were some researches concerning control problem of bicycle. Among these, some researchers utilized fuzzy and inteligent control [7] and others tried to closed-loop, time-invariant and globally stable nonlinear control law for a bicycle-like kinematic model based on Lyapunov stability theorem [8]. But almost all of those papers are focusing on tracking and planning problem and in my opinion no one has yet tried to solve the automatic planning problem for the stabilizing the attitude of the unmanned bicycle while making it be staying in some given region. In this paper, we try to solve the above automatic planning problem with bounded inputs such as bounded steering angle and bounded driving wheel velocity without making the bicycle pass over the given boundary of some region. We also try to find feasible motion to make bounded region as small as possible.

This paper presents a nonholonomic motion planning method applied to the unmanned electric bicycle. We describe the formulation of the problem touched in this paper. In the first, we propose a motion planning which can only guarantee the stabilization of the tilt angle of bicycle without restriction of its location, that is, bicycle can go any where without falling down. Next we suggest a new motion planning strategy which can guarantee not only the stabilization but also the regional boundedness of its location. The computer simulation results show the effectiveness of the proposed control algorithm

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II. Unmanned Electric Bicycle

In this section, we consider the kinematic and dynamic model of electric bicycle.

1. Kinematic and Dynamic Models

A bicycle is depicted in Fig. 1. Let p_f denote the center point of the steering front wheel and p_r the center point of the driving rear wheel. Let L be the distance between p_f and p_r.

The bicycle's position is parameterized by the coordinate x, y of p_r and by the orientation of θ. The configuration of the bicycle can be determined by kinematic parameter vector $q = (x, y, \theta)$. We denote by ω the angular velocity of the driving wheel, by α the steering angle, by ϕ the tilt angle of the center of mass of bicycle with respect to the normal axis of the ground, by h the distance of the center of mass of bicycle from the ground, and by R the radius of driving wheel. Steering angle α is mechanically bounded by α_{max}, and ω is bounded by ω_{max}. ¿From the nonholonomic constraint that the wheels doesn't slide sideways and centrifugal force from the curvature of the path, we obtain the following kinemetic and dynamic equations and refer to [3] for the details. We can see that the kinematic equation (1),(2),(3) is the same as that of equation(3.1) of [5] and equation(1) of [6] and the dynamic equation(4) can be derived by considering centrifugal force from the curvature of the path and the gravitational force. We can also see that the dynamics is independent of the mass of bicycle and we assumed that the bicycle is modeled as point mass in this paper.

$$\dot{x}(t) = R\omega(t)cos\alpha(t)cos\theta(t) \quad (1)$$

$$\dot{y}(t) = R\omega(t)cos\alpha(t)sin\theta(t) \quad (2)$$

$$\dot{\theta}(t) = (R/L)\omega(t)sin\alpha(t) \quad (3)$$

$$h\ddot{\phi}(t) = gsin\phi(t) - (R^2/L)sin\alpha(t)\omega^2(t)cos^2\alpha(t)cos\phi(t) \quad (4)$$

where g denotes the constant of gravity.

2. Problem Formulation

We try to solve the following two types of control problems

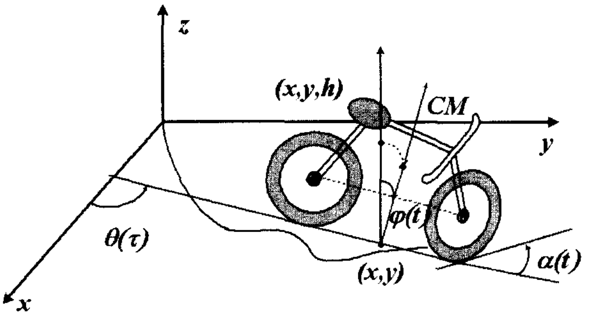

Fig. 1. Kinematic and dynamic parameters of Bicycle.

2.1 Type A

We try to find the angular velocity $\omega(t)$ of the driving wheel and steering angle $\alpha(t)$ that satisfy the following condition

$$\lim_{t\to\infty}\phi(t)=0,$$

that is, we only focus on the stabilization of the bicycle.

2.2 Type B

In this case, we try to find the angular velocity $\omega(t)$ of the driving wheel and steering angle $\alpha(t)$ that satisfy the following conditions

$$\lim_{t\to\infty}\phi(t)=0$$
$$(x,y)\in\Re=\{(u,v)\mid ||(u,v)||<R_1\},$$

that is, we try to make the bicycle do not cross over a bounded area.

III. Stuck and Sliding Patch

In this section, we will summarize the stuck and sliding patch theory which was introduced by W.Ham [4]. Let the following 2-nd order nonlinear system be marginally stable, that is, its state trajectory is oscillating in the phase plane.

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1(t),x_2(t))\\ \dot{x}_2(t)&=f_2(x_1(t),x_2(t))\end{aligned}\tag{5}$$

We summarize all kinds of "stuck" or "sliding patch" of the 2-nd order nonlinear system by adding some kinds of linear and nonlinear switching functions as follows.

1. Linear Stuck and Sliding Patch

Type SL 1:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2)\quad-\gamma_1 sgn(x_1)\\ \dot{x}_2(t)&=f_2(x_1,x_2)\end{aligned}\tag{6}$$

Type SL 2:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2)\\ \dot{x}_2(t)&=f_2(x_1,x_2)\quad-\gamma_2 sgn(x_2)\end{aligned}\tag{7}$$

Type SL 3:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2)-\gamma_1 sgn(\alpha x_1+\beta x_2)\\ \dot{x}_2(t)&=f_2(x_1,x_2)-\gamma_1 sgn(\alpha x_1+\beta x_2)\end{aligned}\tag{8}$$

Type SL 4:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2)-\gamma_1 sgn(\alpha x_1+\beta x_2)\\ &\quad-\gamma_2 sgn(\beta x_1-\alpha x_2)\\ \dot{x}_2(t)&=f_2(x_1,x_2)-\gamma_1 sgn(\alpha x_1+\beta x_2)\\ &\quad-\gamma_2 sgn(\beta x_1-\alpha x_2)\end{aligned}\tag{9}$$

2. Nonlinear Stuck and Sliding Patch

Type SN 1:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2)-\quad\gamma_1\frac{\partial f}{\partial x_1}sgn(f(x_1,x_2))\\ \dot{x}_2(t)&=f_2(x_1,x_2),\quad\gamma_1=\frac{\gamma_3}{\frac{\partial f}{\partial x_1}^2}\end{aligned}\tag{10}$$

Type SN 2:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2),\quad\gamma_1=\frac{\gamma_3}{\frac{\partial f}{\partial x_2}^2}\\ \dot{x}_2(t)&=f_2(x_1,x_2)-\quad\gamma_1\frac{\partial f}{\partial x_2}sgn(f(x_1,x_2))\end{aligned}\tag{11}$$

Type SN 3:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2)\quad-\gamma_1\frac{\partial f}{\partial x_1}sgn(f(x_1,x_2))\\ \dot{x}_2(t)&=f_2(x_1,x_2)\quad-\gamma_1\frac{\partial f}{\partial x_2}sgn(f(x_1,x_2))\end{aligned}\tag{12}$$

Type SN 4:

$$\begin{aligned}\dot{x}_1(t)&=f_1(x_1,x_2)-\gamma_1\tfrac{\partial f}{\partial x_1}sgn(f(x_1,x_2))\\ &\quad-\gamma_2\tfrac{\partial f^\perp}{\partial x_1}sgn(f^\perp(x_1,x_2))\\ \dot{x}_2(t)&=f_2(x_1,x_2)-\gamma_1\tfrac{\partial f}{\partial x_2}sgn(f(x_1,x_2))\\ &\quad-\gamma_2\tfrac{\partial f^\perp}{\partial x_2}sgn(f^\perp(x_1,x_2))\end{aligned}\tag{13}$$

where

$$\gamma_1=\frac{\gamma_3}{\frac{\partial f}{\partial x_1}^2+\frac{\partial f}{\partial x_2}^2},\ \gamma_2=\frac{\gamma_4}{\frac{\partial f^\perp}{\partial x_1}^2+\frac{\partial f^\perp}{\partial x_2}^2}$$

and γ_3,γ_4 are positive constants, $f(0,0)=0, f^\perp(0,0)=0$ and the nonlinear switching functions must be selected to satisfy one of the following conditions if it is possible.

Condition 1: For any c_1 and c_2, the two curves described by $f(x_1,x_2)=c_1$ and $f^\perp(x_1,x_2)=c_2$ must be orthogonal to each other at any intersection points.

In real situation, it is very difficult to find switching functions that satisfy the above condition. So we weaken the above condition as follows.

Condition 2: Two curves described by $f(x_1,x_2)=c_1$ and $f^\perp(x_1,x_2)=c_2$ must be nearly orthogonal to each other at any intersection points in a local region.

Let us take an example that satisfies the condition 2 as follows, where we select nonlinear switching function $f(x_1,x_2)=x_1+x_2^3, f^\perp(x_1,x_2)=x_1^3-x_2$. As we can see in fig. 2, the state trajectory of the above nonlinear system converges to origin and there exists a stable sliding patch on the nonlinear switching curve expressed by $f(x_1,x_2)=x_1+x_2^3=0$.

Example 1:

$$\begin{aligned}\dot{x}_1(t)&=(x_1^2-2x_1+7)x_2-\tfrac{30}{1+9x_2^4}sgn(x_1+x_2^3)\\ &\quad-\tfrac{30x_1^2}{1+9x_1^4}sgn(x_1^3-x_2)\\ \dot{x}_2(t)&=-x_1(x_2^2+1)-\tfrac{90x_2^2}{1+9x_2^4}sgn(x_1+x_2^3)\\ &\quad+\tfrac{10}{1+9x_1^4}sgn(x_1^3-x_2),\\ &\quad x_1(0)=0,\ x_2(0)=10\end{aligned}\tag{14}$$

IV. Control Law

In this section, we will apply the previous stuck and sliding patch theory for the stabilization of the unmanned electric bicycle.

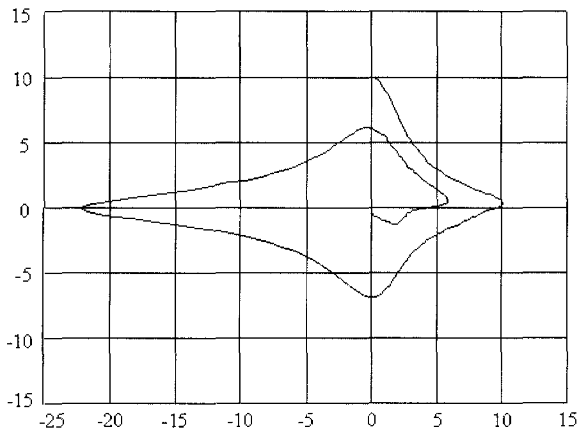

Fig. 2. State trajectory of Example 1.

1. Type A Problem

At first, we propose a control law for the *Type A* problem which was described in section II. In this case, the control problem is to find the control input $u_1(t) = cos^2\alpha(t)sin\alpha(t)\omega^2(t)$ such that $\phi(t)$ converges to zero as time goes to infinity. The dynamic equation (4) can be expressed by the following state equations

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= a_1 sinx_1(t) - b_1 cosx_1(t)\, u_1(t) \end{aligned} \tag{15}$$

where $a_1 = g/h$, $b_2 = (R^2/Lh)$, and $x_1(t) = \phi(t)$. Then we propose a control law through the following theorem.

theorem 1: If we adopt the following control law

$$u_1(t) = \frac{2a_1}{b_1} sinx_1(t) + \frac{\gamma}{b_1} sgn(5x_1(t) + x_2(t)) \tag{16}$$

then the system(15) can be asymptotically stable for some initial condition such that

$$|x_1(0)| = |\phi(0)| < \phi_{max},\ x_2(0) = \phi'(0) = 0,$$

where ϕ_{max} is positive real number between 0 and $\pi/2$.

proof: If we apply the above control law (16) to the dynamic equations (15), we obtain the following equation.

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= a_1 sinx_1(t) - 2a_1 sinx_1(t)cosx_1(t) \\ &- \gamma cosx_1(t)sgn(5x_1(t) + x_2(t)) \end{aligned} \tag{17}$$

If the switching gain $\gamma = 0$, this equation becomes marginally stable and this fact can be easily shown by choosing Lyapunov candidate function

$$V(t) = \frac{x_2^2(t)}{2} + a_1(sin^2 x_1(t) - \int_0^{x_1} sin(\sigma)d\sigma) \tag{18}$$

which is positive for any $|x_1(t)| < \frac{\pi}{2}$ and proving that its time derivative is zero. Therefore we can see that equation (17) is the special case of stable nonlinear stuck and patch *Type SN* 2. Please refer to [4] for more detail analysis of the stability for the general nonlinear stucks and sliding patches.

2. Type B Problem

Now, we propose a control law for the *Type B* problem which was described in section II. It is to find the angular velocity $\omega(t)$ of the driving wheel and steering angle $\alpha(t)$ that satisfy the following conditions

$$\lim_{t\to\infty} \phi(t) = 0$$

$$(x, y) \in \Re = \{(u, v) \mid ||(u, v)|| < R_1\},$$

that is, we try to make the bicycle do not cross over a bounded area. We propose the following control law to make bounded area small from the heuristic control strategy.

Control Law 1:

$$\begin{aligned} \alpha(t) &= \frac{\alpha_{max}}{u_{1,max}} u_1(t) \\ \omega(t) &= \begin{cases} \sqrt{\frac{u_1}{cos^2\alpha sin\alpha}} sgn(u_1(t)) & \text{if } u_1(t) \neq 0 \\ 0 & \text{if } u_1(t) = 0 \end{cases} \end{aligned} \tag{19}$$

We also propose another control law that makes the over all length of the path l expressed by

$$l = \int \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}\, dt \tag{20}$$

as short as possible. By using equation (1)-(2), we obtain

$$l = \int \sqrt{R^2\, \omega(t)^2 cos^2\alpha(t)}\, dt \tag{21}$$

If we use the following fact that the control input is defined as follows

$$u_1(t) = cos^2\alpha(t)sin\alpha(t)\omega^2(t),$$

l can be written by

$$l = \int \sqrt{\frac{R^2\, u_1(t)}{sin\alpha(t)}}\, dt. \tag{22}$$

Now we propose the following control law that makes the over all length of the path be the shortest.

Control Law 2:

$$\begin{aligned} \alpha(t) &= \alpha_{max} sgn(u_1(t)) \\ \omega(t) &= \begin{cases} \sqrt{\frac{u_1}{cos^2\alpha sin\alpha}} sgn(u_1(t)) & \text{if } u_1(t) \neq 0 \\ 0 & \text{if } u_1(t) = 0. \end{cases} \end{aligned} \tag{23}$$

We can see that the control structure of the above optimal control law is very similar to the "bang-bang" control which might be occurred in time optimal control problem.

V. Simulation

In this section, we try to verify the effectiveness of the proposed control laws from the computer simulations. We can easily obtain the simulation results by using Mathlab simulink toolbox. We set the parameters R, h, L, γ, α_{max}, $u_{1,max}$ and initial condition $\phi(0)$ as $0.35m$, $0.4m$, $1.0m$, 2, $\frac{\pi}{4}$, 70 and $0.2rad$ respectively. Fig. 3 shows the the response of the generated tilt angle of the bicycle and the control input $u_1(t)$. As we can see, the tilt angle converges to zero in less than 2.0 sec, and there occurs a chattering in control input. Fig. 4 and Fig. 5 show the trajectories of the paths and their lengths when we apply control law 1 and control law 2 respectively. The length of path obtained by using control law 1 is $6.82m$ and the length of path obtained by using control law 2 is $4.13m$. Therefore, we verify that the length of path obtained by using control law 2 is shorter

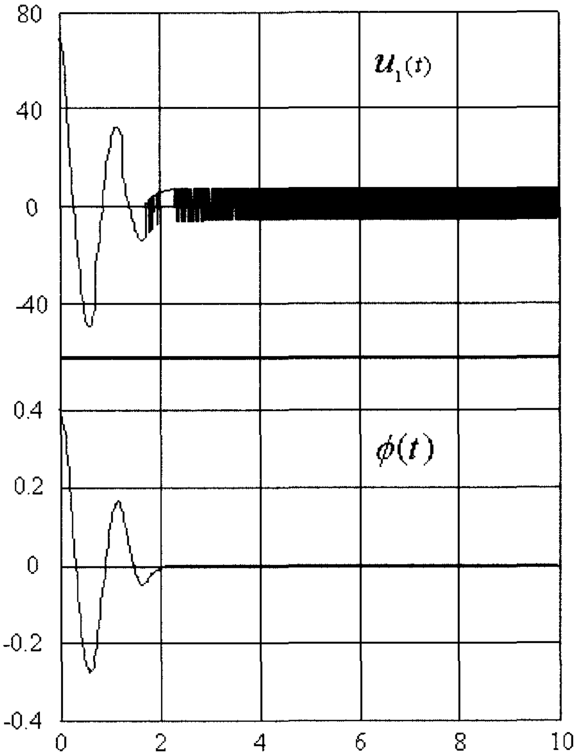

Fig. 3. Control input u_1 and tilt angle ϕ.

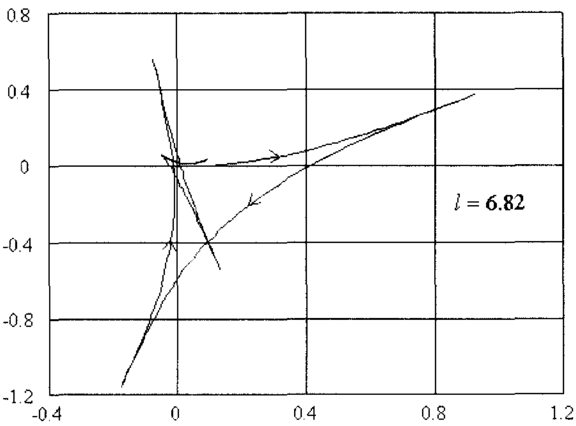

Fig. 4. Trajectoy of the path. (control law 1)

than that of control law 1 as we guessed in section IV. But as you can see in Fig.3, there occurs a chattering problem and in real field, this proposed control algorithm must be a little bit modified to handle this kind of chattering problem and we will tackle this problem in near future. The optimal concept which is dealt in section IV is dependent on the control law expressed by equation (16) and if we can find another better control law which can replace this equation, then we can obtain better short trajectory which also guarantees the stability of tilt angle.

VI. Conclusion

In this paper, we propose nonholonomic motion planning that can stabilize the tilt angle of the unmanned electric bicycle. We solve the two kinds of control problems which can guarantee not only the stabilization but also the regional boundedness of bicycle's location. We suggest a kind of optimal control strat-

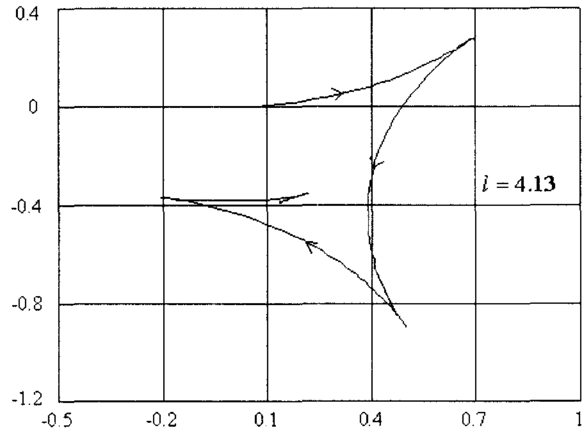

Fig. 5. Trajectoy of the path. (control law 2)

egy that makes the length of the path as short as possible based on the stable control input which is derived by using nonlinear stuck and patch phenomena introduced by W.Ham. The optimal concept which is dealt in this paper is dependent on the control law expressed by equation (16). Computer simulation results show the effectiveness of the proposed control algorithms. I hope this short note can be helpful to the researchers who are eager to find nonlinear control law for stabilization of bicycle problem. The optimal concept which is dealt in this paper is dependent on the control law expressed by equation (16) and if we can find another better control law which can replace this equation, then we can obtain better short trajectory which also guarantees the stability of tilt angle.

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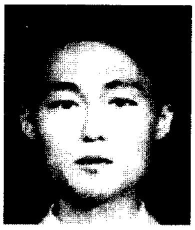

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