Input Constrained Receding Horizon Control with Nonzero Set Points and Model Uncertainties

Young Il Lee

Abstract: An input constrained receding horizon predictive control algorithm for uncertain systems with nonzero set points is proposed. For constant nonzero set points, models with uncertainty can be converted into an augmented incremental system through the use of integrators and the problem is transformed into a zero-state regulation problem for the incremental system. But the original constraints on inputs are converted into constraints on the sum of control inputs at each time instants, which have not been dealt in earlier constrained robust receding horizon control problems. Recursive state bounding technique and worst case minimizing strategy developed in earlier works are applied to the augmented incremental system to yield an offset error free controller. The resulting algorithm is formulated so that it can be solved using LP.

Keywords: constrained input, model uncertainty, tracking control, invariant set, free control moves

I. Introduction

Stability of receding horizon control in the presence of physical constraints has been studied by many researchers. Rawlings and Muske[10] provided stability conditions in the form of equality constraints at the end of a finite horizon N. These equality constraints, which is likely to be overly stringent, can be relaxed by requiring that the state at the end of prediction belongs to a set which is invariant and feasible under a state feedback law, $\mathbf{u} = F\mathbf{x}$. This strategy leads to the dual-mode paradigm in which N free control moves are deployed to steer the state into a feasible and invariant target set, so that all future predicted states $\mathbf{x}$ remain within this set and so that the feedback law itself is feasible. The dual-mode paradigm was suggested by Mayne and Michalska [8] for nonlinear systems and adopted by Lee *et al.*[4] for linear deterministic systems.

In the presence of model uncertainties, application of the N free control moves is likely to result in impracticable computational complexity. Kothare *et al.*[1] avoided the computational complexity by removing the use of N free control moves but instead allowing the state feedback gain to vary. The free control moves were re-introduced by Kouvaritakis *et al.*[2] and Lee and Kouvariatkis[7][6] using ellipsoidal invariant sets and polyhedral invariant sets, respectively. Computational complexity was avoided by employing autonomous augmented system representation[2] and by recursive state bounding[7]. In the work of Lee and Kouvariatkis[6], recursive state bounding technique was improved so that large target invariant sets corresponding to 'detuned' state feedback gain and good predicted performances corresponding to 'tightly tuned' state feedback gain can be combined.

In the above mentioned works on robust receding horizon control, the control objective is to bring the system state to zero state ($\mathbf{x} = 0$), which is a common steady state for all models in the uncertainty set with the same input $\mathbf{u} = 0$. For nonzero setpoints, however, there is no single steady state to serve as a reference in a zero state regulation problem, which makes it difficult to define target invariant sets around a steady state.

In this paper, a constrained receding horizon control with nonzero set point is developed by deploying the dual-mode paradigm. Through the use of integrators, uncertain systems can be augmented so that the augmented incremental dynamics have a common steady state *e.g.* zero steady state error, zero increments on states and inputs, for all models in the uncertainty set. For the incremental systems, original constraints on inputs are converted into constraints on sum of control inputs at each time instants. Application of the recursive bounding technique to the augmented incremental system can be made by taking the constraints on the sum of control inputs into account with care.

II. System description

Consider the system with uncertainty:

$$\mathbf{x}(k+1) = \tilde{A}\mathbf{x}(k) + \tilde{B}\mathbf{u}(k), \quad (1)$$
$$\mathbf{y}(k) = C\mathbf{x}(k) \quad (2)$$
$$-\mathbf{u}_{max} \leq \mathbf{u}(k) \leq \mathbf{u}_{max} \quad (3)$$

where $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{y}$ are state, input and output vectors of dimension n, m and l, respectively, $\tilde{A}$, $\tilde{B}$ and C are matrices of conformal dimensions and vector inequalities apply on an element-by-element basis. $\tilde{A}$ and $\tilde{B}$ belong to the uncertainty class:

$$\Omega = \{(\tilde{A}, \tilde{B}) : (\tilde{A}, \tilde{B}) = \sum_{i=1}^{n_p} \eta_i (A_i, B_i),$$
$$\eta_i \geq 0, \sum_{i=1}^{n_p} \eta_i = 1\}, \quad (4)$$

where $n_p = 2^{n_a} \times 2^{n_b}$, and n_a, n_b are the numbers of uncertain parameters in $\tilde{A}$ and $\tilde{B}$, respectively. It has been known that the model (1- 2) can be transformed into the following incremental state-space model:[3]

$$\mathbf{x}_a(k+1) = \tilde{\mathcal{A}}\mathbf{x}_a(k) + \tilde{\mathcal{B}}\Delta\mathbf{u}(k) \quad (5)$$
$$\mathbf{y}(k) = \mathcal{C}\mathbf{x}_a(k), \quad (6)$$

Manuscript received: Mar. 23, 2001., Accepted: July. 10, 2001.
Young Il Lee: Dept. of Control and Instrumentation, Seoul National University of Technology.(yilee@duck.snut.ac.kr)
* This study was supported financially by the Korea Science and Engineering Foundation through the Research Center for Aircraft Parts Technology(ReCAPT), Gyeongsang National University, Korea.

where $\Delta = 1 - q^{-1}$, $q^{-1}\mathbf{u}(k) = \mathbf{u}(k-1)$ and $\mathbf{x}_a(k)$, $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}$ and $\mathcal{C}$ are defined as:

$$\mathbf{x}_a(k) := [y(k)' \ \Delta\mathbf{x}(k)']', \qquad \tilde{\mathcal{A}} := \begin{bmatrix} I & C\tilde{A} \\ 0 & \tilde{A} \end{bmatrix}$$
$$\tilde{\mathcal{B}} := [(C\tilde{B})' \ \tilde{B}']', \qquad \mathcal{C} := [I \ 0].$$

When we consider constant set points $\mathbf{y}_r$ *i.e.* $\mathbf{y}_r(k) = \mathbf{y}_r(k+1)$, we have:

$$\mathbf{x}_E(k+1) = \tilde{\mathcal{A}}\mathbf{x}_E(k) + \tilde{\mathcal{B}}\Delta\mathbf{u}(k) \tag{7}$$
$$\mathbf{e}(k) = \mathcal{C}\mathbf{x}_E(k), \tag{8}$$

by subtracting $\mathbf{y}_r$ from the top part of $\mathbf{x}_a$, where

$$\mathbf{e}(k) := y(k) - y_r, \quad \mathbf{x}_E(k) := [\mathbf{e}(k)' \ \Delta\mathbf{x}(k)']'. \tag{9}$$

The constraints on inputs in (3) can be rewritten as constraints on the sum of input increments as follows:

$$|\mathbf{u}_0 + \sum_{j=1}^{k} \Delta\mathbf{u}(j)| \le \mathbf{u}_{max}, \quad \forall k > 0, \tag{10}$$

where $\mathbf{u}_0$ is initial value of the actuators and the absolute values of vectors are defined as the vectors of absolute values. Thus a tracking control problem for the original system (1-3) is converted into a regulation problem for the incremental system (7-10).

Recursive state bounding technique [7] and worst case minimizing approach [6], which were used for regulation control of system (1 3), can be deployed to develop a robust predictive control method for the uncertain incremental system (7- 10). However, application of the earlier methods to the incremental system is not trivial because, unlike the earlier approaches, sum of infinite numbers of control increments is involved in defining feasible invariant sets of this incremental system.

III. Robust predictive control method for incremental systems

1. Definition of feasible invariant sets

Application of $\Delta\mathbf{u} = F\mathbf{x}_E$ and use of the state transformation $\mathbf{z}_E = W\mathbf{x}_E$ yieldsthe closed loop state equation:

$$\mathbf{z}_E(k+1) = \tilde{\Phi}^W \mathbf{z}_E(k), \tag{11}$$

where $\tilde{\Phi}^W = W(\tilde{\mathcal{A}} + \tilde{\mathcal{B}}F)V$, $W = V^{-1}$. The matrix W can be used as a degree of freedom to be chosen (off-line) so that there exists an invariant and feasible set for the system (11). Then using (11), it is easy to show the following result.

Theorem 1: [7] The set

$$\hat{\mathcal{R}}_F^W(\alpha) = \{\mathbf{x}_E \in R^{n+l} | \ |\mathbf{z}_E| \le \alpha\}, \tag{12}$$

where α is a positive column vector of length n, $|\mathbf{z}_E|$ represents the modulus of each elements of $\mathbf{z}_E$ and the inequality applies on element-by-element basis, is invariant with respect to the closed-loop dynamics of (11) if and only if

$$|\Phi_i^W|\alpha \le \alpha, \ i = 1, 2, \cdots, n_p, \tag{13}$$

where $\Phi_i^W := W(\mathcal{A}_i + \mathcal{B}_i F)V$ and $(\mathcal{A}_i, \mathcal{B}_i)$ are defined as $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}})$ except that $\tilde{A}$ and $\tilde{B}$ in $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}})$ are replaced by A_i and B_i, respectively. Furthermore if (13) holds with strict inequality, then F is robustly stabilizing.

The set $\hat{\mathcal{R}}_F^W$ is only invariant but can be modified by adding the feasibility condition on α so that the control law $\Delta\mathbf{u} = F\mathbf{x}_E$ satisfies the input constraints for all the elements in the set. Unlike earlier work of Lee and Kouvaritakis [7][6], the feasibility condition is related with bound on sum of $F\mathbf{x}_E(k+i)$ for $i = 0, 1, 2, \cdots$ as:

$$\begin{aligned} &|\mathbf{u}(k-1) + \sum_{j=0}^{\infty} \Delta\mathbf{u}(k+j|k)| \qquad (14)\\ &= |\mathbf{u}(k-1) + \sum_{j=0}^{\infty} F\hat{\mathbf{x}}_E(k+j|k)| \\ &\le \mathbf{u}_{max} \ \forall\mathbf{x}(k) \in \hat{\mathcal{R}}_F^W(\alpha), \end{aligned}$$

where $\hat{\mathbf{x}}_E(k+j|k) = \tilde{\Phi}^j\mathbf{x}_E(k)$. Relation (14) states that when we consider guaranteed feasibility of $\Delta\mathbf{u} = F\mathbf{x}_E$ for the incremental system (7-10), not only the values of the current state $\mathbf{x}_E$ but also current values of the system input $\mathbf{u}$ should be taken into account. Provided that the invariance condition (13) is satisfied with strict inequality *i.e.*

$$|\Phi_i^W|\alpha \le \rho\alpha, \ i = 1, 2, \cdots, n_p, \tag{15}$$

for some positive scalar $\rho(< 1)$, sufficient condition of (14) can be derived as:

$$\begin{aligned} &|\mathbf{u}(k-1) + \sum_{j=0}^{\infty} F\hat{\mathbf{x}}_E(k+j|k)| \qquad (16)\\ &\le |\mathbf{u}(k-1)| + |F^W| \sum_{j=0}^{\infty} |\tilde{\Phi}^W|^j |\mathbf{z}_E(k)| \\ &\le |\mathbf{u}(k-1)| + |F^W| \sum_{j=0}^{\infty} \rho^j \alpha \\ &\le |\mathbf{u}(k-1)| + \frac{1}{1-\rho}|F^W|\alpha \\ &\le \mathbf{u}_{max}, \end{aligned}$$

where $F^W = F \cdot V$. Based on the above arguments, feasibility and invariance of the set $\hat{\mathcal{R}}_F^W(\alpha)$ can be established as per the following theorem.

Theorem 2: Consider uncertain system (1-4) and its incremental error dynamics (7-10) for a constant reference value with a transformed state $\mathbf{z}_E = W\mathbf{x}_E$. For any initial state of $\mathbf{x}_E$ in $\hat{\mathcal{R}}_F^W$, the state is guaranteed to remain in $\hat{\mathcal{R}}_F^W$ and asymptotically converges to zero by a state feedback control $\Delta\mathbf{u} = F\mathbf{x}_E$ provided α and initial system input $\mathbf{u}_0$ satisfy:

$$|\Phi_i^W|\alpha \le \rho \cdot \alpha, \ i = 1, 2, \cdots, n_p \tag{17}$$
$$|\mathbf{u}_0| + \frac{1}{1-\rho}|F^W|\alpha \le \mathbf{u}_{max} \tag{18}$$

for some positive scalar $\rho(< 1)$.

Remark 1: The existence of α satisfying (17) is guaranteed under the assumption that the Perron-frobenius norm of $\max_i |\Phi_i^W|$ is less than or equal to $\rho(< 1)$ [9], where the maximization occur elementwise. This assumption places constraints on the choice of W. If there is no uncertainty and the pair $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}})$ is controllable, we can choose a stabilizing F so that $\tilde{\Phi}$ has n distinct eigenvalues and V can be composed of n-distinct eigenvalues of $\tilde{\Phi}$[5]. In this case the Perron-Frobenius norm of $|\tilde{\Phi}^W|$ would be the maximum magnitude of the eigenvalues, which is less than 1 and guarantee the existence of α satisfying (17-18). It is known that the pair $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}})$ is controllable if and only if $(\tilde{A}, \tilde{B})$ is controllable and the following condition holds [3]:

$$rank \begin{bmatrix} 0 & C \\ \tilde{B} & \tilde{A} - I \end{bmatrix} = n + l. \tag{19}$$

In the sequel $\mathcal{R}_F^W(\alpha(\mathbf{u}_0))$ would be used to denote a set of incremental state $\mathbf{x}_E$ which are feasible and invariant with respect to a state feedback control $\Delta\mathbf{u} = F\mathbf{x}_E$ for a given initial value of input $\mathbf{u}_0$.

2. Constrained receding horizon tracking control

For convenience, the set of pairs $(\alpha, \mathbf{u}_0)$ satisfying (17-18) for a given ρ will be denoted by $S_{(\alpha,\mathbf{u}_0)}$. Here, the closed-loop prediction formulation [11]:

$$\begin{aligned} \Delta\mathbf{u}(k+l|k) &= F\mathbf{x}_E(k+l|k) + \mathbf{c}(k+l|k) \\ &= FV\mathbf{z}_E(k+l|k) + \mathbf{c}(k+l|k) \end{aligned} \tag{20}$$

will be adopted. The perturbation sequence $\mathbf{c}$'s are deployed to steer the current state $\mathbf{x}_E(k)$ into a target invariant set *i.e.*

$$\mathbf{x}_E(k+N|k) \in \mathcal{R}_F^W(\alpha(\mathbf{u}_0)), \tag{21}$$

for some $(\alpha, \mathbf{u}_0) \in S_{(\alpha,\mathbf{u}_0)}$ under the feasibility condition

$$\begin{aligned} &|\mathbf{u}(k+l|k)| && (22) \\ &= |\mathbf{u}(k-1) + \sum_{j=0}^{l}\{F^W\mathbf{z}_E(k+l|k) + \mathbf{c}(k+l|k)\}| \\ &\le \mathbf{u}_{max}(l = 0, 1, \cdots, N-2) \\ &|\mathbf{u}(k+N-1|k)| && (23) \\ &= |\mathbf{u}(k-1) + \sum_{j=0}^{N-1}\{F^W\mathbf{z}_E(k+l|k) + \mathbf{c}(k+l|k)\}| \\ &\le \mathbf{u}_0. \end{aligned}$$

We will search αs satisfying (21) on line to avoid the conservatism we may have when we use $\mathcal{R}_F^W(\alpha(\mathbf{u}_0))$ with some fixed α as our target set. Uncertainty makes the use of condition (21) and (22-23) difficult, but it is possible to use recursive bounding technique (Lee and Kouvaritakis, 2000a,b) to overcome this problem. The recursive bounding technique is based on the following obvious lemma.

Lemma 1: Given a matrix M, let $M^+ = max(M, 0)$ and $M^- = max(-M, 0)$, then

$$\begin{aligned} \underline{\mathbf{y}} \quad &\le \mathbf{y} \le \quad \overline{\mathbf{y}} \ implies \\ M^+\underline{\mathbf{y}} - M^-\overline{\mathbf{y}} \quad &\le M\mathbf{y} \le \quad M^+\overline{\mathbf{y}} - M^-\underline{\mathbf{y}}, \end{aligned} \tag{24}$$

where maximization is conducted on an element-by-element basis.

Linear inequalities which guarantee terminal membership condition (21) and feasibility condition (22-23) can be established as per the following theorem.

Theorem 3: Consider uncertain system (1-4) and its incremental error dynamics (7-10) for a constant reference value with a transformed state $\mathbf{z}_E = W\mathbf{x}_E$. Conditions (21) and (22-23) are satisfied despite the uncertainties if we can find bounds $\underline{\mathbf{z}}_E(\cdot|k)$ and $\overline{\mathbf{z}}_E(\cdot|k)$ such that:

$$\begin{aligned} &\Psi_i^+\underline{\mathbf{z}}_E(k+l|k) - \Psi_i^-\overline{\mathbf{z}}_E(k+l|k) + B_i^W\mathbf{c}(k+l|k) \\ &\quad \ge \underline{\mathbf{z}}_E(k+l+1|k),\ l = 0, 1, \cdots, N-1 && (25) \\ &\Psi_i^+\overline{\mathbf{z}}_E(k+l|k) - \Psi_i^-\underline{\mathbf{z}}_E(k+l|k) + B_i^W\mathbf{c}(k+l|k) \\ &\quad \le \overline{\mathbf{z}}_E(k+l+1|k),\ l = 0, 1, \cdots, N-1, && (26) \\ &\underline{\mathbf{z}}_E(k+N|k) \ge -\alpha && (27) \\ &\overline{\mathbf{z}}_E(k+N|k) \le \alpha && (28) \end{aligned}$$

and

$$\begin{aligned} &\mathbf{u}(k-1) + \sum_{j=0}^{l}\{\mathcal{F}^+\underline{\mathbf{z}}_E(k+j|k) - \mathcal{F}^-\overline{\mathbf{z}}_E(k+j|k) \\ &\quad +\mathbf{c}(k+j|k)\} \ge -\mathbf{u}_{max},\ l = 0, 1, \cdots, N-2 && (29) \\ &\mathbf{u}(k-1) + \sum_{j=0}^{l}\{\mathcal{F}^+\overline{\mathbf{z}}_E(k+j|k) - \mathcal{F}^-\underline{\mathbf{z}}_E(k+j|k) \\ &\quad +\mathbf{c}(k+j|k)\} \le \mathbf{u}_{max},\ l = 0, 1, \cdots, N-2 && (30) \\ &\mathbf{u}(k-1) + \sum_{j=0}^{N-1}\{\mathcal{F}^+\underline{\mathbf{z}}_E(k+j|k) - \mathcal{F}^-\overline{\mathbf{z}}_E(k+j|k) \\ &\quad +\mathbf{c}(k+j|k)\} \ge -\mathbf{u}_0, && (31) \\ &\mathbf{u}(k-1) + \sum_{j=0}^{N-1}\{\mathcal{F}^+\overline{\mathbf{z}}_E(k+j|k) - \mathcal{F}^-\underline{\mathbf{z}}_E(k+j|k) \\ &\quad +\mathbf{c}(k+j|k)\} \le \mathbf{u}_0. && (32) \end{aligned}$$

are satisfied for $i = 1, 2, \cdots, n_p$ with $\underline{\mathbf{z}}_E(k|k) = \overline{\mathbf{z}}_E(k|k) = \mathbf{z}_E(k)$, where $\Psi_i^+ = max(\Phi_i^W, 0)$, $\Psi_i^- = max(-\Phi_i^W, 0)$, $\mathcal{F}^+ = max(F^W, 0)$ and $\mathcal{F}^- = max(-F^W, 0)$.

Proof : Consider the initial bound $\underline{\mathbf{z}}_E(k|k) = \overline{\mathbf{z}}_E(k|k) = \mathbf{z}_E(k)$, which is measurable, then from Lemma 1 and the fact that

$$\mathbf{z}_E(k+l+1|k) = \tilde{\Phi}^W\mathbf{z}_E(k+l|k) + \tilde{B}^W\mathbf{c}(k+l|k), \tag{33}$$

it is possible to derive bounds $\underline{\mathbf{z}}_E(k+1|k)$ and $\overline{\mathbf{z}}_E(k+1|k)$ such that

$$\begin{aligned} \underline{\mathbf{z}}_E(k+1|k) &\le \Psi_i^+\underline{\mathbf{z}}_E(k|k) - \Psi_i^-\overline{\mathbf{z}}_E(k|k) + B_i^W\mathbf{c}(k|k) \\ &\qquad \le \mathbf{z}_E(k+1|k) && (34) \\ \overline{\mathbf{z}}_E(k+1|k) &\ge \Psi_i^+\overline{\mathbf{z}}_E(k|k) - \Psi_i^-\underline{\mathbf{z}}_E(k|k) + B_i^W\mathbf{c}(k|k) \\ &\qquad \ge \mathbf{z}_E(k+1|k), && (35) \end{aligned}$$

for $i = 1, 2, \cdots, n_p$. Recursive application of this argument yields (25-26).

Applying Lemma 1 to conditions (22-23), we have (29-32) and it is easy to see that (27-28) with (31-32) guarantee terminal membership (21). ■

Note that conditions (25)-(32) are linear inequalities on $(\underline{\mathbf{z}}_E(k+j|k), \overline{\mathbf{z}}_E(k+j|k))(j = 1, 2, \cdots, N)$, $\mathbf{c}(k+j)(j = 0, 1, \cdots, N-1)$, $\mathbf{u}_0$ and α. Under the assumption that the underlying state feedback gain F is chosen (off-line) so that the resulting stabilizable set is large, our control strategy is to choose perturbations $\mathbf{c}(\cdot|k)$ so that a worst case performance is optimized. We can concentrate on worst case tracking performance by considering the bounds on the predicted values of

$\mathbf{z}_E(\cdot|k)$ satisfying (25-26) and in particular by considering the maximum of $|\overline{\mathbf{z}}_E(\cdot|k)|$ and $|\underline{\mathbf{z}}_E(\cdot|k)|$. A simple way of achieving this is through the definition of variables $\mathbf{t}(k+l|k)$ which satisfies the linear inequalities:

$$\mathbf{t}(k+l|k) - |\overline{\mathbf{z}}_E(k+l|k)| \geq 0, \quad (36)$$

$$\mathbf{t}(k+l|k) - |\underline{\mathbf{z}}_E(k+l|k)| \geq 0, \quad (37)$$

for $l = 0, 1, \cdots, N$. Then the optimization problem can be formulated as:

$$\min_{\substack{C(N|k), T(N|k), \alpha, \mathbf{u}_o \\ \overline{\mathbf{z}}_E(k+1|k) \cdots \overline{\mathbf{z}}_E(k+N|k) \\ \underline{\mathbf{z}}_E(k+1|k) \cdots \underline{\mathbf{z}}_E(k+N|k)}} J(T(N|k)) \quad (38)$$

$$subject\ to\ (17-18), (25-32)\ and\ (36-37)$$

with $\overline{\mathbf{z}}_E(k|k) = \underline{\mathbf{z}}_E(k|k) = \mathbf{z}_E(k)$, where

$$J(T(N|k)) = \{\sum_{l=0}^{N-1} I_{row} \cdot \mathbf{t}(k+l|k) + \gamma \cdot \mathbf{t}(k+N|k)\}, \quad (39)$$

$I_{row} = [\overbrace{1, \cdots, 1}^{l}, \overbrace{0, \cdots, 0}^{n}]$, $\gamma (\geq 0)$ is a row vector of dimension $n+l$ and $C(N|k), T(N|k)$ are a block vectors containing as block elements the vectors $\mathbf{c}(k+l|k)(l = 0, 1, \cdots, N-1)$, $\mathbf{t}(k+l|k)(l = 1, 2, \cdots, N)$, respectively.

Thus, optimization problem can be solved using LP and a receding horizon tracking control algorithm based on this LP problem can be summarized as:

Algorithm (CRHTC)

Step 1 : Calculate $C^*(N|k)$ by solving the LP (38)

Step 2 : Apply $\mathbf{u}(k) = \mathbf{u}(k-1) + F\mathbf{x}_E(k) + \mathbf{c}^*(0|k)$ to the system.

Stability and offset-error free property of CRHTC can be established as per the following theorem.

Theorem 4: [7] Consider uncertain system (1-4) and its incremental error dynamics (7-10) for a constant reference value $\mathbf{y}_r$ with a transformed state $\mathbf{z}_E = W\mathbf{x}_E$. If algorithm CRHTC has feasible solutions at start time and the terminal weights γ satisfies:

$$\gamma \leq \gamma \overline{\Phi} + I_{row}, \quad (40)$$

where $\overline{\Phi} = \max_i |\Phi_i^W|$, then $\mathbf{x}_E$ converges to zero and $\mathbf{y}$ converges to reference value $\mathbf{y}_r$.

IV. Numerical example

Consider the uncertain system [1] with polyhedral set Ω defined by (4) with $\overline{u} = 1$

$$A_1 = \begin{bmatrix} 0.9347 & 0.5194 \\ 0.3835 & 0.8310 \end{bmatrix} A_2 = \begin{bmatrix} 0.0591 & 0.2641 \\ 1.7971 & 0.8717 \end{bmatrix}$$

$$B = \begin{bmatrix} 1.4462 \\ 0.7012 \end{bmatrix}.$$

Let $A_0 = \frac{1}{2}(A_1 + A_2)$, then a stabilizing state feedback control for its augmented system $(\mathcal{A}_0, \mathcal{B})$ was obtained as $F = -[0.0806\ 0.3491\ 0.3443]$ by designing an LQ controller for augmented states. We use the transform matrix $W = V^{-1}$, where V is composed of eigenvectors of $(\mathcal{A}_0 + \mathcal{B}F)$, where:

$$W = \begin{bmatrix} -1.3810 & -0.6065 & -0.0379 \\ 0.3561 & 1.6018 & -0.0237 \\ 0.9836 & 1.4192 & 1.0051 \end{bmatrix}. \quad (41)$$

The Perron-Frobenius norm of $\overline{\Phi}^W$ is 0.9675 and the existence of γ satisfying (40) is guaranteed and we took γ as [39.1694 24.4308 10.6219]. It is assumed that the system is initially at zero state. Figure 1 shows the output and input trajectory of the system when the reference signal is given as $y_r = 0.5$. During the simulation the underlying system was changed from A_1 to A_2 and then return to A_1 during the steps 1 and 5.

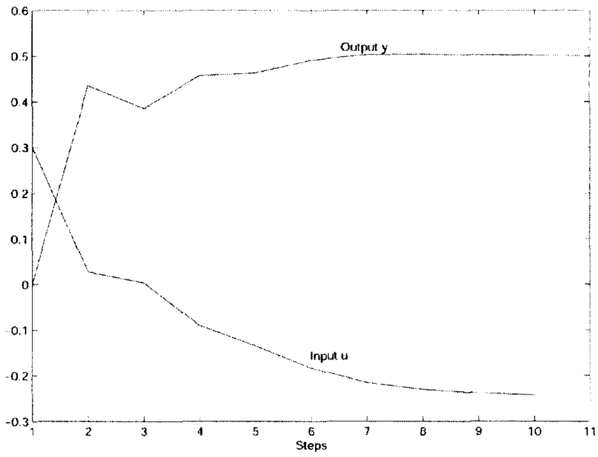

Fig. 1. Time domain response, output and input trajectories for $y_r = 0.5$.

V. Conclusions

In this paper, a receding horizon control method which deals with nonzero set points has been developed for systems with model uncertainty and input constraints. Through the use of integrators, original systems are converted into augmented incremental systems which have constraints on the sum of control inputs at each time instants. Recursive state bounding technique and worst case minimizing strategy, which were developed in earlier works, are applied to the augmented incremental system to yield offset error free control method. The difficulties to deal with sum of infinite numbers of control inputs in defining feasible and invariant sets for the incremental system can be overcome under the assumption that underlying feedback gain F and corresponding transformation matrix W are chosen so that the Perron-Frobenius norm of $\max_i |\Phi_i^W|$ $(i = 1, 2, \cdots, n_p)$ is less than 1. The resulting control algorithm is formulated in the form of Linear Programming which can be solved quite effectively.

References

[1] M. V. Kothare, Balakrishnan, and M. Morari(1996), Robust constrained receding horizon control, *Automatica*, vol. 32, no. 10, pp. 1361-1379.

[2] B. Kouvaritakis, J. A. Rossiter, and J. Schurmans(1999), Efficient robust predictive control, *proceedings of*

[3] W. H. Kwon, & D. G. Byun(1989), *Receding horizon tracking control and its stability properties*, Int. J. Control, vol. 50, no. 5, pp. 1807-1824.

[4] J. W. Lee, W. H. Kwon, and J. Choi(1998), "On stability of constrained receding horizon control with finite terminal weighting matrix", *Automatica*, vol. 34, no. 12, pp. 1607-1612.

[5] Y. I. Lee and B. Kouvaritakis(1999), "Stabilizable regions of receding horizon predictive control with input constraints," *Systems and Control Letters*, vol.38, no.1, pp.13-20.

[6] Y. I. Lee & B. Kouvaritakis(2000a), "Robust receding horizon predictive control for systems with uncertain dynamics and input saturation", *Automatica*, vol. 36, no. 10, pp. 1497-1504.

[7] Y. I. Lee and B. Kouvaritakis(2000b), "A linear programming approach to constrained robust predictive control", *IEEE Trans on Automatic Control*, vol. 45, no. 9. pp. 1765-1769.

[8] D. Q. Mayne and H. Michalska(1993), "Robust receding horizon control of constrained nonlinear systems", *IEEE Trans on Automatic Control*, vol. 38, no. 11, pp. 1623-1633.

[9] A. I. Mees(1981), Achieving diagonal dominances, *Systems and Control Letters*, vol. 1, no. 3, pp. 155-158.

[10] J. B. Rawlings and K. B. Muske(1993), "The stability of constrained receding horizon control", *IEEE Trans. On Automatic Control*, vol. 38, no. 10, pp. 1512-1516.

[11] J. A. Rossiter, B. Kouvaritakis, & M. J. Rice(1997), "A numerically robust state-space approach to state-predictive control strategies", *Automatica*, vol. 34, no. 1, pp. 65-73.

Young Il Lee

He received the B.Sc(1986), M.S.(1988) and Ph.D.(1993) degrees in Control and Instrumentation Engineering from Seoul National University. He was a visiting research fellow of the Department of Engineering Science, Oxford University during Feb. 1998 and July 1999. He was with the Department of Control and Instrumentation at Gyeongsang National University from 1994 till July 2001. His current post is Associate Professor of Seoul National University of Technology.