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A Study on the Development of Creativity in the Secondary Mathematics in Korea

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This study sheds light on the importance of developing creativity in mathematics class by examining the theoretical base of creativity and its relationship to mathematics. The study also reviewed the realities of developing creativity in mathematics courses, and it observed and analyzed the processes in which students and teachers solve the mathematics problems. By doing so, the study examined creative abilities of both students and teachers and suggests what teachers can do to tap the potential of the students.

The subjects of the study are two groups of students and one group of mathematics teachers. These groups were required to solve a particular problem. The grading was made based on the mathematical creativity factors. There were marked differences in the ways of the solutions between of the student groups and the teacher group.

It was clear that the teachers' thinking was limited to routine approaches in solving the given problems. In particular, there was a serious gap in the area of originality. As can be seen from the problem analysis by groups, there was a meaningful difference between the creativity factors of students and those of teachers. This study presented research findings obtained from students who were guided to freely express their creativity under encouragement and concern of their teachers. Thus, teachers should make an effort to break from their routine thinking processes and fixed ideas. In addition, teaching methods and contents should emphasize on development of creativity. Such efforts will surely lead to an outcome that is beneficial to students.

1. INTRODUCTION

Currently there is a growing concern over mathematics education in Korea. The main problem is that the introduction of new context through explanations, exemplified calculation methods, and routine application training of algorithms hinder the flexibility and the possibility of developmental applications, which are the primary characteristics of mathematical thinking. Now it is the time for teachers of mathematics to change their traditional ideas about teaching.

Today the world is moving towards rapid exchange of information, highly advanced science and technology, and internationalization and globalization. Under Korea's current environment, mathematics education is required not just to teach all principles of mathematics, rules and details but also to help students understand basic concepts and thus develop their abilities to cope with the explosive increasing amount of knowledge and information. In this context, the importance of divergent and horizontal thinking has been emphasized, in addition to the realization of creativity in education. Accordingly, it has become necessary than ever before to foster students' abilities to explore and solve problems in a creative way, that is, to become creative citizens of the society.

This study sheds light on the importance of developing creativity in mathematics class by examining the theoretical base of creativity and its relationship to mathematics. The study also reviewed the realities of developing creativity in mathematics courses, and it observed and analyzed the processes in which students and teachers solve the mathematics problems. By doing so, the study examined creative abilities of both students and teachers and suggests what teachers can do to tap the potential of the students.

2. CREATIVITY AND MATHEMATICS

2.1. Definition of creativity as related to mathematics

Mathematical creativity is an ability of analyzing a given problem in a variety of ways, based on existing knowledge and experience, breaking from fixed patterns, and obtaining a result by combining the factors of a problem with mathematical ideas in a new way. That is, creativity in mathematics should first give weight to the correct introduction of existing notions. Even though creativity relates to the production of certain new results, it appears to be more closely related to new analyses of a problem, new approaches and new ways of problem-solving. Accordingly, creativity in mathematics represents mathematical thinking of the highest level.

Mathematical creativity is generated by the interaction of understanding, intuition, insight, and generalization. Understanding refers to the ability to reproduce a theory of a mathematician or some of the orders of a mathematical creation. Here, understanding is not as a tool. Rather, it indicates a relational understanding, as Skemp (1987, pp. 152-

163) maintains, which means a state of knowing a method and reason or a state in which a certain rule or algorithm can be deduced from a rather more general mathematical relation.

2.2. Creativity in mathematics

Literature discusses a variety of factors of creativity. However, this study will only focus on five factors[1] of creativity that directly relate to mathematics in terms of practical problem-solving and which are actually used in evaluating test of creativity about problem-solving.

2.2.1. Diffusion

Diffusion refers to an ability to produce many ideas in a quantitative sense, and it has nothing to do with the quality of the ideas. This means that we accept ideas, irrespective of whether or not they are correct. Many ideas do not necessarily lead to creative solution of a problem, but they can raise such probabilities. Therefore, diffusion constitutes an important factor of creativity, and it is true that ones in the field of mathematics ideas are often found in incorrect answers as well as correct answers. Consequently, it is necessary to give marks without criticism to all ideas in when evaluating creativity.

2.2.2. Logicality

Logicality means an ability to produce an idea that can serve as a solution to a given problem. A meaningless or incorrect idea generated from existing knowledge may not be helpful for problem-solving.

2.2.3. Fluency

Like the concept of diffusion, fluency is concerned with the quantity of ideas. Unlike diffusion, however, fluency only considers the quantity of ideas that are correct in a mathematical sense. Thus, fluency refers to the ability to produce as many ideas as possible that is usable in practice. Ideas that are completely illogical, miscalculated, or hard to accepted are excluded. In reality, a high grade in fluency evaluating creativity indicates a high possibility of problem-solving.

2.2.4. Flexibility

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[1] Saito (2000) regarded the factors of creativity relating mathematics as these five factors.

Flexibility is an ability that relates to a wide variety of more comprehensive ideas. It relates to the ability to make use of ideas by identifying the ideas from diverse points of view. While fluency describes the total number of ideas, flexibility describes the ability to classifying ideas with common properties into common categories. A high grade in flexibility implies a rich pool of ideas and suggests a high-level factor of creativity.

2.2.5. Originality

Originality relates to the manifestation of one's own unique idea. Breaking from existing ways of thinking or problem-solving methods of other people. This represents the most ideal goal of creativity. Originality cannot be obtained all at once. It is attainable only with intentional, conscious effort. To facilitate original ideas, it is necessary to produce as many ideas as possible, to recognize the importance of given problem and to concentrate on it, and to approach a problem in a relaxed manner with sufficient time. Sometimes, it is also necessary to activate a super-logical jump.

The grading of originality depends upon the frequency of answers given by the members of an examinees' group. No originality mark shall be credited to the answer given by most of the members. An answer of a unique idea will receive a high grade. For example, no points shall be given for answers offered by 15% or more of the examinees. 1 point shall be given for answers offered by 7% to 14% of the examinees; 2 points for 3% to 6%; 3 points for 1% to 2%; and 4 points for less than 1%.[2]

3. PRACTICE OF CREATIVE PROBLEM-SOLVING

3.1. Preparations for the study

Many scholars report that creativity is an inherent ability that can be found in everyone, and it can be further developed by training. Therefore, they assert that the roles of teachers are very important for the development of creativity. Accordingly, this study tries to examine what influences teachers' roles have on the creativity of students and compare the creativity of teachers with that of students in problem-solving.

In this study, new types of problems were given to students, who were given sufficient time for solving them. Some of the problems were also given to teachers and they were then asked to solve them.

3.2. The research method

[2] Cropley (1982) transformed the evaluation method of Torrance, based on a normal distribution.

3.2.1. Subjects

Group A: 90 students of 1st-year (grade 10) at the Pusan Electronic Technical High School

From September 1999 to December 1999[3], these students were intentionally encouraged to think creatively by their mathematics teachers who had a great interest in creativity and had read a wide variety of books on the topic. The students were also required to solve the problems[4] in lively class atmosphere. When the students received the problem (see Figure 1) prepared for this study, the students were allowed to exchange their ideas with peers to a certain extent, and this helped them find efficient strategies of problem-solving. The same environment was also applied to Groups B and C, too.

Group B: 60 students of 3rd-year (grade 9) at the Tohyun Middle School in Busan

Unlike the students of Group A, these were ordinary students who had not received any training on creativity or been encouraged to think creatively by their teacher. The problem-solving environment for this group was the same as that for Group A.

Group C: 40 teachers of mathematics

The same problems were given to 40 mathematics teachers of Busan. The teachers' teaching experiences range from 15 to 25 years. During the problem-solving no intentional atmosphere was created for the sake of creativity or creativity training. In addition, the test was conducted in a lecture hall of teachers' training institute and so there were some discussions among the teachers about the problems as in the case of the students.

3.2.2. Time of the tests

Group A: Early-November, 1999.

Group B: Late-November, 1999, when the mathematics course was almost completed in the middle schools (grades 7–9).

Group C: Mid-January, 2000, during the general training session of middle-school mathematics teachers.

3.2.3. Test problem

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[3] During the first semester from March to July, these students did not take a mathematics course, in accordance with the school's curriculum, and so did not study mathematics for six months after they entered high school.

[4] Division of diagrams, rule finding, maze, magic square, match stick activity, square logic, modeling, figure play, etc.

Of the given problems, the problem of making quadrilaterals with two sets of two right triangles with 30°, 60° and 90° angles and another with 45°, 45° and 90° angles was selected and presented to the groups. The tastes were requested to find as many solutions as possible. Some consultations with each other about the problem were allowed.

Problem: Please make quadrilaterals with two sets of two right triangles (cf. Figure 1). Find as many solutions as possible. The hypotenuses have the same length.

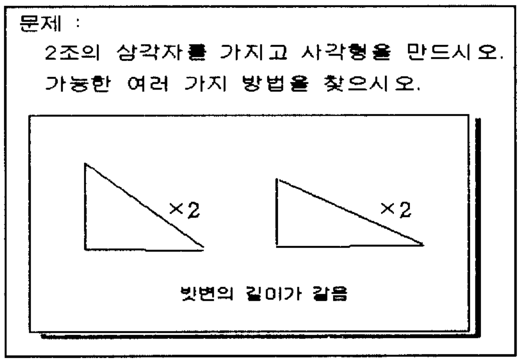

Figure 1. Two right triangles

3.2.4. Grading

Grading was practiced by the list of 5 factors. This list was reorganized moderately to above problems (see Table 1).

3.2.5. Classification of ideas

To give marks in the factor of flexibility (see Table 1), it is necessary that the ideas be classified by their distinguishing marks.

1) An outer frame filled with triangles.
2) An inner frame of triangles.
3) A drawing with the use of one or two triangles or a case using the wall face.
4) A quadrilateral made of outer frames by putting one triangle over another.
5) A quadrilateral made by putting one triangle over another.
6) In case the side of a solid formed by placing triangles on end is a quadrilateral or in case space (a solid) is used.
7) Solutions that are logically correct and original (e.g., a depressed quadrilateral)
8) Solutions that are illogical but have ideas (e.g., cutting or throwing triangles)

3.3. Results of the research

Table 1. Factors[5], contents and points

Factors	Symbols	Contents and Points
Diffusion	x_1	• Total number of solutions, including all correct and incorrect answers. • Two congruent figures shell be considered as one. • Points: 1 point for 1 solution
Logicality	x_2	• Total number of solutions whose procedures are logically correct for more than half of the whole process. • A mark is given to a quadrilateral made of any triangle. • Quadrilaterals made of using not given triangles or diagrams other than quadrilaterals shall not be assigned any mark. • Points: 1 point for 1 solution
Fluency	x_3	• Total number of correct answers • Points: 1 point forl correct answer
Flexibility	x_4	• Total number of different ideas among the correct answers. • Marks shall be given to the solutions with ideas, even if they fall short of completely correct answers. • See the idea classification shown in Section 2. 2. 5. • Points: 1 point for 1 idea
Originality	X_5	• Total number of correct answers with rich originality. • Marks will be assigned based on the percentage of the ideas classified in Section 3. 2. 5. • Points 7%~14%: 1 point 3%~6%: 2 points 1%~2%: 3 points less than1%: 4 points
Total mark $x = \sum w_j x_j$		

3.3.1. Summary of the solutions

Figures 2 and 3 show solutions of the groups.

Upon receiving the problem sheets, many students and teachers asked various questions but no questions were taken because that would lead to additional conditions or rules.

There were marked differences between the solutions of the students and those of the teachers. In Figures 2 and 3, the teachers put forward only 5 to 6 solutions, while the students suggested other solid-type solutions and solutions of marked originality, in addition to the answers shown in Figures 2 and 3. The teachers confined their scope of thinking to the conditions, rules, and fixed ideas (such as "No overlapping is allowed," "It

[5] The items (factors) of Table 1 are due to Saito (2000).

must be filled," etc.), which were not given in the problem but created by the teachers themselves, while the students sought their solutions in a free manner. Some students gave answers without regarding the degrees of 45° and 60°, but in such cases, they only failed to get a mark in the item of logicality. On the whole, the students demonstrated freethinking and great interest in problem-solving.

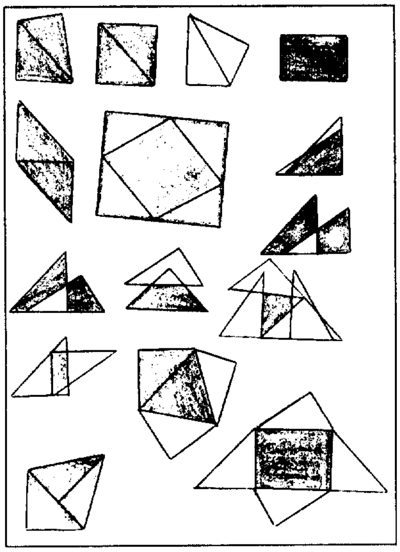

Figure 2. Solutions (1)

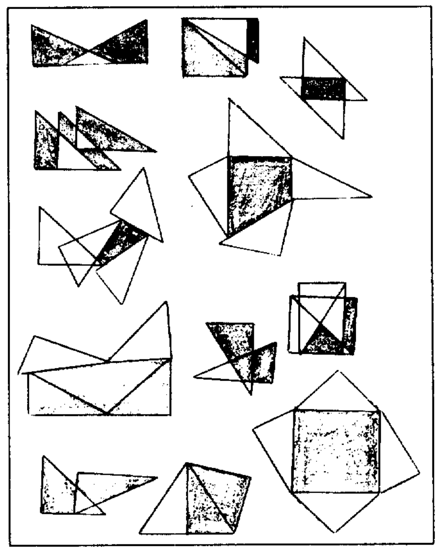

Figure 3. Solutions (2)

Some students said that there was not sufficient time for the test while most of the teachers responded that too much time was given. It appears that felt that too much time was given because they could not find many solutions, due to the conditions and rules set by themselves.

3.3.2. Average marks by the group

Table 2 shows the average marks by the group. We also combined the marks of Group A and Group B to show the difference between the student group and the teacher group.

Table 2. Average marks

Groups	Diffusion	Logicality	Fluency	Flexibility	Originality	Total
A	8.01	7.70	5.70	1.97	1.99	25.37
B	6.90	6.80	5.13	1.42	0.85	21.10
A+B	7.57	7.34	5.47	1.75	1.53	23.66
C	8.03	7.90	7.70	1.15	0.25	25.03

3.3.3. Comparison between Group A and Group B

Group A received higher marks than Group B in all of the three items (see Figure 4). The members of Group A tried to find as many solutions as possible, and they tended to describe their ideas that came to mind without giving much critical thought to them.

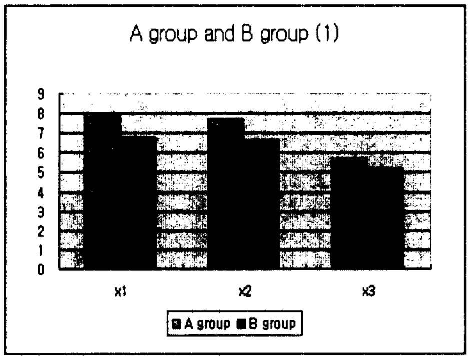

Figure 4. Group A and Group B in Diffusion, Logicality and Fluency

Unlike the three items (x_1, x_2, x_3), marks of x_4 and x_5 were assigned based not on the number of solutions but on the number of ideas. As shown in the graph (See Figure 5), there are differences between Group A+B and Group C in the number of ideas generated. Despite the differences, the marks are relatively low, considering the fact that the total number of ideas was eight. However the composition of the ideas varies greatly, extent and this will be analyzed later. However the composition of the ideas varies greatly, extent and this will be analyzed later.

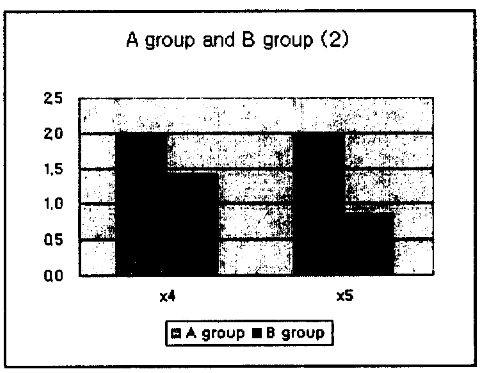

Figure 5. Group A and Group B in Flexibility and Originality

3.3.4. Comparison between Group A+B and Group C

The group C comprised of teachers excelled the groups A and B comprised of students in three items. In particular, there was a big difference in fluency (the number of correct answers, see Figure 6). The students described many kinds of ideas, but they included logically incorrect answers as well as logically correct answers, whereas the teachers seldom gave incorrect answers.

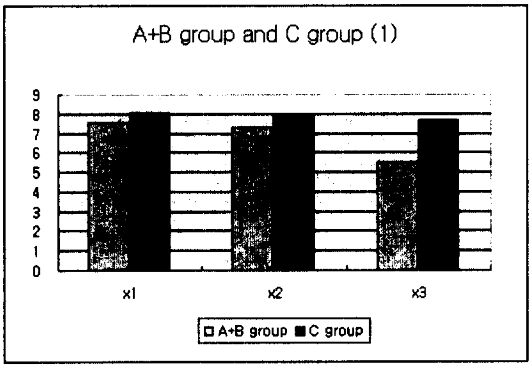

Figure 6. Student Group (A+B) and Teacher Group C in Diffusion. Logicality and Fluency

There was a marked difference in flexibility (kinds of ideas) between the groups comprised of students and the group comprised of teachers. The Group C received a point of nearly zero in originality (see Figure 7). This may indicate that the teachers rarely think differently from the routines.

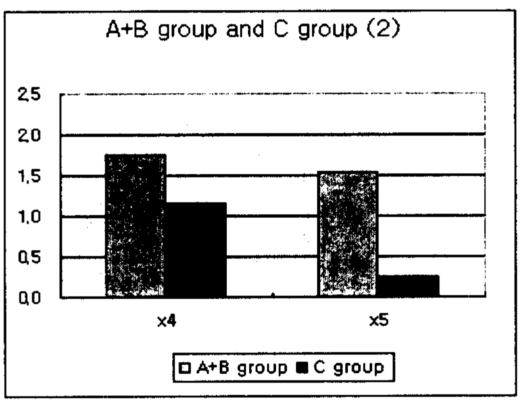

Figure 7. Student Group (A+B) and Teacher Group in Flexibility and Originality

Group C (comprised of teachers) gave more solutions and correct answers while receiving low marks in flexibility and originality. This indicates that their thinking is rigid and bound by fixed concepts and patterns in solving a mathematics problem.

3.3.5. Comparison of the number of ideas among groups

Even though the teachers gave many correct answers, they received low marks in flexibility and originality. To examine the reason of this, the distribution of the solutions, depending upon the classification of ideas, was analyzed. The results are shown in the graphs Figures 8 and 9. These show that the relative frequency of Group C is extremely concentrated on Idea 1 (cf. Section 3. 2. 5). Idea 1 represents basic idea, the outer frame filled with triangles. This demonstrates that Group C, the group of teachers, had few ideas except the Idea 1. In particular, they were not able to produce any creative ideas. That is, the teachers presented almost the same solution for the problem.

On the other hand, Group A shows a very even distribution of relative frequency, except for the basic idea (Idea 1). If they had an opportunity to solve the problems in small groups (to exchange views in a freer manner or to solve the problem using a brainstorming technique), the students were able to share more ideas with their peers and thus present a wider variety of solutions than the teachers could.

Figures 8 and 9 show the distribution of ideas, except Idea 1 (cf. Section 3. 2. 5) and they indicate a considerable difference among the groups. Although it was not a specialized creativity program, there existed a difference in the number (see Figure 8) of ideas between Group A (encouraged by their teacher) and Group B (without such encouragement).

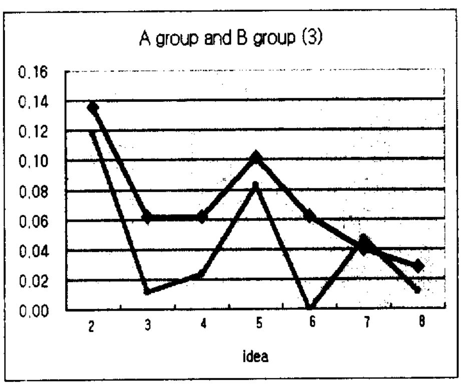

Figure 8. Distribution of ideas of Groups A and B

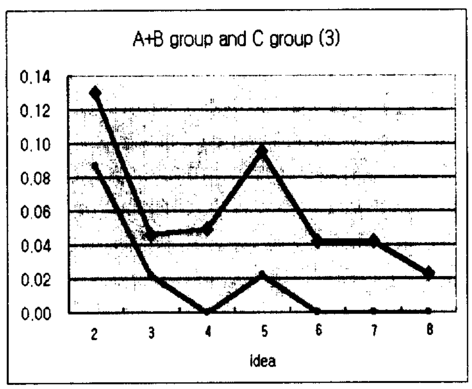

Figure 9. Distribution of ideas Groups A+B and C

3.3.6. Results of the analysis

The results of the analysis demonstrate that there are considerable differences in the possibilities of the level of creativity between the students who received encouragement from the teacher and the students without such encouragement and between the students and the teachers.

Some studies suggest that creative teachers develop creative students (cf. Yoon, 1998). Some even assert that the teacher's interests in creativity have a positive effect on the development of creativity of the students. It appears that the interest in creativity of the part of the teachers can influence atmosphere of question-raising technique. This study supports such assertions in the results of the comparative analysis between Group A and Group B in this study.

A comparison of the student groups with the group of teachers, who had more than 15 years of teaching experiences, shows how rigid teachers' thinking is. The teachers could have given much more correct answers if problems were given with detailed explanations about the types of ideas. However it was revealed that, when the teachers were given the problems only, they showed the tendency to unconsciously create conditions and situations at their options that had not been given in the problem. This hindered the teachers from approaching the problem, in a new way and, unfortunately, caused them to miss an opportunity to look for a novel, proper solution.

There is a concern that the non-creative habits of mathematics teachers may restrict the students' thinking by a pattern of routine problem-solving, even though the students think variously and their ideas are not fixed. Thus, teachers should serve as guides who help the students expand their scope of thinking, and it is necessary for the teachers to make conscious efforts towards this goal.

4. CONCLUSION

It's clear that creativity inherently exits in students. To stimulate and encourage creativity can be of great help to the intellectual development of students. However, many mathematics teachers try to make their students understand problems through a deductive introduction and do repetitive process of solving of similar problems. This teaching method is detrimental to the development of creativity.

As it can be seen from the problem analysis of the groups, there is a meaningful difference of students and teachers in the creativity factors. Teachers are rigid in the flexibility and the originality factors of their thinking, and thus they can undermine the development of students' creativity when they try to limit the active students' thinking into a fixed pattern.

To develop the creativity of students, teachers need to first have an interest in showing creativity in their teaching. This is because teachers' interests have positive influences on the students in terms of creativity development. Also, it is necessary for teachers to participate in only having interest is sufficient for active development of creativity active development of creativity in their students. Practical participation and effort can only make a difference.

This study presented research findings obtained from students who were guided to freely express their creativity under the encouragement and concern of their teachers. (In spite of the students were not those who were trained under a specialized creativity development program). We expect further studies to follow in which students and teachers participate in a systematic program aimed at developing creativity. To this end, there must be a verified program designed for creativity development in the mathematics classes, and it is also necessary to train teachers so that they will be able to conduct such programs in an efficient way.

In conclusion, teachers should make an effort to break their routine thinking processes and fixed ideas. In addition, teaching methods should emphasize on development of creativity. Such efforts will surely lead to a production outcome for students.

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