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WEAK CONVERGENCE TO COMMON FIXED POINTS OF COUNTABLE NONEXPANSIVE MAPPINGS AND ITS APPLICATIONS

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ABSTRACT. In this paper, we introduce an iteration generated by countable nonexpansive mappings and prove a weak convergence theorem which is connected with the feasibility problem. This result is used to solve the problem of finding a solution of the countable convex inequality system and the problem of finding a common fixed point for a commuting countable family of nonexpansive mappings.

1. Introduction

Let $C_1, C_2, \cdots, C_k$ be nonempty closed convex subsets of a Hilbert space H whose intersection C_0 is nonempty. Given metric projections P_i onto C_i for $i = 1, 2, \cdots, k$, find a point of C_0 by an iterative scheme. Such a problem is connected with the feasibility problem. In fact, let $\{g_1, g_2, \cdots, g_k\}$ be a finite family of real valued continuous convex functions on H. Then the feasibility problem is to find a solution of the finite convex inequality system, i.e., to find such a point $x \in C_0$ that

$$C_0 = \{x \in H : g_i(x) \leq 0, \quad i = 1, 2, \cdots, k\}.$$

In 1991, Crombez [4] obtained the following result: Put $T = \alpha_0 I + \sum_{i=1}^{k} \alpha_k (I + \lambda_i (P_i - I))$, where I is an identity operator, $0 < \lambda_i < 2$ for $i = 1, 2, \cdots, k$, $\alpha_i > 0$ for $i = 0, 1, \cdots, k$, and $\sum_{i=0}^{k} \alpha_i = 1$. Then, starting from an arbitrary element x of H, a sequence $\{T^n x\}$ converges weakly to $z \in C_0$. Later, Kitahara and Takahashi [8] dealt with the

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feasibility problem by convex combinations of sunny nonexpansive retractions in uniformly convex Banach spaces.

Recently, motivated by Ishikawa [7] and Das and Debeta [5], Takahashi and Shimoji [15] introduced a new iteration generated by finite nonexpansive mappings and proved weak convergence theorems which are connected with the feasibility problem.

On the other hand, Dye and Reich [6] proved the following result for an unrestricted product of nonexpansive mappings: Let $\{T_n\}$ be nonexpansive mappings on a Hilbert space H with a common fixed point $z \in H$. Suppose that the algebraic semigroup S generated by $\{T_n\}$ satisfies the condition (W) with respect to $p \in H$. Let r be a mapping from the set of natural numbers into itself and let $S_n = T_{r(n)}T_{r(n-1)} \cdots T_{r(2)}T_{r(1)}$. Then $S_n x$ converges weakly for each $x \in H$. An algebraic semigroup S generated by $\{T_n\}$ is said to satisfy the condition (W) with respect to p if for any bounded sequence $\{v_n\}$ of H and a sequence $\{W_n\}$ of words from S with $\|v_n - p\| - \|W_n v_n - p\| \to 0$, it follows that $v_n - W_n v_n$ converges weakly to 0; see [6] for more details. Though they obtained a sufficient condition for which S satisfies the condition (W) with respect to p in the paper, it still seems to be a strong condition.

In this paper, we introduce an iteration generated by countable nonexpansive mappings and prove a weak convergence theorem which is connected with the feasibility problem. This iteration is different from that of Dye and Reich's, and our theorem generalizes Takahashi and Shimoji's result [15] for finite nonexpansive mappings. Using our theorem, we also consider the feasibility problem of finding a solution of the countable convex inequality system and the problem of finding a common fixed point for a commuting countable family of nonexpansive mappings.

2. Preliminaries and lemmas

Throughout this paper, we denote by $\mathbb{N}$ and $\mathbb{R}$ the sets of natural numbers and real numbers, respectively. Let E be a Banach space and let C be a nonempty closed convex subset of E. A mapping T of C into itself is called nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. The set of fixed points of T is denoted by $F(T)$, that is, $z \in F(T)$ if and only if $z = Tz$. Let D be a nonempty subset of C. A mapping P of C onto D is said to be a retraction if $Px = x$ for each $x \in D$. If there exists a nonexpansive retraction of C onto D, then D is said to be a nonexpansive retract of C.

For a Banach space E, we define the modulus δ_E of convexity of E as follows: δ_E is a function of $[0,2]$ into $[0,1]$ such that

$$\delta_E(\epsilon) = \inf\{1 - \|x+y\|/2 : \|x\| \le 1, \|y\| \le 1, \|x-y\| \ge \epsilon\}$$

for each $\epsilon \in [0,2]$. E is called uniformly convex if $\delta_E(\epsilon) > 0$ for each $\epsilon > 0$. E is called strictly convex if $\|x+y\|/2 < 1$ for every $x, y \in E$ with $\|x\| = \|y\| = 1$ and $x \ne y$. It is known that a uniformly convex Banach space is strictly convex and reflexive.

A Banach space E satisfies Opial's condition [10] if, for each sequence $\{x_n\}$ of E converging weakly to x, $x \ne y$ implies

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|.$$

The norm of E is said to be Fréchet differentiable if, for each $x \in E$

$$\lim_{t\to 0} \frac{\|x+ty\| - \|x\|}{t}$$

converges uniformly for y with $\|y\| = 1$.

The following lemma was proved by Schu [12].

LEMMA 2.1 [Schu [12]]. *Let $\{x_n\}$ and $\{y_n\}$ be sequences of a uniformly convex Banach space E and let $\{\alpha_n\}$ be a sequence of real numbers such that $0 < a \le \alpha_n \le b < 1$ for all $n \in \mathbb{N}$. Suppose that $\lim_{n\to\infty} \|\alpha_n x_n + (1-\alpha_n) y_n\| = c$ exists. If $\limsup_{n\to\infty} \|x_n\| \le c$ and $\limsup_{n\to\infty} \|y_n\| \le c$, then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.*

Let C be a nonempty closed convex subset of a Banach space E. Let $\{T_n : n = 1, 2, \cdots, k\}$ be mappings of C into itself and let $\{\alpha_n \in \mathbb{R} : n = 1, 2, \cdots, k\}$ be real numbers satisfying $0 \le \alpha_n \le 1$ for each $n = 1, 2, \cdots, k$. Then, we define a mapping W of C into itself as follows (Takahashi [13]):

$$\begin{cases} U_k = \alpha_k T_k + (1-\alpha_k) I, \\ U_{k-1} = \alpha_{k-1} T_{k-1} U_k + (1-\alpha_{k-1}) I, \\ \vdots \\ U_2 = \alpha_2 T_2 U_3 + (1-\alpha_2) I, \\ W = U_1 = \alpha_1 T_1 U_2 + (1-\alpha_1) I. \end{cases}$$

Such a mapping W is called the W-mapping generated by T_k, T_{k-1}, $\cdots$, T_2, T_1 and α_k, α_{k-1}, $\cdots$, α_2, α_1.

The following lemma was proved by Takahashi and Shimoji [15].

LEMMA 2.2 (Takahashi and Shimoji [15]). *Let C be a nonempty closed convex subset of a strictly convex Banach space E. Let T_1, T_2, $\cdots$, T_k be nonexpansive mappings of C into itself and suppose that $\bigcap_{i=1}^{k} F(T_i)$ is nonempty. Let $\alpha_1, \alpha_2, \cdots, \alpha_k$ be real numbers such that $0 < \alpha_i < 1$ for all $i = 1, 2, \cdots, k$. Let W be the W-mapping generated by T_1, T_2, $\cdots$, T_k and α_1, α_2, $\cdots$, α_k. Then*

$$F(W) = \bigcap_{i=1}^{k} F(T_i).$$

The following lemma was proved by Reich [11]; see also [14].

LEMMA 2.3 (Reich [11]). *Let C be a nonempty closed convex subset of a uniformly convex Banach space E whose norm is Fréchet differentiable. Let $\{W_n\}$ be a sequence of nonexpansive mappings of C into itself such that the set of their common fixed points is nonempty. Choose $x \in C$ arbitrarily and make an iteration $\{x_n\}$ by*

$$x_n = W_n W_{n-1} \cdots W_2 W_1 x$$

for each $n \in \mathbb{N}$. Then the set $\bigcap_{k=1}^{\infty} \overline{co}\{x_m : m \geq k\} \cap \bigcap_{n=1}^{\infty} F(W_n)$ consists of at most one point.

3. The main result

Now we prove a weak convergence theorem for countable nonexpansive mappings in Banach spaces which is connected with the feasibility problem.

THEOREM 3.1. *Let C be a nonempty closed convex subset of a uniformly convex Banach space E. Let $\{T_n\}$ be a sequence of nonexpansive mappings of C into itself and suppose*

$$\bigcap_{n=1}^{\infty} F(T_n) \neq \emptyset.$$

Let $a, b \in \mathbb{R}$ with $0 < a \leq b < 1$ and let $\{\alpha_{n,k} : n, k \in \mathbb{N}, 1 \leq k \leq n\} \subset [a, b]$. For $n \in \mathbb{N}$, let W_n be the W-mapping generated *by $T_n, T_{n-1}, \cdots, T_2, T_1$ and $\alpha_{n,n}, \alpha_{n,n-1}, \cdots, \alpha_{n,2}, \alpha_{n,1}$, that is,*

$$(1)\qquad \begin{cases} U_{n,n} = \alpha_{n,n}T_n + (1-\alpha_{n,n})I, \\ U_{n,n-1} = \alpha_{n,n-1}T_{n-1}U_{n,n} + (1-\alpha_{n,n-1})I, \\ \vdots \\ U_{n,2} = \alpha_{n,2}T_2U_{n,3} + (1-\alpha_{n,2})I, \\ W_n = U_{n,1} = \alpha_{n,1}T_1U_{n,2} + (1-\alpha_{n,1})I. \end{cases}$$

Let $x_1 \in C$ and put $x_{n+1} = W_n x_n$ for each $n \in \mathbb{N}$. Then $\|T_k x_n - x_n\| \to 0$ as $n \to \infty$ for each $k \in \mathbb{N}$. Moreover, if either E satisfies Opial's condition or E has a Fréchet differentiable norm, then $\{x_n\}$ converges weakly to $z \in \bigcap_{n=1}^{\infty} F(T_n)$.

Proof. Let $w \in \bigcap_{n=1}^{\infty} F(T_n)$ and $x_1 \in C$. Then, by the definition of $\{x_n\}$, we obtain

$$\begin{aligned} \|x_{n+1} - w\| &= \|W_n x_n - w\| = \|U_{n,1}x_n - w\| \\ &\le \alpha_{n,1}\|T_1U_{n,2}x_n - w\| + (1-\alpha_{n,1})\|x_n - w\| \\ &\le \alpha_{n,1}\|U_{n,2}x_n - w\| + (1-\alpha_{n,1})\|x_n - w\| \\ &\le \alpha_{n,1}\alpha_{n,2}\|U_{n,3}x_n - w\| + (1-\alpha_{n,1}\alpha_{n,2})\|x_n - w\| \\ &\quad \vdots \\ &\le \alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,k-1}\|U_{n,k}x_n - w\| \\ &\quad + (1-\alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,k-1})\|x_n - w\| \\ &\quad \vdots \\ &\le \alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,n}\|T_n x_n - w\| \\ &\quad + (1-\alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,n})\|x_n - w\| \\ &\le \alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,n}\|x_n - w\| \\ &\quad + (1-\alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,n})\|x_n - w\| \\ &= \|x_n - w\|, \end{aligned}$$

and hence $\lim_{n\to\infty}\|x_n - w\|$ exists. Put $c = \lim_{n\to\infty}\|x_n - w\|$. Fix $k \in \mathbb{N}$. Then for any $n \in \mathbb{N}$ with $n \ge k$, we have

$$\|U_{n,k}x_n - w\| \le \|x_n - w\|$$

and hence

$$\limsup_n \|U_{n,k}x_n - w\| \le c. \tag{2}$$

On the other hand, since

$$\begin{aligned}\|x_{n+1} - w\| \le \alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,k-1}\|U_{n,k}x_n - w\| \\ + (1 - \alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,k-1})\|x_n - w\|,\end{aligned}$$

we obtain

$$\begin{aligned}\|x_n - w\| &\le \|U_{n,k}x_n - w\| + \frac{\|x_n - w\| - \|x_{n+1} - w\|}{\alpha_{n,1}\alpha_{n,2}\cdots\alpha_{n,k-1}} \\ &\le \|U_{n,k}x_n - w\| + \frac{\|x_n - w\| - \|x_{n+1} - w\|}{a^{k-1}}\end{aligned}$$

and hence

$$c \le \liminf_n \|U_{n,k}x_n - w\|. \tag{3}$$

From (2) and (3), we have $c = \lim_{n\to\infty} \|U_{n,k}x_n - w\|$. So, we have

$$c = \lim_{n\to\infty} \|\alpha_{n,k}T_kU_{n,k+1}x_n + (1 - \alpha_{n,k})x_n - w\|.$$

Since

$$\limsup_{n\to\infty} \|T_kU_{n,k+1}x_n - w\| \le \limsup_{n\to\infty} \|U_{n,k+1}x_n - w\| = c$$

and $\limsup_{n\to\infty} \|x_n - w\| = c$, from Lemma 2.1, we get

$$\lim_{n\to\infty} \|T_kU_{n,k+1}x_n - x_n\| = 0.$$

For each $n \ge k + 2$, we have

$$\begin{aligned}\|T_kx_n - x_n\| &\le \|T_kx_n - T_kU_{n,k+1}x_n\| + \|T_kU_{n,k+1}x_n - x_n\| \\ &\le \|x_n - U_{n,k+1}x_n\| + \|T_kU_{n,k+1}x_n - x_n\| \\ &= \|x_n - (\alpha_{n,k+1}T_{k+1}U_{n,k+2}x_n + (1 - \alpha_{n,k+1})x_n)\| \\ &\quad + \|T_kU_{n,k+1}x_n - x_n\| \\ &= \alpha_{n,k+1}\|T_{k+1}U_{n,k+2}x_n - x_n\| + \|T_kU_{n,k+1}x_n - x_n\|.\end{aligned}$$

Consequently, we obtain $\limsup_{n\to\infty} \|T_k x_n - x_n\| \le 0$ and hence

$$\lim_{n\to\infty} \|T_k x_n - x_n\| = 0.$$

Suppose that E satisfies Opial's condition. Since $\{x_n\}$ is bounded and E is reflexive, we can choose a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ converging weakly to z. We shall show that z is a common fixed point of $\{T_k\}$. Suppose $z \notin \bigcap_{k=1}^{\infty} F(T_k)$. Then $z \neq T_k z$ for some $k \in \mathbb{N}$. Using Opial's condition, we have

$$\begin{aligned}
\liminf_i \|x_{n_i} - z\| &< \liminf_i \|x_{n_i} - T_k z\| \\
&\le \liminf_n (\|x_{n_i} - T_k x_{n_i}\| + \|T_k x_{n_i} - T_k z\|) \\
&\le \liminf_n (\|x_{n_i} - T_k x_{n_i}\| + \|x_{n_i} - z\|) \\
&= \liminf_n \|x_{n_i} - z\|.
\end{aligned}$$

This is a contradiction and hence we have $z \in \bigcap_{k=1}^{\infty} F(T_k)$.

Next, we shall show that the set of weakly cluster points of $\{x_n\}$ consists of one point. Let $\{x_{n_i}\}$ and $\{x_{n_j}\}$ be subsequences of $\{x_n\}$ converging weakly z_1 and z_2, respectively. Then both z_1 and z_2 are common fixed points of $\{T_k\}$ and thus $\lim_{n\to\infty} \|x_n - z_1\|$ and $\lim_{n\to\infty} \|x_n - z_2\|$ exist. We claim $z_1 = z_2$. If not, we have

$$\begin{aligned}
\lim_{n\to\infty} \|x_n - z_1\| &= \lim_{i\to\infty} \|x_{n_i} - z_1\| \\
&< \lim_{i\to\infty} \|x_{n_i} - z_2\| \\
&= \lim_{n\to\infty} \|x_n - z_2\| \\
&= \lim_{j\to\infty} \|x_{n_j} - z_2\| \\
&< \lim_{j\to\infty} \|x_{n_j} - z_1\| \\
&= \lim_{n\to\infty} \|x_n - z_1\|.
\end{aligned}$$

This is a contradiction and we get $z_1 = z_2$. Therefore $\{x_n\}$ converges weakly to $z \in \bigcap_{k=1}^{\infty} F(T_k)$.

On the other hand, suppose that E has a Fréchet differentiable norm. Without loss of generality, we may assume that C is bounded because a common fixed point of $\{T_k\}$ exists. Let $\{x_{n_i}\}$ be a subsequence of $\{x_n\}$ converging weakly to z. Then, since $\lim_{n\to\infty} \|T_k x_{n_i} - x_{n_i}\| = 0$

for each $k \in \mathbb{N}$ and $I - T_k$ is demiclosed, it follows from Browder [1] that z is a common fixed point of $\{T_k\}$. Since $F(W_n) = \bigcap_{k=1}^{n} F(T_k)$ from Lemma 2.2, we have $z \in \bigcap_{k=1}^{\infty} F(T_k) = \bigcap_{n=1}^{\infty} F(W_n)$ and $x_n = W_n W_{n-1} \cdots W_1 x_1$ for each $n \in \mathbb{N}$. Using Lemma 2.3, we obtain

$$\bigcap_{n=1}^{\infty} \overline{co}\{x_m : m \geq n\} \cap \bigcap_{k=1}^{\infty} F(T_k) = \{z\}.$$

Consequently, $\{x_n\}$ converges weakly to $z \in \bigcap_{k=1}^{\infty} F(T_k)$. □

4. Applications

In this section, using Theorem 1, we consider the feasibility problem of finding a solution of the countable convex inequality system. Further, we consider the problem of finding a common fixed point for a commuting countable family of nonexpansive mappings.

THEOREM 4.1. *Let C be a nonempty closed convex subset of a uniformly convex Banach space whose norm is Fréchet differentiable and let $a, b \in \mathbb{R}$ with $0 < a \leq b \leq 1$. Let $\{C_n\}$ be a sequence of nonexpansive retracts of C such that $\bigcap_{n=1}^{\infty} C_n$ is nonempty. For $n \in \mathbb{N}$, let W_n be the W-mapping generated by $P_n, P_{n-1}, \cdots, P_2, P_1$ and $\alpha_{n,n}, \cdots, \alpha_{n,2}, \alpha_{n,1}$, where P_k is a nonexpansive retraction of C onto C_k for each $k \in \mathbb{N}$ and $0 < a \leq \alpha_{n,k} \leq b < 1$ for $n, k \in \mathbb{N}$ with $0 \leq k \leq n$. Consider an iteration $\{x_n\}$ defined by (1). Then $\{x_n\}$ converges weakly to $z \in \bigcap_{n=1}^{\infty} C_n$.*

Proof. From Lemma 2, we have

$$F(W_n) = \bigcap_{k=1}^{n} F(P_k) = \bigcap_{k=1}^{n} C_k.$$

This implies

$$\bigcap_{n=1}^{\infty} F(W_n) = \bigcap_{n=1}^{\infty} F(P_n) = \bigcap_{n=1}^{\infty} C_n.$$

Using Theorem 1, $\{x_n\}$ converges weakly to $z \in \bigcap_{n=1}^{\infty} C_n$. □

REMARK 4.1. Let C be a nonempty closed convex subset of a uniformly convex Banach space E and let T be a nonexpansive mapping of C into itself with $F(T) \neq \emptyset$. Then, it follows from Bruck [2, 3] that $F(T)$ is a nonexpansive retract of C. We also know that every nonempty closed convex subset of a Hilbert space H is a nonexpansive retract of H and the norm of H is Fréchet differentiable.

THEOREM 4.2. *Let C be a nonempty bounded closed convex subset of a uniformly convex Banach space E whose norm is Fréchet differentiable. Let $\{T_n\}$ be a sequence of nonexpansive mappings of C into itself such that $T_iT_j = T_jT_i$ for all $i, j \in \mathbb{N}$. Let a and b be real numbers with $0 < a \leq b < 1$ and let $\{\alpha_{n,k} : n, k \in \mathbb{N}, 1 \leq k \leq n\} \subset [a, b]$. Then the set of common fixed points of $\{T_n\}$ is nonempty, and an iteration $\{x_n\}$ defined by (1) converges weakly to an element of $\bigcap_{n=1}^{\infty} F(T_n)$.*

Proof. Since E is uniformly convex, from [1] or Kirk's fixed point theorem [9], we obtain that each T_i has a fixed point. We also know from commutativity of $\{T_n\}$ that $\{F(T_n)\}$ is a sequence of nonempty closed convex subsets which has the finite intersection property; for more details, see [8]. Since C is bounded and E is uniformly convex, C is weakly compact. Hence we have

$$\bigcap_{n=1}^{\infty} F(T_n) \neq \emptyset.$$

Therefore, by Theorem 3.1, $\{x_n\}$ converges weakly to an element of $\bigcap_{n=1}^{\infty} F(T_n)$. □

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