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Double Unit Root Tests Based on Recursive Mean Adjustment and Symmetric Estimation†

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ABSTRACT

Symmetric estimation and recursive mean adjustment are considered to construct tests for the double unit root hypothesis for both parametric and semiparametric time series models. It is shown that simultaneous application of symmetric estimation and recursive mean adjustment yields the most powerful test. Moreover, size property of the semiparametric test based on the simultaneous application is best among all semiparametric tests.

Keywords: Double unit roots; recursive mean adjustment; symmetric estimation.

1. INTRODUCTION

Now days, it is a cliche to say that major economic or financial time series are well represented by nonstationary time series models. One large class of nonstationary models is that of first order integrations, which can be characterized by one autoregressive unit root. However, several authors claimed that double unit root models are more suitable for some time series such as the commercial bank real-estate loans (Dickey and Pantula, 1987), the U.S. population (Sen and Dickey, 1987), the Latin American exchange rates and relative prices (Haldrup, 1994), and the Korean price indices (Shin and Kim, 1999).

In the literature, several methods are available for testing the double unit root hypothesis. Hasza and Fuller(1979) and Sen and Dickey(1987) constructed F-type tests and Dickey and Pantula(1987) proposed a sequential procedure. Haldrup(1994) and Shin and Kim(1999) constructed semiparametric tests.

We consider two approaches to improve powers of double unit root tests: the symmetric estimation and the recursive mean adjustment. The symmetric

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estimation proves to be effective in improving powers of double unit root tests as shown by Sen and Dickey(1987) and Shin and Kim(1999). We expect that the recursive mean adjustment of Shin and So(2001) and So and Shin(1999) is also effective in improving powers of double unit root tests. We find that simultaneous application of the symmetric estimation and the recursive mean adjustment is more effective both in improving sizes of semiparametric tests and in enhancing powers of all tests than single applications.

In Section 2, tests are constructed for the eight combinations of (ordinary estimation, symmetric estimation), (ordinary mean adjustment, recursive mean adjustment), and (parametric model, semiparametric model). In Section 3, Monte-Carlo experiments are conducted to compare the tests.

2. MODELS AND TESTS

2.1. Parametric tests

We first consider a parametric regression model considered by Hasza and Fuller (1979) and Sen and Dickey(1987) given by

$$\Delta^2 y_t = \pi_1(y_{t-1}-\mu)+\pi_2\Delta y_{t-1}+\alpha_1\Delta^2 y_{t-1}+...+\alpha_p\Delta^2 y_{t-p}+e_t, \tag{1}$$

$t = p+3, ..., n$, where $\{y_t, t = 1, ..., n\}$ is the set of observations, $p \geq 0$ is a given integer, $\pi = (\pi_1, \pi_2)'$, $\alpha = (\alpha_1, ..., \alpha_p)'$ are vectors of unknown coefficients, and Δ is the difference operator such that $\Delta y_t = y_t - y_{t-1}$. For model (1), we assume:

C1. e_t is a sequence of independent random variables having mean zero and variance σ_e^2;

all roots of the characteristic equation $1 - \alpha_1 B$ - ... $- \alpha_p B^p = 0$ lie outside the unit circle.

We are interested in testing the null hypothesis of double unit roots

$$H_0 : \pi_1 = \pi_2 = 0.$$

Let $\theta = (\pi', \alpha')' = (\pi_1, \pi_2, \alpha_1, ..., \alpha_p)'$. A common method for adjusting the mean parameter μ is to use the mean adjusted observations $y_t - \bar{y}$, where $\bar{y} = n^{-1}\sum_{t=1}^n y_t$ is the sample mean. The ordinary least squares estimator (OLSE) is given by

$$\hat{\theta}_O = (X_O' X_O)^{-1} X_O' Y,$$

where X_O is the $(n-p-2) \times (p+2)$ row vector whose $(t-p-2)-th$ row is

$$(y_{t-1} - \bar{y},\ \Delta y_{t-1},\ \Delta^2 y_{t-1},\ \dots\ ,\Delta^2 y_{t-p}),\ t = p+3,\ \dots,\ n,$$

Y is the $(n-p-2) \times 1$ row vector whose $(t-p-2)-th$ element is $\Delta^2 y_t$, $t = p+3, \dots, n$. Another method for mean adjustment is to use recursively adjusted observations $y_{t-1} - \bar{y}_{t-1}$, where $\bar{y}_t = t^{-1}\sum_{i=1}^{t} y_t$. In the recursive mean adjustment, a datum y_t at a time t is adjusted for the mean through the partial sample mean $\bar{y}_t$, the average of the data up to the time point. The OLSE based on the recursive mean adjustment is given by

$$\hat{\theta}_R = (X_R' X_R)^{-1} X_R' Y,$$

where X_R is the $(n-p-2) \times (p+2)$ row vector whose $(t-p-2)-th$ row is

$$(y_{t-1} - \bar{y}_{t-1},\ \Delta y_{t-1},\ \Delta^2 y_{t-1},\ \dots\ ,\Delta^2 y_{t-p}),\ t = p+3,\ \dots,\ n.$$

See Shin and So(2001) and So and Shin(1999) for properties of the recursive mean adjustment. The estimator based on the symmetric regression and the ordinary mean adjustment is

$$\hat{\theta}_S = (X_S' X_S)^{-1} X_S' Y_S,$$

where

$$X_S = (X_1^s|\ X_2^s|\ Z_1^s|\ \dots\ |Z_p^s),$$

$$X_1^s = (y_{p+2} - \bar{y},\ \dots,\ y_{n-1} - \bar{y},\ y_{n-p-1} - \bar{y},\ y_{n-p-2} - \bar{y},\ \dots,\ y_2 - \bar{y})',$$
$$X_2^s = (\Delta y_{p+2},\ \dots,\ \Delta y_{n-1},\ -\Delta y_{n-p},\ -\Delta y_{n-p-1},\ \dots,\ -\Delta y_3)',$$
$$Z_k^s = (\Delta^2 y_{p+3-k},\ \dots,\ \Delta^2 y_{n-k},\ \Delta^2 y_{n-p+k},\ \dots,\ \Delta^2 y_{k+3})',\ k = 1,\ \dots,\ p,$$

and

$$Y_S = (\Delta^2 y_{p+3},\ \dots,\ \Delta^2 y_n,\ \Delta^2 y_{n-p},\ \dots,\ \Delta^2 y_3)'.$$

See Fuller(1976, p. 60) and Pantula et al.(1994) for discussions on symmetric estimation. The estimator based on the symmetric regression and the recursive mean adjustment is

$$\hat{\theta}_{SR} = (X_{SR}' X_{SR})^{-1} X_{SR}' Y_S$$

where X_{SR} is the same as X_S except that $y_t - \bar{y}_t$ is used in place of $y_t - \bar{y}$.

Now, the F-tests for H_0 based on $\hat{\theta}_O = (\hat{\pi}_O',\ \hat{\alpha}_O')'$, $\hat{\theta}_R = (\hat{\pi}_R',\ \hat{\alpha}_R')'$, $\hat{\theta}_S = (\hat{\pi}_S',\ \hat{\alpha}_S')'$, and $\hat{\theta}_{SR} = (\hat{\pi}_{SR}',\ \hat{\alpha}_{SR}')'$ are

$$F_O = \hat{\pi}_O'[(X'X)^{11}]^{-1}\hat{\pi}_O/(2\hat{\sigma}_e^2),$$

$$F_R = \hat{\pi}_R'[(X_R'X_R)^{11}]^{-1}\hat{\pi}_R/(2\hat{\sigma}_e^2),$$

$$F_S = \hat{\pi}_S'[(X_S'X_S)^{11}]^{-1}\hat{\pi}_S/(2\hat{\sigma}_e^2),$$
$$F_{SR} = \hat{\pi}_{SR}'[(X_{SR}'X_{SR})^{11}]^{-1}\hat{\pi}_{SR}/(2\hat{\sigma}_e^2),$$

respectively, where $(X'X)^{11}$ is the upper left 2×2 submatrix of $(X'X)^{-1}$ and etc and $\hat{\sigma}_e^2$ is a consistent estimator of σ_e^2 for which we may use the OLS variance estimator. The test F_O is the same as the F-type test ($\Phi_2(2)$) of Hasza and Fuller(1979) except for mean adjustment. In constructing $\Phi_2(2)$, ordinary least square regression is performed to the intercept model

$$\Delta^2 y_t = c + \pi_1 y_{t-1} + \pi_2 \Delta y_{t-1} + \alpha_1 \Delta^2 y_{t-1} + \dots\ + \alpha_p \Delta^2 y_{t-p} + e_t,\ t = p+3,\ \dots,$$

In this OLS regression of Hasza and Fuller(1979), the regressor Δy_{t-1} is adjusted by $(n-p-2)^{-1}\sum_{t=p+3}^{n}\Delta y_{t-1} = (n-p-2)^{-1}(y_{n-1} - y_{p+1})$. On the other hand, in constructing F_O, no adjustment is made for Δy_{t-1}. The test F_S is considered by Sen and Dickey(1987). The tests F_R and F_{SR} are the same as F_O and F_S except that the recursive mean adjustment is used instead of the ordinary mean adjustment. In the following theorem, we give the limiting null distributions of the test statistics.

Theorem 1. *Consider model (1) with C1. Assume* $\pi_1 = \pi_2 = 0$. *Then*

$F_O \Rightarrow 2^{-1}B_O'A_O^{-1}B_O,$

$F_R \Rightarrow 2^{-1}B_R'A_R^{-1}B_R,$

$F_S \Rightarrow \{\int_0^1 W_{O2}^2(s)ds\}^{-1}\{\int_0^1 W_{O2}(s)dW(s)\}^2 + 4^{-1}\{\int_0^1 W^2(s)ds\}^{-1},$

$F_{SR} \Rightarrow \{\int_0^1 W_{R2}^2(s)ds\}^{-1}\{\int_0^1 W_{R2}(s)dW(s)\}^2 + 4^{-1}\{\int_0^1 W^2(s)ds\}^{-1},$

where $\Rightarrow$ *denotes convergence in distribution,* $W(s)$ *is a standard Brownian motion on [0, 1],*

$$A_O = \begin{pmatrix} \int_0^1 W_{O2}^2(s)ds & \int_0^1 W_{O2}(s)W(s)ds \\ \int_0^1 W_{O2}(s)W(s)ds & \int_0^1 W^2(s)ds \end{pmatrix},\ B_O = \begin{pmatrix} \int_0^1 W_{O2}(s)dW(s) \\ \int_0^1 W(s)dW(s) \end{pmatrix},$$

$$A_R = \begin{pmatrix} \int_0^1 W_{R2}^2(s)ds & \int_0^1 W_{R2}(s)W(s)ds \\ \int_0^1 W_{R2}(s)W(s)ds & \int_0^1 W^2(s)ds \end{pmatrix},\ B_R = \begin{pmatrix} \int_0^1 W_{R2}(s)dW(s) \\ \int_0^1 W(s)dW(s) \end{pmatrix},$$

$W_2(s) = \int_0^s W(r)dr,$

$W_{O2}(s) = W_2(s) - \int_0^1 W_2(r)dr,$

$W_{R2}(s) = W_2(s) - s^{-1}\int_0^s W_2(r)dr.$

Proof. The results are consequences of the invariance principle $n^{-1/2}\sum_{t=1}^{[ns]} e_t \Rightarrow \sigma_e W(s)$ and the continuous mapping theorem, where $[ns]$ is the integer part of ns. Details are omitted.

Table 1. Null right percentiles of the test statistics adjusted for mean.

	probability of larger value					
n	10%	5%	1%	10%	5%	1%
	F_O, $Z(\bar{F}_O)$			F_S, $Z(\bar{F}_S)$		
25	3.88	4.96	7.60	7.55	9.72	15.13
50	3.89	4.86	7.15	7.44	9.30	13.63
100	3.90	4.82	6.94	7.43	9.15	13.05
250	3.90	4.80	6.72	7.40	9.05	12.73
500	3.90	4.80	6.72	7.36	9.02	12.75
	F_R, $Z(\bar{F}_R)$			F_{SR}, $Z(\bar{F}_{SR})$		
25	2.48	3.28	5.23	5.46	7.29	11.80
50	2.56	3.30	5.15	5.60	7.22	11.21
100	2.59	3.34	4.97	5.63	7.27	10.86
250	2.64	3.38	5.03	5.68	7.33	11.10
500	2.63	3.35	5.04	5.71	7.33	10.92

In Table 1, some percentage points of the test statistics are presented. These percentiles are constructed from 50,000 simulated test statistics for model $\Delta^2 y_t = \pi_1(y_{t-1} - \mu) + \pi_2\Delta y_{t-1} + e_t, t = 1, ..., n$ with $y_{-1} = y_0 = 0$, $\pi_1 = \pi_2 = 0$, and standard normal errors e_t generated from RNNOA, a FORTRAN subroutine in IMSL(1989).

2.2. Semiparametric tests

We next consider a semiparametric regression model considered by Haldrup(1994) and Shin and Kim(1999) given by

$$\Delta^2 y_t = \pi_1(y_{t-1} - \mu) + \pi_2\Delta y_{t-1} + u_t, \tag{2}$$

where u_t is a stationary process satisfying the following condition:

C2. $E(u_t) = 0$ for all t;

$\sup_t E[|u_t|^{\eta+\epsilon}] < \infty$ for some $\eta > 2$ and $\epsilon > 0$;

$\sigma^2 = \lim_n n^{-1}E[\sum_{t=1}^n u_t]^2$ exists and $\sigma^2 > 0$;

$\{u_t\}$ is a strong mixing, with mixing coefficients γ_m satisfying $\sum_{m=1}^\infty \gamma_m^{1-2/\eta} < \infty$.

This condition is required for invariance principle of the partial sums of u_t.

We now construct semiparametric tests. Let $\bar{\pi}_O = (\bar{\pi}_{O1}, \bar{\pi}_{O2})'$, $\bar{\pi}_R = (\bar{\pi}_{R1}, \bar{\pi}_{R2})'$, $\bar{\pi}_S = (\bar{\pi}_{S1}, \bar{\pi}_{S2})'$, $\bar{\pi}_{SR} = (\bar{\pi}_{SR1}, \bar{\pi}_{SR2})'$, $\bar{F}_O$, $\bar{F}_R$, $\bar{F}_S$, $\bar{F}_{SR}$ be the same as

$\hat{\pi}_O$, $\hat{\pi}_R$, $\hat{\pi}_S$, $\hat{\pi}_{SR}$, F_O, F_R, F_S, F_{SR}, respectively, in Section 2.1 constructed with $p=0$. Also let $\bar{X}_O$, $\bar{X}_R$, $\bar{X}_S$, $\bar{X}_{SR}$ be the same as X_O, X_R, X_S, X_{SR}, respectively, constructed with $p=0$. The semiparametric tests are

$$Z(\bar{F}_O) = (\bar{\sigma}_u/\bar{\sigma})^2\bar{F}_O - 2^{-1}M_O^{-1}[2\bar{\lambda}(m_{\Delta y\Delta^2 y}m_{Oyy} - m_{Oy\Delta y}m_{Oy\Delta^2 y}) - (\bar{\lambda}\bar{\sigma})^2 m_{Oyy}],$$
$$Z(\bar{F}_R) = (\bar{\sigma}_u/\bar{\sigma})^2\bar{F}_R - 2^{-1}M_R^{-1}[2\bar{\lambda}(m_{\Delta y\Delta^2 y}m_{Ryy} - m_{Ry\Delta y}m_{Ry\Delta^2 y}) - (\bar{\lambda}\bar{\sigma})^2 m_{Ryy}],$$
$$Z(\bar{F}_S) = [(\bar{\sigma}_u/\bar{\sigma})\bar{\pi}_{S1},\ (\bar{\sigma}/\bar{\sigma}_u)\bar{\pi}_{S2}](\bar{X}_S'\bar{X}_S)[(\bar{\sigma}_u/\bar{\sigma})\bar{\pi}_{S1},\ (\bar{\sigma}/\bar{\sigma}_u)\bar{\pi}_{S2}]'\ /(2\bar{\sigma}^2),$$
$$Z(\bar{F}_{SR}) = [(\bar{\sigma}_u/\bar{\sigma})\bar{\pi}_{SR1},\ (\bar{\sigma}/\bar{\sigma}_u)\bar{\pi}_{SR2}](\bar{X}_{SR}'\bar{X}_{SR})[(\bar{\sigma}_u/\bar{\sigma})\bar{\pi}_{SR1},\ (\bar{\sigma}/\bar{\sigma}_u)\bar{\pi}_{SR2}]'\ /(2\bar{\sigma}^2),$$

where $(\bar{\sigma}^2,\ \bar{\sigma}_u^2)$ are consistent estimators of $(\sigma^2,\ \sigma_u^2)$,

$$m_{\Delta y\Delta^2 y} = n^{-3}\sum \Delta y_t\Delta^2 y_t, \qquad m_{\Delta y\Delta y} = n^{-2}\sum \Delta y_t\Delta y_t,$$
$$m_{Oyy} = n^{-4}\sum(y_t-\bar{y})^2, \qquad m_{Oy\Delta y} = n^{-3}\sum(y_t-\bar{y})\Delta y_t,$$
$$m_{Oy\Delta^2 y} = n^{-2}\sum(y_t-\bar{y})\Delta^2 y_t, \qquad M_O = m_{Oyy}m_{\Delta y\Delta y} - m_{Oy\Delta y}^2,$$
$$m_{Ryy} = n^{-4}\sum(y_t-\bar{y}_t)^2, \qquad m_{Ry\Delta y} = n^{-3}\sum(y_t-\bar{y}_t)\Delta y_t,$$
$$m_{Ry\Delta^2 y} = n^{-2}\sum(y_t-\bar{y}_t)\Delta^2 y_t, \qquad M_R = m_{Ryy}m_{\Delta y\Delta y} - m_{Ry\Delta y}^2,$$
$$\bar{\lambda} = 2^{-1}(\bar{\sigma}^2 - \bar{\sigma}_u^2)/\bar{\sigma}^2,$$

ℓ is an integer that increases with n and $w_{h\ell} = 1-h\ /(\ell+1)$. The test $Z(\bar{F}_O)$ and $Z(\bar{F}_S)$ were introduced by Haldrup(1994) and Shin and Kim(1999), respectively. The other two tests $Z(\bar{F}_R)$ and $Z(\bar{F}_{SR})$ are the recursive counterparts of $Z(\bar{F}_O)$ and $Z(\bar{F}_S)$, respectively.

Theorem 2. *Consider model (2) with condition C2. Let $\pi_1 = \pi_2 = 0$. Then the limiting distributions of $Z(\bar{F}_O)$, $Z(\bar{F}_R)$, $Z(\bar{F}_S)$, $Z(\bar{F}_{SR})$ are the same as those F_O, F_R, F_S, F_{SR}, respectively, given in Theorem 1.*

Proof. The results are consequences of the invariance principle $n^{-1/2}\sum_{t=1}^{[ns]} u_t \Rightarrow \sigma W(s)$ and the continuous mapping theorem together with consistency of $(\bar{\sigma}^2, \bar{\sigma}_u^2)$. Details are omitted.

Since the limiting null distributions of $Z(\bar{F}_O)$, $Z(\bar{F}_R)$, $Z(\bar{F}_S)$, $Z(\bar{F}_{SR})$ are the same as those of F_O, F_R, F_S, F_{SR}, respectively, we can use the percentage points in Table 1 for testing purpose. For consistent estimators of $(\sigma^2,\ \sigma_u^2)$, we may use

$$\bar{\sigma}_O^2 = n^{-1}\sum_{t=1}^n \bar{u}_t^2 + 2n^{-1}\sum_{h=1}^{\ell} w_{h\ell}\sum_{t=h+\ell}^n \bar{u}_t\bar{u}_{t-h}, \qquad \bar{\sigma}_{Ou}^2 = n^{-1}\sum_{t=1}^n \bar{u}_t^2,$$

or

$$\bar{\sigma}_0^2 = n^{-1}\sum_{t=1}^n(\Delta^2 y_t)^2 + 2n^{-1}\sum_{h=1}^{\ell} w_{h\ell}\sum_{t=h+\ell}^n(\Delta^2 y_t)(\Delta^2 y_{t-h}), \qquad \bar{\sigma}_{0u}^2 = n^{-1}\sum_{t=1}^n(\Delta^2 y_t),$$

where $\bar{u}_t = \Delta^2 y_t - \bar{\pi}_{O1}(y_{t-1}-\bar{y}) - \bar{\pi}_{O2}\Delta y_{t-1}$ are residuals in the OLS fitting to model (2). The estimators $(\bar{\sigma}_0^2,\ \bar{\sigma}_{0u}^2)$, being based on the null model, would give better size performances and the other estimators $(\bar{\sigma}_O^2,\ \bar{\sigma}_{Ou}^2)$, being based on OLS-residuals, would give better power performances for the test statistics.

3. A MONTE CARLO STUDY

We compare size and power properties of the eight tests through Monte-Carlo experiments. Data are simulated from model

$$\Delta^2 y_t = \pi_1(y_{t-1}-\mu)+\pi_2\Delta y_{t-1}+\alpha\Delta^2 y_{t-1}+e_t, \tag{3}$$

with the standard normal errors e_t generated by RNNOA and $y_0 = y_{-1} = 0$. The parametric tests F_O, F_R, F_S, F_{SR} are constructed by fitting model (3). The semiparametric tests $Z(\bar{F}_O)$, $Z(\bar{F}_R)$, $Z(\bar{F}_S)$, $Z(\bar{F}_{SR})$ are constructed with lag-window $\ell = 4(n\ /\ 100)^{1/4}$ and variance estimators $(\bar{\sigma}_0^2, \bar{\sigma}_{0u}^2)$. The nominal level of the tests is set to 5% and the number of replications is 10,000.

We first investigate size performances for which we consider the following parameter combination: n = 25, 50, 100, 250; α = .8, .4, 0, -.4, -.8. Table 2 reports empirical sizes of the test statistics. We observe the following points. The parametric tests F_O, F_R, F_S, F_{SR} have reasonable sizes regardless of type of mean adjustment except for (n = 25, 50 and $\alpha = 0.8$). Sizes of the semiparametric tests $Z(\bar{F}_O)$, $Z(\bar{F}_R)$, $Z(\bar{F}_S)$, $Z(\bar{F}_{SR})$ seem to be affected by type of mean adjustment: compared with $Z(\bar{F}_O)$, $Z(\bar{F}_R)$ has better size for $\alpha > 0$ but has worse size for $\alpha < 0$; compared with $Z(\bar{F}_S)$, $Z(\bar{F}_{SR})$ has better size for $\alpha > 0$ while having similar size for $\alpha \leq 0$. Among all the semiparametric tests, the test $Z(\bar{F}_{SR})$ based on symmetric estimation and recursive mean adjustment has the best size performance. For example, if $n = 100$, $\alpha = 0.8$, the size value 13.8% of $Z(\bar{F}_{SR})$ is much better than the corresponding size values (30.3%, 31.3%, 22.1%) of ($Z(\bar{F}_O)$, $Z(\bar{F}_R)$, $Z(\bar{F}_S)$).

We next look into power performances for which we consider the following parameter configuration: $n = 100$; $(1+\pi_1, 1+\pi_2) = (1, .95, .9)$, $\pi_1 \geq \pi_2$; α = .8, .4, 0, -.4, -.8. Table 3 reports empirical powers of the test statistics. We find the following facts. The tests F_R, F_{SR}, $Z(\bar{F}_R)$, $Z(\bar{F}_{SR})$ based on the recursive mean adjustment have higher powers than the corresponding tests F_O, F_S, $Z(\bar{F}_O)$, $Z(\bar{F}_S)$, respectively, based on the ordinary mean adjustment. For example, if $(1+\pi_1 = 1, 1+\pi_2 = 0.9, \alpha = 0)$, the power values (43.8%, 47.0%, 51.5%, 52.1%) of (F_R, F_{SR}, $Z(\bar{F}_R)$, $Z(\bar{F}_{SR})$) are greater than the corresponding power values (36.2%, 37.5%, 38.1%, 41.2%) of (F_O, F_S, $Z(\bar{F}_O)$, $Z(\bar{F}_S)$), respectively. This power comparison is meaningful because size values of the tests at $(1+\pi_1 = 1+\pi_2 = 1,\ \alpha = 0)$ are all close to the nominal level 5%. Among the four parametric tests (F_O, F_S, F_R, F_{SR}), the test F_{SR} has the highest power and among the four semiparametric tests ($Z(\bar{F}_O)$, $Z(\bar{F}_R)$, $Z(\bar{F}_S)$, $Z(\bar{F}_{SR})$), the test $Z(\bar{F}_{SR})$ has the highest power.

From this Monte-Carlo study, we can say that the recursive mean adjustment and symmetric estimation substantially improve powers of tests. The two methods have "cocktail effect" in that joint application of these two methods are more effective in improving powers of tests than single applications. In addition to power improvement, joint application of the two method enhance size properties of the semiparametric tests.

Table 2. Empirical sizes(%) of tests for double unit roots adjusted for mean.

n	α	F_O	F_R	F_S	F_{SR}	$Z(\bar{F}_O)$	$Z(\bar{F}_R)$	$Z(\bar{F}_S)$	$Z(\bar{F}_{SR})$
25	.8	12.4	10.8	4.6	3.0	58.1	48.3	20.4	13.7
25	.4	8.0	7.8	4.8	3.9	20.2	14.6	11.4	5.7
25	.0	7.3	7.2	4.7	4.4	10.4	7.4	6.9	5.0
25	-.4	6.2	6.6	4.3	4.2	8.6	18.5	7.7	9.4
25	-.8	6.8	6.9	4.8	4.8	31.9	68.4	33.1	35.3
50	.8	8.2	8.1	4.9	3.7	48.4	39.6	22.4	13.8
50	.4	6.3	6.4	5.0	4.8	15.2	10.4	10.1	6.0
50	.0	6.6	6.2	5.3	4.7	8.9	6.9	6.9	5.6
50	-.4	5.9	5.9	4.8	4.7	7.9	19.4	7.7	8.8
50	-.8	5.8	5.7	4.6	4.9	34.3	74.8	34.0	32.5
100	.8	6.5	6.3	4.7	4.2	38.1	31.3	22.1	12.4
100	.4	5.7	5.6	4.9	4.5	11.3	7.9	8.5	5.7
100	.0	5.6	5.5	4.8	4.6	7.1	6.2	6.1	5.2
100	-.4	5.1	5.3	4.6	5.1	7.5	17.9	6.9	8.5
100	-.8	5.8	5.3	5.2	5.0	32.9	75.6	31.9	31.3
250	.8	5.9	5.4	5.3	4.3	30.3	33.3	19.8	11.4
250	.4	5.3	5.6	5.2	5.0	7.8	8.9	7.1	5.4
250	.0	5.4	5.5	5.1	5.4	6.3	5.5	5.7	5.6
250	-.4	5.0	5.5	5.0	4.8	8.0	29.5	6.1	6.9
250	-.8	5.0	5.2	4.8	5.0	33.3	92.6	25.4	23.5
500	.8	5.1	4.9	5.1	4.6	27.4	32.4	19.0	11.6
500	.4	5.3	5.1	5.2	4.9	7.7	8.2	7.2	5.2
500	.0	5.3	4.9	5.2	4.5	5.5	5.1	5.5	4.8
500	-.4	5.0	5.0	5.1	5.1	7.6	30.4	5.4	6.3
500	-.8	4.8	4.8	4.9	4.4	31.4	93.3	18.6	18.1

Table 3. Empirical powers(%) of tests for double unit roots adjusted for mean.

π_1	π_2	α	F_O	F_R	F_S	F_{SR}	$Z(\bar{F}_O)$	$Z(\bar{F}_R)$	$Z(\bar{F}_S)$	$Z(\bar{F}_{SR})$
1.00	1.00	.8	6.6	6.6	5.0	4.3	37.5	31.2	21.6	12.4
1.00	.95	.8	15.3	14.7	14.9	16.8	30.7	20.7	30.0	19.0
1.00	.90	.8	24.8	26.8	26.9	32.5	28.1	19.1	29.5	24.6
.95	.95	.8	39.8	41.6	48.4	56.2	48.4	39.7	54.7	49.6
.95	.90	.8	57.7	66.0	70.8	80.6	53.9	50.8	64.9	66.6
.90	.90	.8	77.2	85.6	89.2	94.6	66.3	68.5	80.2	85.2
1.00	1.00	.4	5.9	4.9	4.9	4.2	11.2	8.0	8.3	5.3
1.00	.95	.4	16.5	17.5	17.0	21.8	17.0	11.6	17.5	18.2
1.00	.90	.4	34.7	40.1	37.1	47.9	26.7	23.8	30.0	36.0
.95	.95	.4	47.3	54.2	58.2	70.6	40.9	40.7	51.9	60.7
.95	.90	.4	72.6	82.8	85.1	92.9	61.5	68.6	77.4	86.6
.90	.90	.4	92.7	97.1	97.9	99.5	84.5	91.8	95.5	98.5
1.00	1.00	.0	5.3	5.7	4.8	4.8	7.2	6.6	5.9	5.4
1.00	.95	.0	15.8	17.3	15.8	22.4	16.8	18.1	17.3	22.9
1.00	.90	.0	36.2	43.8	38.1	51.5	37.5	47.0	41.2	52.1
.95	.95	.0	49.0	56.6	60.1	73.8	50.1	58.3	62.4	74.6
.95	.90	.0	77.6	86.3	89.1	94.9	79.7	87.9	91.6	95.6
.90	.90	.0	95.8	98.6	99.2	99.9	96.9	99.1	99.5	99.9
1.00	1.00	-.4	5.7	5.7	5.0	5.4	7.8	18.3	7.7	8.5
1.00	.95	-.4	16.3	17.5	16.3	23.0	33.8	53.4	35.0	40.9
1.00	.90	-.4	37.6	45.7	40.0	53.7	74.9	89.5	76.8	81.4
.95	.95	-.4	50.2	57.4	60.3	74.9	80.5	89.2	87.7	93.1
.95	.90	-.4	79.8	88.5	90.4	96.1	98.0	99.4	99.4	99.8
.90	.90	-.4	96.6	99.0	99.3	99.9	99.9	100.0	100.0	100.0
1.00	1.00	-.8	5.3	5.4	4.6	4.5	34.0	75.4	33.3	31.0
1.00	.95	-.8	17.0	17.8	17.1	23.2	90.5	98.0	90.5	89.0
1.00	.90	-.8	38.9	47.6	41.7	55.1	99.8	100.0	99.8	99.8
.95	.95	-.8	50.4	58.3	61.0	74.8	99.8	100.0	99.9	100.0
.95	.90	-.8	80.8	89.8	91.2	97.0	100.0	100.0	100.0	100.0

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