

Practical Use of Technology for Mathematics Education¹

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The purposes of this paper are to understand the functions of technology TI92 graphing calculator and to develop some typical examples showing that current Korean secondary school mathematics curriculum can be treated with the technology. This study is consisted with five chapters. In Chapter 1, the background and purposes for this study is described. In Chapter 2, the textbooks developed by Core-Plus Mathematics Project to reform mathematics education in the United States are analyzed to find the possibilities for practical uses in mathematics classrooms. In Chapter 3, the functions of TI-92 are analyzed with regard to how they can be applied to mathematics education. In Chapter 4, some examples are developed to show that TI-92 can be used for Korean secondary school mathematics education. Chapter 5 is consisted with summaries of this study and some suggestions for further study.

1. INTRODUCTION

The trends of traditional mathematics instruction are such that a teacher writes mathematical concepts, principles, or rules on blackboard and explains students about the contents, and students note them on their notebooks, listen the teacher to understand, and ask some questions about them. In this case, the main tools for teaching and learning mathematics are pencils, notebooks, or a few simple hand-held manipulative materials, and mathematics education has been weighted on training students' skillful computational abilities. But, nowadays by using technology such as computers or calculators, students can compute complex expressions speedily and accurately, collect and analyze various statistical data, and transform dynamically geometric figures. So, learning activities to develop computational skills can be replaced by investigating activities to understand mathematical concepts and problem solving. Thus, technology can be used as important

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tools for mathematics education.

The movement of using computers and calculators for mathematics education began to form early in the 1960s. Most of the early studies on teaching and learning of mathematics with technology had focused on computer programming and CAI (computer assisted instruction) materials. In this time, since computers were not improved much, about half of the studies showed negative responses in using computers for mathematics education (Blume, 1984). From 1990s, since technology has been improved so much and it can be used effectively in most areas of real life, each level of schools has equipped classrooms with computers and multi-systems and many studies have suggested a lot of ideas and materials on how to use technology in mathematics education.

Nowadays, Computers and calculators are common technology that can be used effectively for mathematics education. With computers we can use CAI materials, programming, micro world software, and internet-systems. With using internet-systems we can communicate various helpful information through whole of the world. There are three types of calculators such as common calculators, scientific calculators and graphic calculators. With graphing calculators, called microcomputers, we can compute symbolic expressions such as differentiation, integration, matrices, and statistics, and we can draw graphs of functions, plots of statistical data, and geometric figures. In Korea, nowadays a lot of studies have taken interest in using computers for education, but few studies has focused on calculators and internet-systems. Moreover, it is hard to find any systematic researches on using graphing calculators for learning and teaching mathematics in classrooms. Therefore, this study aims to analyze the functions of TI-92 for mathematics education and to develop some examples to solve typical mathematical tasks from Korean secondary school mathematics textbooks with TI-92.

2. TECHNOLOGY FOR MATHEMATICS EDUCATION

2.1. The Characteristics of Technology

Technology is essential in teaching, learning, and doing mathematics. The main characteristics of computers and graphing calculators are as follows: Both computers and graphing calculators can be used to compute numerical or symbolic expressions, store data to use them any time later, represent data by graphs or statistical plots, and draw and measure geometric figures. When technological tools are available, students can focus on decision-making, reflection, reasoning, and problem solving, rather than hard and boring computational work.

Students can learn more mathematics more deeply by using the appropriate technology (NCTM, 2000). Technology should not be used as a replacement for basic

understandings and intuitions. It can and should be used to foster those understanding and intuitions. In teaching and learning of mathematics programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics. Currently since students can access technology with various functions, we need to rethink the contents of school mathematics and instructional methods.

1) Technology enhances mathematics learning.

Technology can help students learn mathematics. For example, with calculators and computers students can examine more examples or representational forms. Students can make and explore conjectures easily by using computers. The graphic power of technological tools extends the range of problems accessible to students and also enables them to execute routine procedures quickly and accurately, thus allowing more time for conceptualizing and modeling.

By using technology, students can see mathematical ideas from various aspects. Students can communicate their ideas on the effects of some objects or on the dynamic changes from the screen produced by the technology.

Technology offers teachers options for adapting instruction to special student needs. Students who are easily distracted may focus more intently on computer tasks, and those who have organizational difficulties may benefit from the constraints imposed by a computer environment. Students who have trouble with basic procedures can develop and demonstrate other mathematical understandings, which in turn can eventually help them learn the procedures.

2) Technology supports effective mathematics teaching.

The effective use of technology in mathematics classroom is dependent on teachers. Technology is not always good for any subjects. Technology, as with any teaching tool, can be used well or poorly. Teachers should select or create suitable mathematical topics that students can learn well with technology. For examples, teachers can use simulations that are difficult to create without technology, or they can use data and sources from the "Internet" and the "World Wide Web" to design student tasks. Spreadsheets, dynamic geometry software, and computer micro-worlds are also useful tools for posing worthwhile problems.

Technology cannot replace mathematics teachers. Teachers can perform various important roles in mathematics classes by using technology. For example, teachers make decisions that affect students' learning in important ways. Teachers should decide if, when, and how technology will be used. While students use calculators or computers in mathematics classes, teachers can have opportunities to observe the students and to focus

on their thinking. As students work with technology, they may show ways of thinking about mathematics that are otherwise often difficult to observe. Thus, technology aids in assessment, allowing teachers to examine the process used by students in their mathematical investigations as well as the results, thus enriching the information available for teachers to use in making instructional decisions.

3) Technology influences what mathematics is taught.

Technology not only influences how mathematics is taught and learned but also affects what is taught and when a topic appears in the curriculum. By using technology at hand, young children can investigate and solve problems involving large numbers, or they can investigate some characteristics of shapes using dynamic geometry software. Secondary school students can study linear relationships and the ideas of slope and uniform change with computer representations and by performing physical experiments with calculator-based-laboratory systems. They can use simulations to study sample distributions, and they can work with computer algebra systems that efficiently perform most of the symbolic manipulation that is the focus of traditional high school mathematics program. Using technology tools, students can reason about more general issues, such as parameter changes, and they can model and solve complex problems.

Technology can help teachers connect the development of skills and procedures to the more general development of mathematical understanding. As some skills that were once considered essential are rendered less necessary by technological tools, students can be asked to work at higher levels of generalization or abstraction.

2.2. Technology Used in Mathematics Textbooks

In contemporary mathematics textbooks used in Korea, there are some symbolic marks to encourage students to use calculators or computers. Those marks are usually on exercises or extra topics located at the end of each unit. Using technology is still optional. But most secondary school mathematics textbooks developed currently in the United States of America treat technology as a necessary tool to develop new mathematical concepts for problem solving. Moreover, a few national standardized tests such as National Achievement Educational Progress test issued in 1988 by Educational Testing Service involve items for which students have to use calculators and items for which students should decide to whether use technology or not.

Currently Core-Plus Mathematics Project (CPMP, 1988a; 1988b; 1998), funded by National Science Foundation, has developed three courses of mathematics textbooks for grades 9 through 11, and the course 4 textbook for 12th grade college bound students has been developed recently. These textbooks have been developed to reform mathematics

education for the information society after 1990s. These materials are very popular and used nationwide in the United States of America. Graphing calculator and computer software developed for these textbooks are used as basic tools in these textbooks. So, this study, to figure out how technology is used practically in the current mathematics textbooks, analyzes typical types of using technology through the three courses of textbooks developed by CPMP.

2.2.1. Course 1 textbook

Type 1: Listing and plotting data.

Students use graphing calculators in collecting data such as the orders of qualities of cars produced in the United States, in listing the data in order, and in representing the data with box plots and histograms. By using a box plot minimum, maximum, median, lower quartile value, and upper quartile value of the data are visually represented.

Type 2: Finding patterns of change.

The populations of whales are investigated only by pressing the [enter] key of a graphing calculator repeatedly. The formula to find the changed population is

“answer + growth rate \times answer + constant”.

Several kinds of populations are investigated by this method.

Type 3: Students draw graphs of a few functions on the screen of a graphing calculator and investigate some properties of the graphs.

Type 4: A few numbers are selected randomly in two ways; one is to use random number generator function of a graphing calculator and the other is to use the software ‘Collections’ developed for this textbook.

2.2.2. Course 2 textbook

Type 1: With graphing calculators, students evaluate inverse matrices, generate Markov processes, and apply them to solve real world problems.

Type 2: With graphing calculator, computer software, or the software ‘Geoxplor’ developed for this textbook, students draw, measure, and transform geometric figures, and investigate some properties of the figures. By using the software ‘Flagturn’ developed for this book, geometric figures can be moved dynamically.

Type 3: With graphing calculators or computer software, students solve systems of linear equations. They draw the graphs of the systems of equations and compare the coordinates of the intersected points by the solution generated by an algebraic method.

Type 4: Data having two variables are entered into a calculator and some properties of the data are investigated such as correlation of the data. A few types of regression equations are investigated.

Type 5: A few types of functions such as polynomial functions and rational functions are graphed and their properties are investigated by using graphing calculators or computers.

Type 6: Trigonometric ratios and inverse trigonometric ratios are evaluated by using calculators. In this case, both degree unit and radian unit can be used.

2.2.3. Course 3 textbook

Type 1: By using 'preferential voting' calculator software, students investigate how to decide the winner with the information of the ranks voted by each group and the number of voters. There are a few ways to decide winners; majority winner, plurality winner, run-off method, pairwise-comparison method, points-for-preferences method, and approval voting.

Type 2: Three internet-sites related to voting are introduced. They are

<http://www.vote-smart.org>,

<http://www.doftdisk.com/hits/vote.html>,

<http://www.nytimes.com>.

Type 3: Several types of functions such as polynomial function, rational function, exponential function, logarithmic function, and trigonometric function are investigated focused on the domains, codomains, ranges, and values of the functions.

Type 4: Students investigate a few types of circular equations and list some values of the equations.

3. THE FUNCTIONS OF TI-92 GRAPHING CALCULATOR

The TI-92 is a kind of technology developed to be used in studying secondary school and college level mathematics such as calculus, linear algebra, and statistics. This technology is one of the most advanced graphing calculators. In this chapter, the functions of TI-92 that can be applied for Korean secondary school mathematics were analyzed as follows.

3.1. Basic Function Graphing

1) Setting the graph mode

To display the MODE dialog box, press [MODE] key. Before graphing $y(x)$ functions, we must select FUNCTION graph mode. We may also need to set the Angle mode, which affects how the TI-92 graphs trigonometric functions. For graphs that do not use complex numbers, set Complex Format = REAL.

2) Defining functions for graphing.

In FUNCTION graphing mode, we can graph functions named $y_1(x)$ through $y_{99}(x)$. To define and edit these functions, use the Y= Editor.

3) Selecting functions to graph.

Regardless of how many functions are defined in the “Y= Editor”, we can select the ones we want to graph by pressing F4 key. To cancel the selected function, press F4 key again. To select or cancel all of the functions, we may use F5 key.

4) Setting the display style for a function.

For each defined function, we can set a style that specifies how that function will be graphed. This is useful when graphing multiple functions. For example, we can set one graph as a solid line and another as a dotted line, etc.

5) Defining the viewing Window.

The viewing window represents the portion of the coordinate plane displayed on the graph screen. By setting Window variables, we can define the viewing window's boundaries and other attributes. Function graphs, parametric graphs, etc., have their own independent set of Window variables.

6) Changing the graph format.

We can set the graph format to show or hide reference elements such as the axes, a grid, and the cursor's coordinates. Function graphs, parametric graphs, etc., have their own independent set of formats.

7) Graphing the selected functions.

When we are ready to graph the selected functions, display the graph screen by

pressing GRAPH key or APPS key.

To display the exact coordinates of any plotted point on a graphed function, we can use the 'F3 Trace' tool. The trace cursor moves only along a function's plotted points.

8) *Using zooms to explore a graph.*

The 'F2 Zoom' toolbar menu has several tools to adjust the viewing window. 'ZoomStd' sets window variables to their default values, and 'ZoomSqr' adjusts window variables so that a square or circle is shown in correct proportion instead of a rectangle or ellipse.

9) *Using Math toolbar to analyze functions.*

On the graph screen, the 'F5 Math' toolbar menu has several tools that help we analyze graphed functions. The elements of Math menu are value, zero, minimum, maximum, intersection, derivatives, integration, distance, tangent, arc, and shade.

3.2. Tables

1) *Setting up table parameters.*

To set up the initial parameters for a table, use the TABLE SETUP dialog box by pressing 'TblSet' key. After the table is displayed, we can also use this dialog box to change the parameters.

2) *Displaying an automatic table.*

If Independent=Auto on the TABE SETUP dialog box, a table is generated automatically when we display the Table Screen.

3) *Building a manual (ask) table.*

If Independent=Ask on the TABLE SETUP dialog box, the TI-92 lets us build a table manually by entering specific values for the independent variable.

3.3. Symbolic Manipulation

1) *Using undefined or defined variables.*

When performing algebraic or calculus operations, it is important to understand the effect of using undefined and defined variables. If the variable is undefined, it is treated as an algebraic symbol, and if it is defined its value replaces the variable. To determine if a variable is undefined we have to enter a variable. If the variable is defined the

variable's value is displayed, and if undefined the variable name is displayed on the screen. To delete a defined variable, use the "DelVar" function in F4 of home screen like as "delvar x, y". We can also delete by using the VAR-LINK key. We can delete all one-letter variables ($a - z$) in the current folder by pressing F6 in the home screen.

2) *Using exact, approximate, and auto modes.*

We can select one of the three modes exact, approximate, and auto when the TI-92 calculates a result. When we type an expression on the entry line and press the [ENTER] key, the TI-92 automatically simplifies the expression according to its default simplification rules.

3) *Substituting values and setting constraints.*

The "with" operator (I) key lets we temporarily substitute values into an expression or specify domain constraints. For example, ' $3x + 2$ I $x = 2$ ' outputs 8, but it does not define the variable x .

4) *Using algebra menu.*

We can use the "F2 Algebra" toolbar menu to select the most commonly used algebraic functions. 'solve', 'factor', 'expand', 'zeros', 'approx', 'comDenom', 'proFrac', 'nSolve', 'Trig', 'Complex', and 'Extract' are algebraic menu items.

5) *Using calculus menu.*

We can use "F3 Calc" toolbar menu to select commonly used calculus functions. 'differentiate', 'integrate', 'limit', 'sum', 'product', 'fMin', 'fMax', 'arcLen', 'taylor', 'nDeriv', and 'nInt'" are calculus menu items.

3.4. **Geometry**

We can open geometry session by pressing the [APPS] key, function number 8.

1) *Setting application preferences.*

The "F8 File" toolbar menu contains the Format command that opens a dialog box to specify application preferences, such as angles in degrees or radians, and the display precision of calculations.

2) *Selecting and moving objects.*

The “F1 Point” toolbar menu contains the tools associated with geometry pointer features. These features allow us to select objects and perform freehand transformations. We can select an object by pointing to it and pressing [ENTER] when the cursor message appears for that object, and deselect an object by pointing to unoccupied location and pressing [ENTER]. We can move an object by selecting it, holding, and pressing cursor pad to the direction we want.

3) *Deleting objects from a construction.*

The “F8 File” toolbar menu contains commands that let us delete selected objects or all objects from a construction.

4) *Creating geometric figures.*

We can create points, circles, lines, segments, rays, vectors, arcs, triangles, and polygons. We can construct perpendicular lines, parallel lines, perpendicular bisectors, and angle bisectors.

5) *Translating, rotating, and dilating objects.*

The “Translation” tool creates the image of an object translated by a specified previously defined vector. To translate an object, select the object first and select the vector that defines the translation direction and distance. We can change the vector by grab and drag the vector head.

To rotate an object about a defined point, create an object and a point. Select rotate menu in F1. Select the rotation point, point to the object and drag the direction that we want to rotate the object. We can rotate objects by creating and specifying rotating point and angle value.

We can dilate an object by doing as follows. Create an object, a point, and a numerical value. Press F5 and select “3:Dilation”. Select the object to dilate. Select the point of dilation and then select the factor of dilation.

6) *Creating reflections and symmetries.*

The “F5 Transformations” toolbar menu contains the tools associated with transformational geometry for creating reflections and inverse objects. The Reflection tool creates a mirror image of an object that can be reflected across a line, segment, ray, vector, axis, or side of a polygon. The symmetry tool creates the image of an object that is rotated 180 degrees around a point.

7) *Measuring objects.*

We can measure length, perimeter, circumference, area, or angle of an object by using the “F6 Measurement” toolbar menu.

8) *Determining equations and coordinates.*

The “equation & coordinates” tool in F6 displays the equation of a line, circle, or coordinates of a point with respect to a default coordinate system. The equation or coordinates are updated when the object is modified or moved.

9) *Putting objects in motion.*

The Animation tool in “F7 Display” menu automatically moves an independent object along a specified path. If the Point tool is visible in the toolbar and the object does not lie on a defined path, the animated direction is 180degrees from the spring. Otherwise, the object is animated along its defined path. If the Rotate, Dilate, or Rotate & Dilate tool is visible in the Pointer toolbar and the object can be transformed, the animation will be relative to the visible Pointer tool. For example, if the Rotate tool is visible, the object is rotated automatically. To animate an object, construct the object, select animation tool, select the point of the object to animate, and drag the animation spring in the opposite direction of the intended animation, and then release the grab key (hand shaped).

10) *Controlling how objects are displayed.*

The “F7 Display” toolbar menu contains tools to display features of objects. The F8 file toolbar menu contains several tools that determine how objects are viewed. The Thick tool in the display toolbar menu changes the outline thickness of an object between normal (one pixel) and thick (three pixels) outlines. The Dotted tool in the display toolbar menu changes the outline pattern of objects between solid and dotted outlines. The ‘Show Page’ commands in the File toolbar menu allow us to view an entire construction, which can be larger than the drawing window. It displays the full-page picture of the construction in miniature.

11) *Creating macros.*

The Macro Construction item contains the tools for constructing macros in the geometry application. A macro is a sequence of interdependent constructions. Macros are useful for creating new tools that construct unique objects or perform repetitive tasks.

A macro constructs final objects based on initial objects. Intermediate objects are not constructed. This feature allows for easy construction of complex figures and is the

primary method for constructing fractals. We can save macros for later use.

3.5. Data/Matrix Editor

When we start the Data/Matrix editor in the [APPS] key, we can create a new variable, resume the current variable, or open an existing variable. Because all Data/Matrix editor variables are saved automatically, we can accumulate quite a few variables, which take up memory. To delete a variable, use the VAR-LINK key.

1) Entering and viewing cell values.

Any type of expressions can be entered in a cell such as number, variable, function, string, etc. The cell width affects how many characters are displayed in any cell. We can change the cell width in the Data/Matrix Editor by selecting “9: Formats” dialog box in F1. We can clear a column by pressing F6, 5:Clear Column, and can clear all columns by pressing F1, 8:Clear Editor.

2) Inserting and deleting a row, column, or cell.

The new row or column is inserted before the row or column that contains the highlighted cell by pressing F6 and selecting suitable tools.

3) Sorting columns.

After entering information in a data, list, or matrix variable, we can easily sort a specified column in numeric or alphabetical order by using suitable tools in F6 toolbar menu. We can also sort all columns as a whole, based on a “key” column.

3.6. Statistics and Data Plots

1) Performing a statistical calculation.

From the “Data/Matrix Editor”, we can use the ‘F5 Calc’ toolbar menu to perform statistical calculations. We can analyze one-variable or two-variable statistics, or perform several types of regression analyses. We must have a data variable. The data/Matrix Editor will not perform statistical calculations with a list or matrix variable. The Calculate dialog box (F5) lets us specify the statistical calculation we want to perform. Statistical calculation results are stored to variables. To access these variables, type the variable or use the VAR-LINK screen. All statistical variables are cleared when we edit the data or change the calculation type.

2) Defining a statistical plot.

From the 'Data/Matrix Editor', we can define several types of statistical plots by using the entered data. The statistical plot types in TI-92 are Scatter, xy-line, Box Plot, and Histogram.

3.7. 3D Graphing

We can graph 3D graph equations by the same general steps used for $y(x)$ functions as described before. We can select 3D mode from [MODE] window.

4. DEVELOPMENT AND PRACTICAL USE OF MATERIALS

Typical examples using TI-92 to solve some mathematical tasks selected from Korean secondary school mathematics textbooks are developed in this study. These examples are aimed to show the possibilities of using TI-92 for Korean mathematics education.

4.1. Number and Operation

Example 1: If we divide 50 by a number then the remainder is 2, and if we divide 89 by the number then the remainder is 5. By what number did we divide?

Teaching strategies and students' activities: Before using technology, students should plan to solve the problem. Small groups of students discuss about the numbers satisfying the given conditions. Suppose there were no remainders, then the number we want to find are factors of 50 and 89. So, $50 - 2$ and $89 - 5$ are divisible by the numbers we want to find.

Since the bigger remainder is 5, the factor we want to find should be bigger than 5. That is, the numbers are factors of the greatest common factor of the two numbers $50 - 2$, and $89 - 5$.

Solution: Find GCD of $50 - 2$, $89 - 5$ by using technology, and then find factors of the GCD. Select [MATH], 1: Number, C: GCD, and then execute GCD (48, 84). The answer is 12. The factors of 12 are 2, 3, 4, 6, and 12. But the number we find should be bigger than 5. So, the answer is 6 and 12.

Example 2: Evaluate the following expressions with complex numbers.

$$\begin{array}{ll} (1) (5 + 6i) + (7 - 8i) & (2) (5 + 6i) - (7 - 8i) \\ (3) (5 + 6i) \times (7 - 8i) & (4) (5 + 6i) / (7 - 8i) \end{array}$$

Teaching strategies and students' activities: Students have to know the concept of

complex number before using technology. Teacher checks students' knowledge by letting students solve simple examples such as

$$(1+i) \times i, \quad \frac{1+i}{i}$$

And then students use TI-92 to evaluate more complex numerical expressions.

Solution: Enter each expression into the entry line in [HOME] window. Type i by pressing [2nd] and [I] key. Answers are as follows.

$$(1) 12 - 2i. \quad (2) -2 - 2i. \quad (3) 83 + 2i. \quad (4) -\frac{13}{113} + \frac{82}{113}i$$

4.2. Letters and Expressions

Example 1: Factor

$$x^2 + xy - 6y^2.$$

Teaching strategies and students' activities: Students have to understand that the result of factoring should be expressed by multiplication of two polynomials. Factoring is important in doing mathematics. Sometimes it is not easy to factor a polynomial by using 'trying and error' strategy. Before factoring a polynomial we have to determine the set of coefficients.

Solution: Use factor menu 'factor (' in "F2 Algebra" which treats coefficients as real numbers.

Answer is $(x - 2y)(x + 3y)$.

Example 2: Solve $x^3 - 4x^2 = -8$.

Teaching strategies and students' activities: Let students find all of the possible ways to solve the equation. Students should determine appropriate ways according to the types of roots they want to find.

To find accurate roots we can use algebraic way, and to find real roots we can use graphic functions. 'Guessing and checking' strategy is another way to find estimated roots. And we can also find estimated real roots by making a table.

Divide students into three groups: One group for algebraic method, another group for geometric method, and the other group for making a table.

First group activities: Use cSolve menu in F2 Algebra.

Answers are $x = -(\sqrt{5} - 1)$, $x = \sqrt{5} - 1$, $x = 2$

Second group activities: Graph the two functions $y = x^3 - 4x^2$, $y = -8$

On the same coordinate plane, and find the coordinates of the intersecting points of the two graphs.

Third group activities: Make a table and find the x -values where the function $f(x) = x^3 - 4x^2 + 8$ changes the signs. By controlling the x -intervals we can get more accurate roots.

Example 3: Find the number of real roots.

$$x^4 + 4x + 2 = 0$$

Teaching strategies and students' activities: Make students plan possible strategies for this problem. The most effective strategies for this problem are using “algebraic tool” menu and using the graph of the give function. Students compare the results they solved by different ways.

Solution 1: Using algebraic tools: [HOME], F2: Algebra, 1: solve ($x^4 + 4x + 2=0$, x), Press the [ENTER] key. We may have two negative real roots -0.518 , -1.363 .

Solution 2: Using graphs:

In [Y=] window, input $y=x^4 + 4x + 2$, press the [GRAPH] key.

We may find the graph intersects x -axis at two points in left side of the origin point.

The equation has two different negative real numbers.

Answers are two different real roots.

Example 4: Solve the system of equations.

$$x + 2y - 3z = 22, \quad 2x - 3y + z = 9, \quad 3x + y + 2z = -1$$

Teaching strategies and students' activities: First, make students plan appropriate strategies for this task. One way is to delete one variable and then another one as we do in traditional textbooks. The other one is to use a tool menu in [MATH] window.

Solution: [MATH], 4: Matrix, 5:simult([1, 2, -3; 2, -3, 1; 3, 1, 2], [22; 9; -1]), [ENTER]. The answer is $x = 5$, $y = -2$, $z = -7$.

4.3. Patterns and Functions

Example 1: Find the minimum value and maximum value of the following function.

$$y = 2x^3 - 3x^2 + 12x, \quad -2 < x < 4$$

Teaching strategies and students' activities: Students consider first the ways to solve this problem. One way is to use the graph of this function; the other way is to use the toolbar menu in TI-92.

Solution 1: Using the graph of the function. In [Y=] window, input $y=2x^3 - 3x^2 + 12x$, and then graph this function. Control the window such as $\min x = -2$, $\max x = 4$. Then,

find the minimum and maximum values of the graph.

Solution 2: Use the toolbar menu 'fMin(' and 'fMax(' in [HOME], F3 Calc.

fMin ($2x^3-3x^2+12x, x$) | $x > -2$ and $x < 4$, [ENTER], then the answer is -2.0 .

fMax ($2x^3-3x^2+12x, x$) | $x > -2$ and $x < 4$, [ENTER], then the answer is 4.0 .

Example 2: Graph $y = 3^x$ and its inverse function, and $y = x$ on the same set of coordinate axes.

Teaching strategies and students' activities: Let students solve the inverse function of $y = 3^x$. And then let them input the three functions in [Y=] window. Graph the functions on the same coordinate plane. Guide students to find the relationships between the three graphs.

Solution: Calculator can graph logarithmic functions having e or 10 as basis. So, we have to change the logarithmic function to have 10 or e as its base. Guide students to find that the graph of $y = 3^x$ and its inverse graph are symmetric about the line of $y = x$.

Example 3: Differentiate

$$(1) y = -2x^2 + 1 \quad (2) y = 2^{x-1} \quad (3) y = \ln(\log x)$$

Teaching strategies and students' activities: Make students solve (1) and (2) by hands and by TI-92, and then compare the results. Let students solve (3) by using TI-92.

Solution: [HOME], F3 Calc,

(1) 1: d ($-2x^2 + 1$), [ENTER]. Answer is $-4x$.

(2) Answer is $\frac{1}{x \cdot \ln x}$

(3) Answer is $\frac{2^x \cdot \ln 2}{2}$

Example 4: Find the slope of the tangent of the following function at $x = \frac{\pi}{6}$.

$$y = x \sin 2x$$

Teaching strategies and students' activities: Make students think appropriate ways to solve the problem. In this case, TI-92 may be much convenient than paper and pencils.

Solution: Use 'differentiate' menu in Calculus toolbar in [HOME], F3 Calc,
1:d ($x \sin 2x, x$) | $x = \pi / 6$. The answer is 0.03655 .

Example 5: Integrate

$$\int_1^3 \frac{x}{x^2 + 1} dx$$

Teaching strategies and students' activities: Make students think the ways to evaluate this integration. Let advanced students evaluate the integration by papers and pencils and let lower level of students evaluate the integration by using TI-92. Make students compare the two results of the two works.

Solution: [HOME], F3 Calc. Answer is $\frac{\ln \frac{5}{2}}{2}$

4.4. Probability and Statistics

Example 1: The data collected from the distances each student's throwaway a ball, are listed below.

26.4, 15.7, 23.2, 20.8, 13.5, 14.3, 20.4, 21.5, 19.8, 21.5

- (1) List the data in order.
- (2) Find the mean and standard deviation.
- (3) Construct a histogram of the distribution of the data.

Teaching strategies and students' activities: Make students discuss the effectiveness of using TI-92 than paper and pencil. Students should understand the concepts of mean, standard deviation, and histogram before using technology.

Solution: Select [APPS], 6:Data/Matrix Editor, New, list type. Enter the data in the list screen.

- (1) Select F6 Util, 3:Sort Column and list the data in order.
- (2) Mark a cell on the column C1, select F5 Calc, and input necessary information needed, and then execute to get results involving the answers. Answers are mean 19.7 and standard deviation 4.067882
- (3) Select F2 Setup and then F1 Define. Select 4:Histogram in Plot Type. Type x in c1. Check the plot number we built in [Y=] screen and then select 9:ZoomData in F2 Zoom.

Example 2: Test scores of mathematics and English of a group are shown in the below table.

Student No.	1	2	3	4	5	6	7	8	9	10
Mathematics	80	75	90	86	74	83	76	80	93	96
English	73	81	86	90	91	86	68	85	90	88

- (1) Find the mean and standard deviation of the scores of mathematics and English.
- (2) Find the regression equation and correlation coefficient of the two variables.

Teaching strategies and students' activities: Check all of the students can understand the

statistical concepts and can use those statistical functions of TI-92 for this task.

Solution: Select [APPS], 6: Data/Matrix Editor. Input the score of mathematics into c1 column and that of English into c2 column. Select F5 Calc and then do as follows.

- (1) Input c1 into x and c2 into y and press [ENTER] key. We get mean and standard deviation for each of the two subjects.
- (2) Select F5 Calc, 5: LinReg. Input c1 in x and c2 in y, and press [ENTER] key. Correlation coefficient is 0.462942, and the regression equation is $y = ax + b$, where $a = 0.460138$, $b = 45.470546$

4.5. Geometry

Example 1: Draw regular pentagon.

Teaching strategies and students' activities: Guide students to understand that drawing regular pentagon by hand with a ruler and a compass is pretty hard, but using TI-92 is very easy and funny.

Solution: Select [APPS] in TI-92, 8:Geometry, 3:New. Select 5:Regular polygon in F3. Replace 6 by 5 and press [ENTER] key. Then we may get a regular pentagon.

Example 2: Draw a triangle ABC and find the circumscribed circle and circumcenter of the triangle.

Teaching strategies and students' activities: Guide students to do this task in two ways; using manipulative tools and TI-92. Let students compare the two processes are same basically. Students find that using technology is much convenient in transforming the figures.

Solution: Use F3 toolbar to draw a triangle. Draw the perpendicular bisectors of any two sides of the triangle by using toolbar menus in F4. The intersection point is the circumcenter of the triangle ABC.

5. CONCLUSION AND SUGGESTION

This study looked over the necessary backgrounds and purposes of using technology for mathematics education. Technology pervades most area of current society and it is improved so much that we cannot avoid using it for learning and teaching mathematics. Now, what we can do is to develop more suitable materials for mathematics education.

In Chapter 2, typical types of using technology were analyzed through those textbooks

developed currently by Core-Plus Mathematics Project funded by National Science Foundation of the United States to reform mathematics education. This study found that most topics of the materials are consisted with technology focused on graphic calculators. Without using TI graphing calculators or some software developed for these textbooks, it may be impossible to understand comprehensively mathematical concepts or principles.

In Chapter 3, some typical characteristics and functions of TI-92 were analyzed. The main features of TI-92 are as follows. Students can evaluate and compute symbolic equations or expressions easily; they can collect and analyze statistical data, draw geometry figures, animate the figures dynamically, and analyze their properties. It can be as useful as any computer software for pedagogical purposes for mathematics education.

In Chapter 4, a few typical examples to show that TI-92 can be used to solve most mathematical problems based on Korean secondary school mathematics textbooks were developed. These examples are to show the possibilities for which TI-92 graphing calculator can be used for Korean secondary school mathematics education. To use technology more conveniently for mathematics education, we may have to develop more enriched and comprehensive materials similar to those examples developed in this study.

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