MLFMA for Capacitance Extraction using Adaptive Triangular Mesh

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Abstract

For fast capacitance computation, a simple mesh refinement technique on MLFMA(Multi-Level Fast Multipole Algorithm) is proposed. The triangular meshes are refined mainly in the area which has heavy charge density. The technique is applied to the capacitance extraction of three dimensional conductors. The results show good convergence with comparable accuracy. An adaptive technique concerned with MLFMA is useful to reduce computation time and the number of elements without additional computational efforts in large three dimensional problems.

Key words : Capacitance, multi-pole algorithm, mesh refinement

Ⅰ. INTRODUCTION

There are many methods to compute the capacitance of complicated three-dimensional structures. Algorithms using MOM (method of moments[1]) or BEM(boundary-element methods[2]) are commonly used to compute the capacitances of three-dimensional structures.

Especially, recently developed MLFMA(Multi-Level Fast Multipole Algorithm) is effectively used to solve capacitance of large systems[3][4]. The computation time complexity for the algorithm is shown to grow nearly as O(mn), where n is the number of elements used to discretize a conductor surfaces, and m is the number of conductors. Although the algorithm is very efficient, conventional uniform surface tessellation is not efficient for interpolating the charge distribution. However, in authors' knowledge, only a few work concerned with the adaptive mesh MLFMA has been reported[5]. To enhance the advantage of MLFMA, mesh refinement techniques with no further additional computation effort are indispensable. In this paper, the efficiency of the automatic surface mesh tessellation and refinement technique is verified.

Ⅱ. THEORY

The capacitance of an m-conductor geometry can be summarized by an m by m symmetric matrix C, where the j th column of C has a positive entry C_{jj}, representing the self-capacitance of conductor j, and negative entries C_{ij}, representing coupling capacitance between conductors j and i ($i \neq j$). To determine the jth column of the capacitance matrix, the short-circuit capacitance technique can be used. Then C_{ij} is numerically equal to the charge on conductor i($i = 1, 2, \cdots, m$). Repeating this procedure m times gives the m columns of C.

The methods to extract the capacitance of arbitrary conducting structures are based on Poisson's equation. Considering that the medium surrounding the conductors is homogeneous, it is necessary to solve the integral equation for the surface charge density

$$\varnothing(r) = \int_{surfaces} \sigma(r')\, G(r,\ r')\ ds', \tag{1}$$

where $\varnothing(r)$ is the surface potential, $\sigma(r')$ is the surface charge density, and $G(r', r)$ is the Green's function corresponding to the given structure. The integral equations in (1) can be reduced to a matrix equation through a mathematical procedure called the method of moments. Equation (1) can be written as a vector-matrix expression

$$\vec{\varnothing} = P\vec{q}, \tag{2}$$

where $\vec{\varnothing} = [\varnothing_1, \varnothing_2, \cdots \varnothing_N]^T$ and $\vec{q} = [q_1, q_2, \cdots q_N]^T$ represent the discrete element potentials and element charges, respectively. P is the potential or influence matrix between these elements. The entries of the N by N matrix P are defined by

$$P_{ij} = \frac{1}{a_j} \int_{element\,j} G(r, r')\, da', \tag{3}$$

which depend only on the geometrical properties of the two elements i and j.

If the number of the discrete element, N, is small, the system of linear equations (2) can be solved using a direct scheme such as Gaussian elimination. In practice, however, since N is very large, it takes long computation time to use a direct method in which the computational time for the solution is $O(N^3)$. In the

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case of using iterative method, the time complexity for matrix -vector multiplication can be reduced to $O(N \log N)$ with the multipole expansion methods. Especially, MLFMA (Multi-Level Fast Multipole Algorithm) is effectively used to solve this problem in $O(N)$.

Ⅲ. MULTI-LEVEL FAST MULTIPOLE ALGORITHM

The key idea in the Fast Multipole Algorithm is to develop a scheme to calculate far-field interactions. In the N-body problem, the strength of the interaction is inversely proportional to the distance between the two particles, commonly referred to as a Coulombic interaction. As two particles get far away, the strength of the interaction diminishes. In these hierarchical methods the far-field particles are grouped together so that the number of interactions can be reduced.

A complete description of the Fast Multipole Algorithm is not given here; the original description is in [6], and its application to capacitance extraction is described in [3]~[5],[7],[8]. Here, brief description for the adaptive algorithm is given.

3-1 Multipole Expansions

Multipole expansions are often used to approximate the far field due to a confined charge distribution. As shown in Fig 1, the potential at the center of a panel i, due to the surface charges on those d panels are approximately given by the truncated multipole expansion

$$\Psi(r_i, \varnothing_i, \theta_i) \approx \sum_{n=0}^{l} \sum_{m=-n}^{n} \frac{M_n^m}{r_i^{n+1}} Y_n^m(\varnothing_i, \theta_i), \tag{4}$$

where the spherical coordinates of the evaluation location are measured relative to the origin of the multipole expansion. $Y_n^m(\varnothing_i, \theta_i)$ are the surface spherical harmonics, M_n^m are the multipole coefficients determined from the panel charges, and l is the expansion order. The multipole expansion coefficients are given by

$$M_m^m = \sum_{j=1}^{d} q_j \rho_j^n Y_n^{-m}(\alpha_n, \beta_j) \tag{5}$$

where ρ_j, α_j and β_j are the spherical coordinates of the jth charge relative to the sphere's center. Given the multipole coefficients, the same multipole expansion can be used to evaluate the potential at many panel centers. For example, in Fig. 1 there are d charged panels. The potentials of the panel centers must be evaluated. A direct calculation of those potentials requires order d^2 operations, but only order d operations are needed if the multipole expansion is used(assuming the expansion order l is fixed).

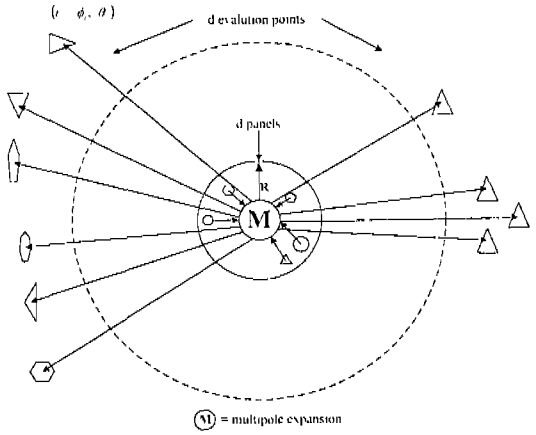

Fig. 1. The evaluation of the potential at $(r_i, \varnothing_i, \theta_i)$.

3-2 Local Expansions

In general, the truncated local expansion approximation for the exact potential in a sphere caused by charges outside the radius of the sphere is given by

$$\Psi(r_j, \varnothing_j, \theta_j) \approx \sum_{n=0}^{l} \sum_{m=-n}^{n} L_n^m Y_n^m(\varnothing_j, \theta_j) r_j^n \tag{6}$$

r_j, $\varnothing_j$ and θ_j are the spherical coordinates of the j th evaluation location with respect to the sphere's center, and the L_n^m factors are the complex local expansion coefficients. For a set of d charges outside the sphere, the local expansion coefficients are given by

$$L_n^m = \sum_{i=1}^{d} \frac{q_i}{\rho_i^{n+1}} Y_n^{-m}(\alpha_i, \beta_i), \tag{7}$$

where ρ_i, α_i and β_i are the spherical coordinates of the ith charge relative to the sphere's center of the sphere containing the evaluation points.

3-3 The Multipole Algorithm

To explain the multipole algorithm, the top view of a simple three dimensional example(two separated bars) is shown in Fig. 2. The biggest cube that contains the entire collection of panels for the problem of interest is determined. This cube will be referred to as the level 0, or root cube. Then, the volume of the cube is broken into eight child cubes of equal size, referred to as level 1 cubes. Each has the level 0 cube as its parent. Each of the level 1 cubes is then subdivided into eight level 2 child cubes. By the same procedure, the cube is divided again into eight equally sized cubes, which are at level 3, and the panels are again distributed based on their center point locations. The number of levels can be further raised depending on the problem size.

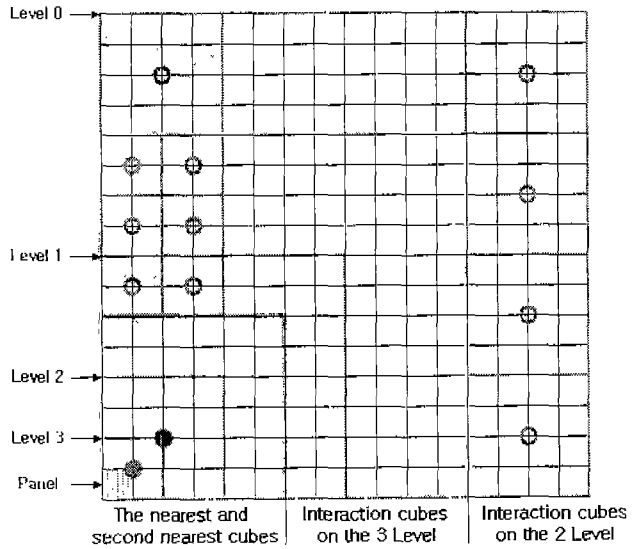

Fig. 2. Cubes of the simple three dimensional example(Top view of two separated bars).

The evaluation points of a cube are the center points of the panels associated with the cube.The nearest neighbors of a cube are those cubes on the same level which have a corner in common with the given cube. The second nearest neighbors of a cube are those cubes on the same level which are not nearest neighbors but have a corner in common with a nearest neighbor. The interaction cubes of a given cube are those cubes which are either the second nearest neighbors of the given cube's parent, or children of the given cube's parent's nearest neighbors, excluding nearest or second nearest neighbors of the given cube.

The structure of the multipole algorithm for computing the n panel potentials form n panel charges is given below.

- Computes the potential due to nearby charges directly.
- Computes the multipole expansion coefficients for every cube at every level.
- Shift the multipole expansion about the child cube's center to a multipole expansion about the cube's center and add it to the multipole expansion coefficient for the cube.
- Convert the multipole expansion about the center of the interaction cube to a local expansion about the cube's center and add it to the local expansion for the cube.
- Shift that expansion to a local expansion about the cube's center, if the cube's parent has a local expansion.
- Evaluates the local expansions at the finest level.

The conversion and shift operations are linear functions of the charges or the expansion coefficients when the geometry is fixed. All coefficients matrices are computed once and stored. These geometry-dependent matrices are used repeatedly in subsequent matrix-vector product calculations.

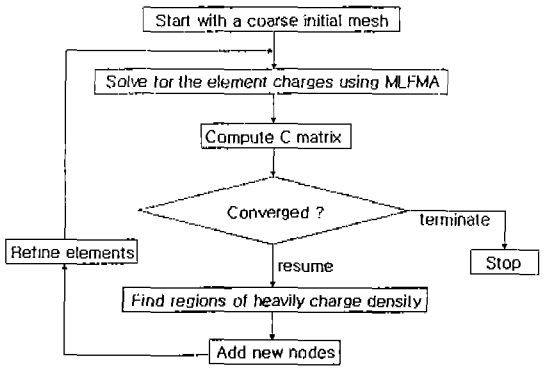

Fig. 3. Diagram of adaptive refinement algorithm.

Ⅳ. ADAPTIVE REFINEMENT TECHNIQUE

Many error estimations and mesh refinement methods are researched and applied to the capacitance computation problem, mainly for the numerical methods based on differential equation, e.g. FEM. To fully take advantage of the MLFMA, simple error estimation and mesh refinement technique with least additional computational cost is required. In this calculation, it is supposed that the errors are mainly spread near edges, corners, and surfaces faced each other which have large charge density than other region, and mesh can be readily refined by using the automatic mesh algorithm. The flow chart of the adaptive refinement algorithm is shown in Fig. 3. At first an initial mesh is generated, and the integral equation is solved using MLFMA. In the following, the capacitance matrix is computed and the refinement of the mesh follows in regions of heavily charge density (see Fig. 4). The problem is solved repeatedly until converged.

Ⅴ. NUMERICAL RESULTS AND CONCLUSION

To test this proposed technique, we calculated the capacitances of the simple two 3-dimensional conducting bars shown

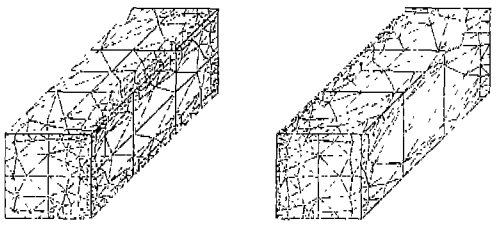

Fig. 4. The refined meshes of two conducting bars.

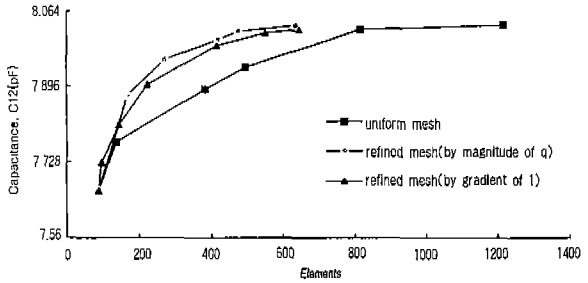

Fig. 5. Convergence of capacitance value using adaptive triangular mesh.

Table 1. Capacitance values

	Capacitance Matrix Entry (pF)	
	C11	C12
Uniform Triangular Mesh (816 elements)	2548.76	−802.26
Adaptive Triangular Mesh (by magnitude of q) (646 elements)	2549.4	−802.14
Adaptive Triangular Mesh (by gradient of q) (636 elements)	2549.3	−803.05

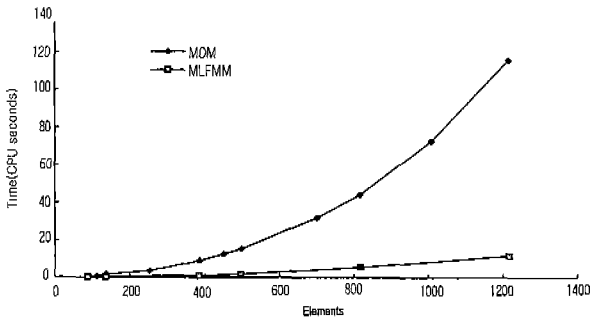

Fig. 6. Comparison of the CPU times(on P-III 800 MHz).

in Fig. 4(two bars spaced 60 m apart, both have 20 m × 20 m cross section and 60 m length). In Fig. 5, the results using the adaptive meshes are compared with one using uniform triangular mesh. The errors are estimated by two different schemes, one is by using the gradient of the charge density, and the other is by using the magnitude of the charge density. The results using these two different methods show very similar convergence. Therefore, for this capacitance extraction, the simple error estimation by using the magnitude of the charge density is more efficient than the one by using the gradient of the charge density which requires additional computation time. The converged capacitance values are in Table 1 for this model. Fig. 6 shows the computation time of the matrix solving in MOM and MLFMA.

By using the mesh refinement technique applied to MLFMA, we can further reduce the computation time(i.e., time complexity) as well as the unknown variables preserving the same accuracy. This makes it possible to efficiently solve very large problem.

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