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A Case Study on Gifted Education in Mathematics[1]

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The Center for Science Gifted Education (CSGE) of Chongju National University of Education was established in 1998 with the financial support of the Korea Science & Engineering Foundation (KOSEF). In fact, we had prepared mathematics and science gifted education program beginning in 1997. It was possible due to the commitment of faculty members with an interest in gifted education.

Now we have 5 classes in Mathematics, two of which are fundamental, one of which is a strengthened second-grade class gifted elementary school students, and one a fundamental class, and one a strengthened class for gifted middle school students in Chungbuk province. Each class consists of 16 students selected by a rigorous examination and filtering process. Also we have a mentoring system for particularly gifted students in mathematics.

We have a number of programs for Super Saturday, Summer School, Winter School, and Mathematics and Science Gifted Camp. Each program is suitable for 90 or 180 minutes of class time. The types of tasks developed can be divided into experimental, group discussion, open-ended problem solving, and exposition and problem solving tasks. Levels of the tasks developed for talented elementary students in mathematics can be further divided into grade 5 and under, grade 6, and grade 7 and over. Types of the tasks developed can be divided into experimental, group discussion, open-ended problem solving, and exposition and problem solving task. Also levels of the tasks developed for talented elementary students in mathematics can be divided into the level of lower than grade 5, level of grade 6, and level of more than grade 7. Three tasks developed and practiced are reported in this article.

I. INTRODUCTION

Before 1998, we had about 15 science high schools in Korea, but nothing for

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mathematically talented middle and elementary students. However, the Korea Science & Engineering Foundation (KOSEF) has helped to establish the Centers for Science Gifted Education in 15 universities since 1998 (for details see Shin & Han, 2000, pp. 81–84). Also, sooner or later, the Ministry of Education plans to establish a special school, and classes for gifted children, as described in the Talent Education Act of 1999. Such fundamental and systematic support at the national level can be helpful in realizing a desirable education system for a gifted students, but what is more important for its success is national corresponding government financial support and the commitment of the persons concerned in gifted education. It is a good opportunity to realize a successful gifted education system in Korea.

II. MATHEMATICS GIFTED EDUCATION PROGRAM IN CSGE

The Center for Science Gifted Education (CSGE) of Chongju National University of Education was established in 1998 with the financial support of the Korea Science & Engineering Foundation (KOSEF). In fact, we had prepared mathematics and science gifted education program beginning in 1997. It was possible due to the commitment of faculty members with an interest in gifted education.

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III. EXAMPLES OF 2-HOUR ACTIVITIES

1. Experimental Task: Pattern Block

Purpose To create an enhanced learning experience using Enactive, Iconic, and Symbolic Representation and Mathematical Reasoning

Table 1. Types and Levels of Mathematical Tasks Developed

Task Type	Levels		
	Grade 5 and under	Grade 6	Grade 7 and over
Experimental tasks (15)	Calculating Several Numbers by Calculator	Tower of Hanoi Pattern Block Top of the Mountain in the Map Simulation of Probability Circle Mystery Mathematical Puzzle	Six Cards Sand and Mathematics The Number of Cases Point, Line, and Face in Figure Exploration of Symmetry by GSP A Problem for Construction I am Pythagoras
Group Discussion tasks (15)	Gulliver's Travels Geoboard Tic-Tac-Toe and Mathematics	The Classification of Graphs Mathematics in Football Discussion in Mathematics Who is the Winner? Making the Best Basketball Team	Magic Card Mystery The Principle of Graphic Calculator Archimedian Weight and Volume Truth and Falsity of Statement From Natural to Real Number Eratosthenes
Open-Ended Problem Solving Tasks (32)	Logo Programming (1), (2), (3) Tangrams The World of 2,2,2	Pascal's Triangle Toothpick Puzzle Camel's Allotment Circular Tangrams Making Cubes and Polydrons Understanding 3-Dimension Angle in the Clock Graph by the Table of Frequency Distribution Hockey Problem Chess Problem Mathematical Puzzles Problem Solving on Some Topics Pentomino Problem	Theory of Probability Fibonacci Sequence Data Processing by Calculator Find the Shortest Way Polydron and Euler's Formula Who is the liar? Making Various Shapes Mathematical Puzzles Patterns in Number Painting the Map Magic Square Volume Problem Pyramid Exploration Factorization by Base Ten Block
Exposition Tasks (7)	The Foundation of Logo Programming	A Maze Problem An Interesting Mathematical Puzzle	Exploration with Tiles Exploration of Fractals The Foundation of Functions Distance Problems

Preparation Activity Free Play

Pattern Block: 250 pieces of six geometric shapes in six colors;
50 green triangle, 25 orange squares, 50 blue parallelograms, 50 tan rhombuses,
50 red trapezoids, and 25 yellow hexagons

Exploration Activity

1. Scoop and Sort
   (1) Making Concrete Graphs
   (2) Discussing the Graphs
      ① Which block do you have most or least of?
      ② How many blocks did you scoop in all?
   (3) Making Pictorial Representations of the Graphs (see Figure 1)

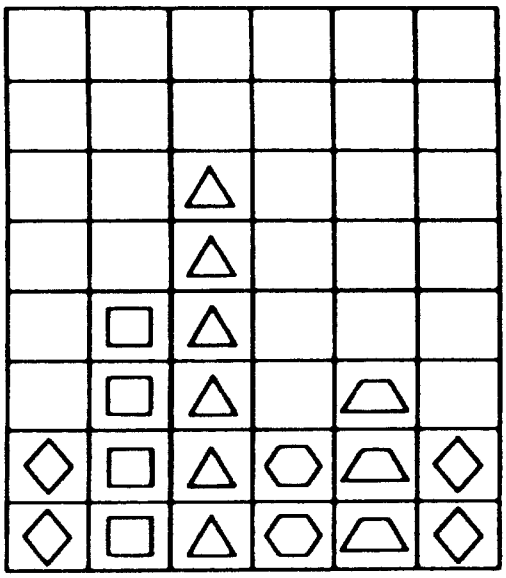

Figure 1. Pictorial Representations of Graphs

2. Building the Yellow Hexagon

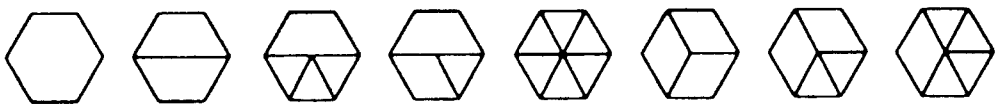

Figure 2. Making Hexagons

(1) Finding the possibility (Figure 2)

① Two red trapezoids: 1/2 +1/2 =1
② Six green triangles: 1/6 +1/6 +1/6 +1/6 +1/6 +1/6 = 1
③ 1/3 + 1/3 + 1/6 + 1/6 = 1
④ 2/3 + 2/6 = 1

(2) A follow-up discussion
(3) Connecting the activity to fractions

Enrichment Activity

"Animal Pattern Card" (refer to IV. RESULTS AND DISCUSSIONS).

2. Discussion Task: Magic Card Mystery

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31
33	35	37	39
41	43	45	47
49	51	53	55
57	59	61	63

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31
34	35	38	39
42	43	46	47
50	51	54	55
58	59	62	63

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31
36	37	38	39
44	45	46	47
52	53	54	55
60	61	62	63

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31
40	41	42	43
44	45	46	47
56	57	58	59
60	61	62	63

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63

32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63

Figure 3. Magic Cards

Purpose Understanding the binary system which underlies the magic card mystery

Preparation Activity

(1) Play the game of magic card.
(2) Think about the principle of making the magic card.

Exploration Activity Find the principle of making 6 magic cards by way of the

following table (Table 2).

Table 2. Decimal Number and Binary Number

Decimal		Binary	Decimal		Binary
1	=	1	32	=	100000
2	=	10	33	=	100001
3	=	11	34	=	100010
4	=	100	35	=	100011
5	=	101	36	=	100100
6	=	110	37	=	100101
7	=	111	38	=	100110
8	=	1000	39	=	100111
9	=	1001	40	=	101000
10	=	1010	41	=	101001
11	=	1011	42	=	101010
12	=	1100	43	=	101011
13	=	1101	44	=	101100
14	=	1110	45	=	101101
15	=	1111	46	=	101110
16	=	10000	47	=	101111
17	=	10001	48	=	110000
18	=	10010	49	=	110001
19	=	10011	50	=	110010
20	=	10100	51	=	110011
21	=	10101	52	=	110100
22	=	10110	53	=	110101
23	=	10111	54	=	110110
24	=	11000	55	=	110111
25	=	11001	56	=	111000
26	=	11010	57	=	111001
27	=	11011	58	=	111010
28	=	11100	59	=	111011
29	=	11101	60	=	111100
30	=	11110	61	=	111101
31	=	11111	62	=	111110
			63	=	111111

Enrichment Activity

(1) Make 5 magic cards.
(2) Make 7 magic cards.

3. Open-ended Problem Solving Task: Pascal's Triangle

Purpose Encouraging students to develop broader perspectives with respect to number tables

Preparation Activity Find the increasing number pattern in the following six figures (omitted).

Exploration Activity See the Pascal's Triangle below (Figure 4) and find as many rules or patterns in columns, rows, and diagonals as you can find among the numbers.

1											
1	1										
1	2	1									
1	3	3	1								
1	4	6	4	1							
1	5	10	10	5	1						
1	6	15	20	15	6	1					
1	7	21	35	35	21	7	1				
1	8	28	56	70	56	28	8	1			
1	9	36	84	126	126	84	36	9	1		
1	10	45	120	210	252	210	120	45	10	1	
-	-	-	-	-	-	-	-	-	-	-	-

Figure 4. Pascal's Triangle

Enrichment Activity Discuss your own findings and debate another's, if possible.

Ⅳ. RESULTS AND DISCUSSION

1. Experimental Task: Pattern Block

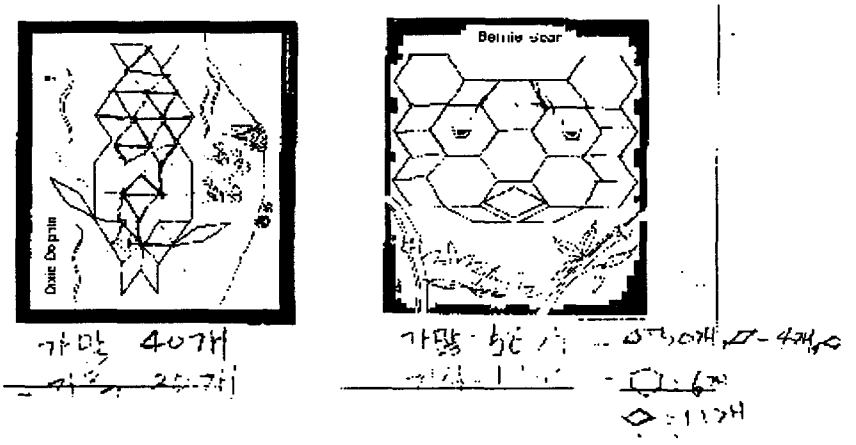

Figure 5. Pattern Blocks

Students covered "Dixie Dolphin" (see Figure 5) with the most blocks (40), and the fewest blocks (20).

Students covered "Bernie Bear" (see Figure 5) with the most blocks (56), and the fewest blocks (19).

A student created a figure (for example, a poodle or flower) by pattern blocks in his own creative way.

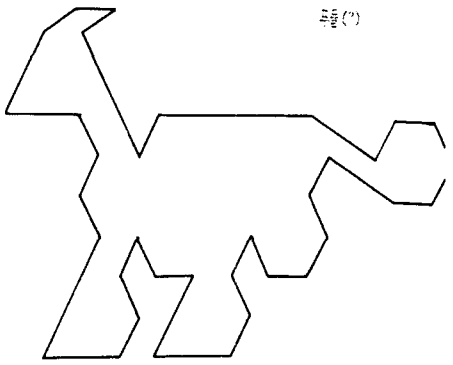

Figure 6. Poodle

2. Discussion Task: Mystery of Magic Card

They were very interested in the binary system, which underlies the magic card mystery. A student made 5 magic cards as follows.

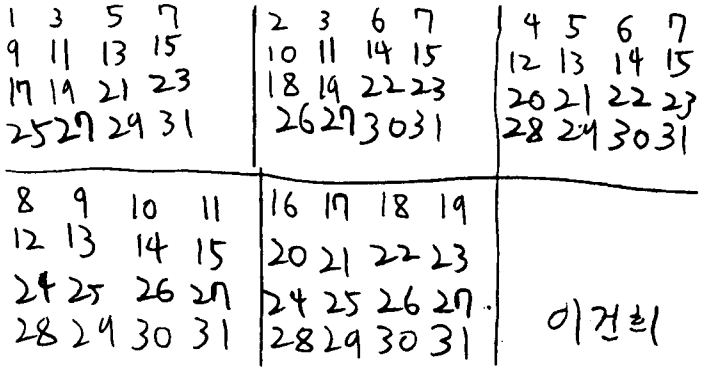

Figure 7. 5 Magic Cards

3. Open-ended Problem Solving Task: Pascal's Triangle

A really special gifted student's response is as follows.

1					$=11^0$
1	1				$=11^1$
1	2	1			$=11^2$
1	3	3	1		$=11^3$
1	4	6	4	1	$=11^4$

He did not know the binomial theorem, but I thought that he must be another young Pascal. It can be interpreted as follows.

1					$=11^0$	$=(10+1)^0$
1	1				$=11^1$	$=(10+1)^1$
1	2	1			$=11^2$	$=(10+1)^2$
1	3	3	1		$=11^3$	$=(10+1)^3$
1	4	6	4	1	$=11^4$	$=(10+1)^4$

A student's response is as follows (see Figure 7).

Columns

(1) All the terms in the first column are 1.
(2) Numbers in the second column are natural numbers.
(3) Numbers in the third column are triangular numbers.
(4) Differences in the fourth column are the numbers in the third column.
(5) The sum of the first several consecutive numbers in any column is equal to the number located to the lower right of the last number.
(6) The numbers in the nth column are the same as the numbers in the nth diagonal going down from the left.
(7) Every number is larger than the number above it.
(8) Numbers in the seventh column are all multiple of 7, except for 1.

Rows

(1) Every number is the sum of the number just above it and the number to its left.
(2) The arrangement of numbers in each row is symmetrical.
(3) Numbers at both ends are 1.
(4) The sum of the numbers in the $(n+1)$th row is 2^n
(5) The numbers in the middle of the rows are the largest.

Diagonals

(1) The sums of numbers along diagonals drawn upward from the 1's in the left column form the "Fibonacci sequence" which is a sequence of numbers obtained by summing the preceding two terms, starting from 1.

(2) All the terms in the right first diagonal are 1.
(3) Numbers in the right second diagonal are natural numbers.
(4) Numbers in the right third diagonal are triangular numbers.

(2) 지금까지 자기가 발견한 규칙성을 다음과 같은 표로 작성하여라.

분류	번호	규칙성
세로줄	1	모두 10다 (첫째줄)
	2	자연수 (둘째줄)
	3	삼각수 (셋째줄)
	4	0 에서부터 삼각수는 차례로 더해 나간 숫자들. (넷째줄)
	5	한 세로줄의 맨 위부터 그 줄의 어떤 수까지 모두 더한 수는
	6	그 어떤 수의 오른쪽 아래에 있는 수와 같다.
	7	세그줄의 첫째 줄과 오른쪽으로 향해 내려가는 대각선의 첫째 줄이
	8	같다. (둘째 줄 셋째 줄도 마찬가지이다)
	9	세그줄의 둘째 수는 오른쪽으로 갈수록 같은하다.
	10	모든 수는 바로 위의 수보다 크다.
		연필밑에 세로줄의 수들은 1을 제외하면 모두 짝수 배수이다.
가로줄	1	나란히 있는 두 수를 더하면 그 두 수들 중 오른쪽 수의 밑에
	2	있는 수가 나온다.
	3	왼쪽부터 읽어도, 오른쪽부터 읽어도 수들의 배치 순서가 같다.
	4	맨 바깥쪽 끝에 수는 모두 1이다.
	5	한 가로줄의 모든 수를 더하면 그의 가로줄수가 된다. (첫째줄은 1',
	6	둘째줄은 2', 셋째줄은 3' ...)
	7	가로줄의 수가 점점 크다.
	8	
대각선	1	수들을 대각선으로 이어 그 대각선의 수들 모두 더하면
	2	피보나치 수자가 된다.
	3	대각선상의 맨 왼쪽 끝에에 있는 수들은 모두 피보나치 수이다.
	4	맨 바깥 오른쪽 대각선은 모두 1 이다.
	5	오른쪽에서 두번째 대각선은 자연수이다.
		오른쪽에서 세번째 대각선은 삼각수이다.

Figure 8. Finding rules for Pascal's triangle

V. CONCLUSION

The fundamental rules of creating a program for talented children in mathematics are as follows.

Firstly, create a program, which consists of Learning Purposes, Preparation Activities, Exploration Activities, Enrichment Activities, and Portfolio Assessment.

Secondly, present an open-ended problem situation, which motivates active exploration and problem solving on the basis of students' own creative ideas, if possible.

Thirdly, develop some programs, which require using technology such as calculators and computers.

Fourthly, develop programs that produce numerous opportunities for group discussion and discourse using mathematical language, symbols, and expressions for problem solving.

Fifthly, develop tasks, which produce an experience of the power of mathematical reasoning and mathematical problem solving.

Sixthly, develop tasks that cover the contents of the mathematics curriculum, which consists of numbers and operations, geometry, measurement, probability and statistics, letters and expressions, patterns and functions (problem solving).

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