Efficient Derivation of Closed-Form Green's Functions for a Microstrip Structure

Byoung-Hee Oh[1] · Eui-Joong Kim[1] · Young-Soon Lee[1] · Young-Ki Cho[2]

Abstract

In order to derive simple and accurate closed-form spatial Green's functions for the thick microstrip substrate, an efficient method based on the two-level approach, which circumvents the burdensome steps (i.e., without necessity of extraction of quasi-static contributions and subsequent determination of approximation parameters) in the previous complex image method, is considered in conjunction with the use of the original Prony's method. The present method is observed to give more accurate results for the evaluation of the Green's functions over wider frequency range independently of the source-to-field distances than the previous method.

Key words : open microstrip structure, complex images, two-level approach

Ⅰ. INTRODUCTION

The spatial Green's function for the open microstrip structure is generally represented by time-consuming Sommerfeld integrals. Recently, in order to avoid the numerical integration of Sommerfeld integrals in evaluating the vector and scalar potential Green's functions for the open microstrip structure, the closed-form Green's functions methods[1],[2] were proposed. Some have dealt with closed-form Green's functions method for the case of thick dielectric substrate[1]. On the other hand, the others have dealt with that for the case of thin substrate[2]. In case of thin dielectric substrate as a usual microstrip antenna, the contribution of the surface wave pole(SWP) is not so significant. However, even though microstrip antennas have usually a thin substrate, the contribution of SWP may become significant as in case of thick substrate, as the frequency region of interest becomes higher. And so, in this study, a modified closed-form Green's functions method that is useful over the wide frequency range (i.e., useful in case of thick dielectric substrate) is considered.

In the previous method[1], the spatial closed-form Green's functions for the case of thick substrate consist of three parts : a) quasi-static, b) surface waves, c) complex images. However, in general, the previous method should take into account quasi-static terms as many as possible to reduce inaccuracy due to slow convergence of the quasi-static Green's function in the series form for the scalar potential, in particular, at low frequencies. The purpose of this study is to consider an efficient method which circumvents the burdensome steps of the previous method such as the re-examination of the approximation parameters according to variation of the frequency region of interest and extraction of quasi-static contributions in the spectral domain.

As a result of adoption of the present method, it is observed that the present closed-form Green's functions give more accurate results for the evaluation of the Green's functions over the wide frequency range.

Ⅱ. THEORY

Consider an x-directed electric dipole located above a microstrip substrate, as shown in Fig. 1. By use of the integral transformation technique, the vector and scalar potential Green's functions in spatial domain in the air region are represented as follows :

$$G_A^{xx} = \frac{\mu_0}{4\pi}\left(\frac{e^{-jk_0 r_0}}{r_0} + \int_{SIP} \frac{1}{j2k_{z0}} R_{TE} e^{-jk_{z0}(z+z')} H_0^{(2)}(k_\rho \rho) k_\rho dk_\rho\right) \tag{1}$$

$$G_q = \frac{1}{4\pi\varepsilon_0}\left(\frac{e^{-jk_0 r_0}}{r_0} + \int_{SIP} \frac{1}{j2k_{z0}} (R_{TE}+R_q) e^{-jk_{z0}(z+z')} H_0^{(2)}(k_\rho \rho) k_\rho dk_\rho\right) \tag{2}$$

where, $H_0^{(2)}$ is the Hankel function of the second kind, SIP is the Sommerfeld integration path defined in Fig. 2(a),

$$R_{TE} = -\frac{r_{10}^{TE} + e^{-j2k_{z1}h}}{1 + r_{10}^{TE} e^{-j2k_{z1}h}},$$

$$R_q = \frac{2k_{z0}^2(1-\varepsilon_r)(1-e^{-j4k_{z1}h})}{(k_{z1}+k_{z0})(k_{z1}+\varepsilon_r k_{z0})(1+r_{10}^{TE}e^{-j2k_{z1}h})(1-r_{10}^{TM}e^{-j2k_{z1}h})},$$

$$r_{10}^{TE} = \frac{k_{z1}-k_{z0}}{k_{z1}+k_{z0}}, \quad r_{10}^{TM} = \frac{k_{z1}-\varepsilon_r k_{z0}}{k_{z1}+\varepsilon_r k_{z0}},$$

$$k_{z0}^2 + k_\rho^2 = k_0^2, \quad k_0 = w\sqrt{\mu_0\varepsilon_0}, \text{ and } \quad k_{z1}^2 + k_\rho^2 = \varepsilon_r k_0^2 .$$

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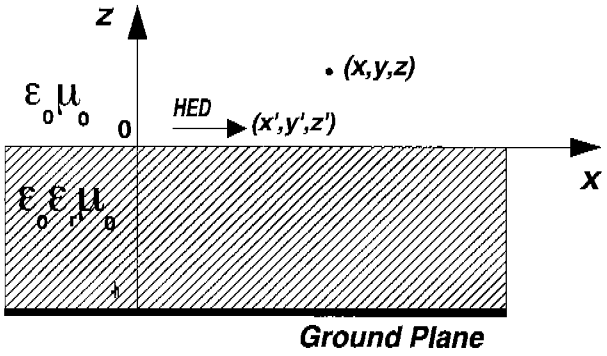


Fig. 1. An x-directed horizontal electric dipole (HED) located over the grounded thick dielectric substrate.

In order to derive simple and accurate closed-form Green's functions from eqns. (1) and (2), we are to consider a method based on the two-level approach[2], which is described as follows :

(1) Find the contribution of only the surface waves(real poles) from the spectral functions R_{TE} and ($R_{TE}+R_q$) and subtract the contribution from the spectral functions, differently from the previous method in which the quasi-static as well as surface wave contributions are extracted at this stage. Let the remaining spectral functions for the vector potential and the scalar potential be denoted by F_{Ar} and F_{qr} respectively.

(2) Then the Prony's method[3] is adopted to approximate the F_{Ar} and F_{qr}. To begin with, the spectral functions are approximated along the first approximation path C_{ap1} given by a parametric equation, $k_{z0}/k_0=-j[T_{02}+t], 0\le t\le T_{01}$, as shown in Fig. 2(b), as follows :

$$f_{Ar1}(\cong F_{Ar} \text{ for } C_{ap1})=\sum_{n=1}^{N_1} a_{1n}e^{-b_{1n}k_{z0}},\quad N_1\le 3 \tag{3}$$

$$f_{qr1}(\cong F_{qr} \text{ for } C_{ap1})=\sum_{n=1}^{N_1} a_{1n}'e^{-b_{1n}'k_{z0}},\quad N_1\le 3 \tag{4}$$

where for a chosen N_1, $a_{1n}(a_{1n}')$ and $b_{1n}(b_{1n}')$ are coefficients and exponents calculated by use of the Prony's method along the C_{ap1} for the $F_{Ar}(F_{qr})$, and N_1 is the number of complex exponentials used in this approximation. For the case of a numerical example in ref[1].($\varepsilon_r=12.6$, $h=1$ mm), $T_{02}=5$ is chosen, for which $[1+T_{02}^2]^{1/2}>\sqrt{\varepsilon_r}$. Here the choice of $T_{01}=50$ is enough to ensure that the contributions of the spectral functions for large k_ρ (i.e., the quasi-static contributions) are mostly considered. This choice is not critical, 40 or 60 could have been chosen instead. However the optimum choice is desired between certain lower and upper bounds.

(3) In order to make the original remaining functions

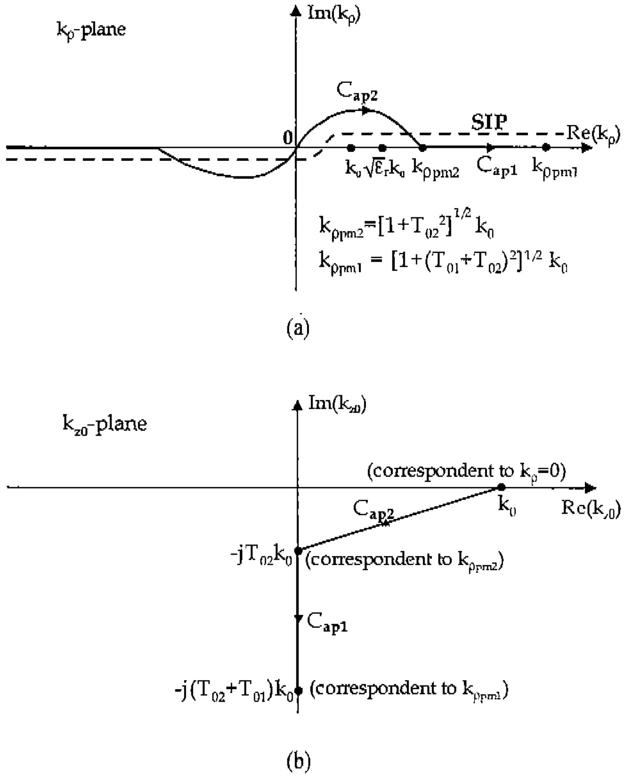


Fig. 2. (a) The approximation path C_{ap1} and C_{ap2} used in two-level approximation in k_ρ-plane (b) The path C_{ap1} and C_{ap2} in k_{z0}-plane correspondent to those in k_ρ-plane. (——— : approximation path, — — — : original SIP).

(F_{Ar} and F_{qr}) fast-converging, subtract the approximating functions(f_{Ar1} and f_{qr1}) from the original remaining functions. Then the remaining functions are approximated along the second approximation path C_{ap2} given by a parametric equation, $k_{z0}/k_0=-jt+(1-t/T_{02}), 0\le t\le T_{02}$, as follows:

$$f_{Ar2}(\cong F_{Ar}-f_{Ar1} \text{ for } C_{ap2})=\sum_{n=1}^{N_2} a_{2n}e^{-b_{2n}k_{z0}},\quad N_2\le 4 \tag{5}$$

$$f_{qr2}(\cong F_{qr}-f_{qr1} \text{ for } C_{ap2})=\sum_{n=1}^{N_2} a_{2n}'e^{-b_{2n}'k_{z0}},\quad N_2\le 4 \tag{6}$$

where for a chosen N_2, $a_{2n}(a_{2n}')$ and $b_{2n}(b_{2n}')$ are coefficients and exponents obtained along the C_{ap2} for the $F_{Ar}-f_{Ar1}(F_{qr}-f_{qr1})$, and N_2 is the number of complex exponentials.

Thus, the spectral functions R_{TE} and ($R_{TE}+R_q$) are composed of three parts : a) complex images corresponding to quasi-static contributions (i.e., f_{Ar1} and f_{qr1}), b) complex images (i.e., f_{Ar2} and f_{qr2}), c) surface waves unlike the previous method mentioned above. Subsequently substitute such R_{TE}

and $(R_{TE}+R_q)$ into eqn. (1) and (2) respectively, and use the Sommerfeld identity and residue theorem. Then following spatial closed-form expressions to be desired are obtained as :

$$G_A^{xx}=G_{As}^{xx}+G_{A,ci1}^{xx}+G_{A,ci2}^{xx}+G_{A,sw}^{xx} \quad (7)$$

$$G_q = G_{qs}+G_{q,ci1}+G_{q,ci2}+G_{q,sw} \quad (8)$$

where

$$G_{As}^{xx}=\frac{\mu_0}{4\pi}\frac{e^{-jk_0r_0}}{r_0},\ G_{qs}=\frac{1}{4\pi\varepsilon_0}\frac{e^{-jk_0r_0}}{r_0},\ r_0=\sqrt{\rho^2+(z-z')^2},$$

$$G_{A,ci1}^{xx}=\frac{\mu_0}{4\pi}\sum_{i=1}^{N_1}a_{1n}\frac{e^{-jk_0r_{1n}}}{r_{1n}},\ G_{q,ci1}=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N_1}a_{1n}'\frac{e^{-jk_0r_{1n}'}}{r_{1n}'},$$

$$G_{A,ci2}^{xx}=\frac{\mu_0}{4\pi}\sum_{i=1}^{N_2}a_{2n}\frac{e^{-jk_0r_{2n}}}{r_{2n}},\ G_{q,ci2}=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N_2}a_{2n}'\frac{e^{-jk_0r_{2n}'}}{r_{2n}'},$$

$$r_{1n}=\sqrt{\rho^2+(z+z'-jb_{1n})^2},\ r_{1n}'=\sqrt{\rho^2+(z+z'-jb_{1n}')^2},$$

$$r_{2n}=\sqrt{\rho^2+(z+z'-jb_{2n})^2},\ r_{2n}'=\sqrt{\rho^2+(z+z'-jb_{2n}')^2}.$$

Here $G_{A,sw}^{xx}$ and $G_{q,sw}$ follow the same meanings as those in ref[1]. While the first(fourth) parts in the right sides of eqns. (7) and (8) are the sources (surface wave) terms which are dominant in the near(far) field region, the second and third parts correspond to the complex images, which are important in the intermediate region, being related to the leaky waves.

Ⅲ. NUMERICAL RESULTS

In order to demonstrate the efficiency of the present method, the results for $(F_{qr}-f_{qrl})$ obtained by the present method are compared with the corresponding previous results for F_2 obtained by the extraction of both surface wave poles and quasi-static contributions from $(R_{TE}+R_q)$ in Fig. 3. From this figure, the present method based upon the two-level approach is observed to give much more rapidly decaying(convergent) results in comparison with the previous method in dealing with the quasi-static contributions.

In order to further examine the validity of the proposed

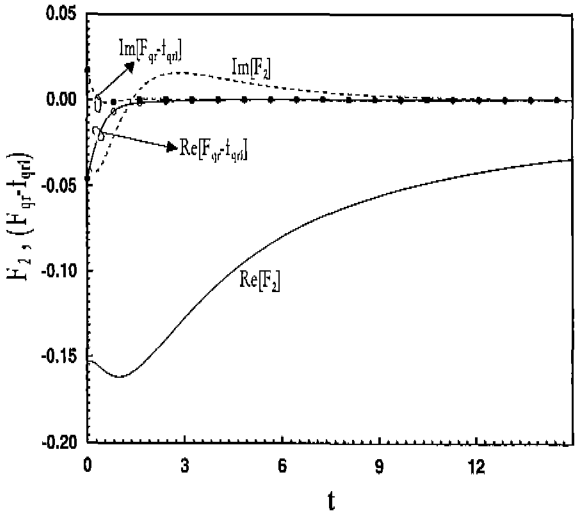
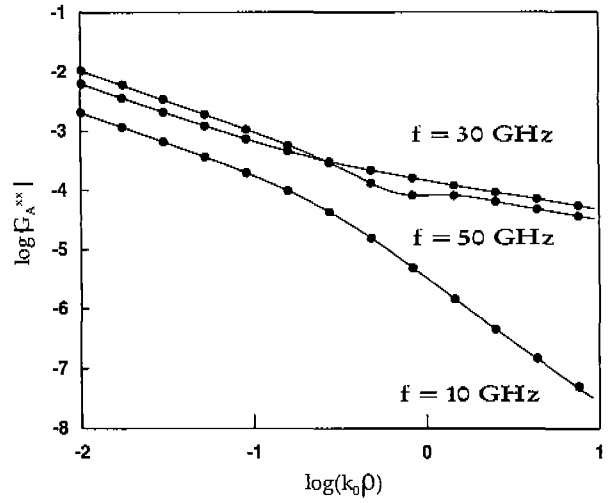
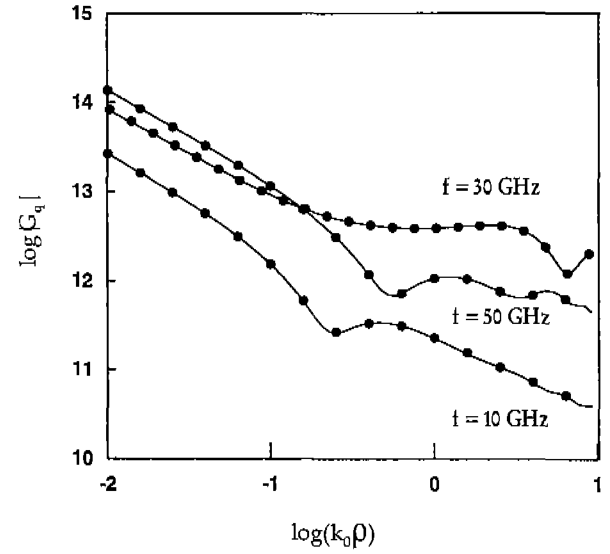


Fig. 3. Comparison between the behaviors of the present spectral function $(F_{qr}-f_{qrl})$ and those of the corresponding function F_2 in the previous work[1] along the path C_{ap2} with $T_{02}=15$ for the case that $\varepsilon_r=12.6$, $h=1$ mm, $f=30$ GHz.



(a)



(b)

Fig. 4. (a) The amplitude of the vector potential G_A^{xx}. (b) The amplitude of the scalar potential G_q. ——— : exact results · · · : present results.

method, the present results for the evaluation of Green's functions are compared with exact results obtained by numerical integration for the case that $\varepsilon_r=12.6, h=1$ mm, $z=z'=0$ at three sampled frequencies (10 GHz, 30 GHz, 50 GHz). Excellent correspondence between them is observed over the wide frequency range independently of source-to-field distance as shown in Fig. 4. Here it is worthy of stressing that even choosing only the one set of approximation parameters ($T_{01}=50$, $T_{02}=5$, $N_1=3$, $N_2=4$) in the present two-level approach allows us to obtain accurate results over the wide frequency range.

Ⅳ. CONCLUSION

In conclusion, a modified complex image method, which circumvents the burdensome steps of the previous method, has been considered based upon the two-level approach by introduction of the term $G_{q,ci1}$, which is newly described here in terms of the complex images for the large k_ρ contribution. Considering that the term $G_{q,ci1}$ corresponds, in essence, to the previous quasi-static Green's function G_{q0} in the prior work[1], the complex images can be called quasi-static complex images[4].

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REFERENCES

[1] Y. L. Chow, J. J. Yang, D. G. Fang, and G. E. Howard, "A closed-form spatial green's function for the thick microstrip substrate", *IEEE Trans. MTT.*, vol. 39, no. 3, pp. 588-592, 1991.
[2] M. I. Aksun, "A robust approach for the derivation of closed-form green's functions", *IEEE Trans. MTT.*, vol. 44, no. 5, pp. 651-658, 1996.
[3] R. W. Hamming, *Numerical Methods for Scientists and Engineers*, Dover, NY, pp. 620-622, 1973.
[4] Y. S. Lee, J. K. Kim, H. S. Kim and Y. K. Cho, "Improved complex image method for a horizontal magnetic dipole in a parallel-plate waveguide", *Microwave and Optical Technology Letters*, vol. 16, no. 1, pp. 30-34, 1997.

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