광 베니언-형 교환 망에서의 누화를 회피하기 위한 교환소자를 달리하는 멀티캐스트 스케줄링 (제1부) : 누화 관계의 그래프 이론적 분석

(Switching Element Disjoint Multicast Scheduling for Avoiding Crosstalk in Photonic Banyan-Type Switching Networks(Part I) : Graph Theoretic Analysis of Crosstalk Relationship)

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(Yeonghwan Tscha)

요 약 본 논문에서는 방향성 커플러를 이용하여 구성한 광 베니언-형 교환 망에 있어 교환소자를 달리하는 멀티캐스팅의 스케줄링을 고려한다. 임의의 주어진 시각에 최대한 하나의 접속만이 각각의 교환소자를 점유하기 때문에 블럭킹은 물론 누화가 발생하지 않도록 보장된다. 이러한 멀티캐스팅에서는 대개 수 차례에 걸친 라우팅이 수반되므로 라우팅 횟수(즉, 스케줄링 길이)를 최소한으로 하는 것이 바람직하다.

먼저 일-대-다 접속 능력을 제공하는 광 베니언-형 망에서 접속들이 동일한 교환소자를 경유(즉, 누화를 유발)하기 위한 필요충분 조건을 제시한다. 멀티캐스트 접속을 일정한 부분접속들로 분할하는 규칙을 정의하고, 부분접속들간의 누화 관계를 그래프로 표현한다. 최악의 경우의 누화를 분석하기 위해 그래프 차수의 상한을 제시한다. 후속 논문(제2부)[14]에서는 스케줄링 알고리즘과 스케줄링 길이의 상한을 고찰하고, 관련 연구결과와의 상세한 비교를 다룬다.

Abstract In this paper, we consider the scheduling of SE(switching element)-disjoint multicasting in photonic Banyan-type switching networks constructed with directional couplers. This ensures that at most, one connection holds each SE in a given time thus, neither crosstalk nor blocking will arise in the network. Such multicasting usually takes several routing rounds hence, it is desirable to keep the number of rounds(i.e., scheduling length) to a minimum.

We first present the necessary and sufficient condition for connections to pass through a common SE(i.e., make crosstalk) in the photonic Banyan-type networks capable of supporting one-to-many connections. With definition of uniquely splitting a multicast connection into distinct subconnections, the crosstalk relationship of a set of connections is represented by a graph model. In order to analyze the worst case crosstalk we characterize the upper bound on the degree of the graph. The successor paper(Part II)[14] is devoted to the scheduling algorithm and the upper bound on the scheduling length. Comparison with related results is made in detail.

1. 서 론

With the advent of optical wide-band technologies, it has been shown that large-scale photonic switching networks can be constructed using 2×2 $LiNbO_3$ directional couplers as the basic switching elements(SE's)[4],[13]. Banyan, Shuffle, and Baseline networks and theirs reverse versions are typical examples of a backbone for such networks extensively used in parallel processing and high-speed switching systems[4],[5],[11],[13].

The photonic networks promise virtually

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unlimited bandwidth. However, they possess their own problems, for instance, path-dependent signal loss and crosstalk. Studies show that the crosstalk is more severe than the loss [13]. The photonic crosstalk occurs when two or more connections pass through a directional-coupler-based SE in common. A typical system-level approach to the zero-crosstalk is to ensure that *at most one input of each SE will be used at any given time*, i.e., *SE-disjoint routing*1) [4],[6].

Qiao and Zhou[6] and Pan *et al*[4] studied various scheduling algorithms for establishing SE-disjoint one-to-one connections in the photonic Banyan network and discussed the scheduling lengths in terms of average number of routing rounds. Vaez and Lea[9],[10] presented wide-sense nonblocking networks and strictly nonblocking networks2), respectively, under various crosstalk constraints. However, the results are applicable to the multi-Banyan networks and multi-Benes networks when they are loaded with *one-to-one connections only*. Pankaj[7] proposed asymptotic upper bounds on the number of wavelengths which are needed to make *multi-Benes multicasting* networks nonblocking, respectively, in the strict-sense and in the wide-sense. In [8], we studied the wide-sense nonblocking and the strictly nonblocking Banyan-type multicasting networks. The works are based on the necessary condition that connections to be intersected in a common link thus, the optimality of the nonblocking conditions would not be justified.

In this paper we study the problem of *scheduling SE-disjoint multicasting in the directional-coupler-based Banyan-type switching networks*3). Such scheduling usually needs several routing rounds therein, the key factor pertaining to it is to keep the number of rounds(i.e., scheduling length) as few as possible. Depending upon switching technologies considered, the scheduling length can be interpreted as the number of time slots in the time domain approach[13], as the number of copies of the switching network in the space domain approach[9], and as the number of wavelengths in the wavelength domain approach[7], all required to make crosstalk-free. Multicasting(i.e., one-to-many connection) is a communication primitive used in high-speed switching and routing in order to simultaneously send data to more than one output for multi-party communications like video distribution and teleconference.

Thus, our study deserves to receive attention, due to the generality of multicast connection, the abstraction of the scheduling length, and the extensive application of the Banyan-type networks to high-speed switching and parallel processing systems. The study consists of two papers. The first(Part I) is devoted to the graph theoretical analysis on the worst case crosstalk and blocking among the multicast connections. Part II gives a scheduling algorithm for the SE-disjoint multicasting and presents various nonblocking networks under crosstalk-free constraint.

The rest of this paper is as follows. Preliminaries on the Banyan-type switching network are introduced in the next section. We present the necessary and sufficient condition for connections to make crosstalk in the multicasting networks. In Section 3, a graph model is introduced for the representation of the crosstalk relationship of connections. In order to analyze the worst case crosstalk we consider the upper bound on the

1) Even if only one signal passes through a photonic SE, a small portion of it may leave at the other unintended output. Hence, this stray signal may arrive at the input of the next stage SE and will result in the second order crosstalk[4]. Nevertheless, this is much smaller than the *first order crosstalk. In this paper, we will consider the* first order crosstalk only, as in [4],[6],[9],[10]. Distinction between crosstalk avoidance and prevention is meaningless in this paper.

2) [1] In the rearrangeable nonblocking network, some of existing connections may be reconfigured for it to be nonblocking. The wide-sense nonblocking is achieved by a clever algorithm that carefully selects the forthcoming connection paths in order to avoid all the blocking states of the network without disturbing existing connections. The strictly nonblocking network has no any constraints.

3) Throughout this paper we refer to Banyan, Shuffle, Baseline networks and their topological equivalents[12] as Banyan-type networks.

degree of the graph. Extension to the graph representing the link-disjoint multicasting is given in Section 4. Section 5 conclude this paper.

2. Preliminaries

Without loss of generality, we use the $N \times N(N=2^n)$ *reverse Baseline network*[12] as the representative of Banyan-type networks. Fig. 1 depicts a 16×16 network. There are four stages, 1, 2, 3, 4(=$\log_2 16$), respectively, from left to right. Sixteen inputs (outputs) are numbered 0 thru 15, respectively, from top to bottom. We assume every photonic SE(switching element) holds the multicasting capability supporting lower and upper broadcasts, as well as one-to-one functions, straight and cross. A multicasting SE can be constructed with directional couplers and other devices like splitters or combiners(for instance, see[3],[13]).

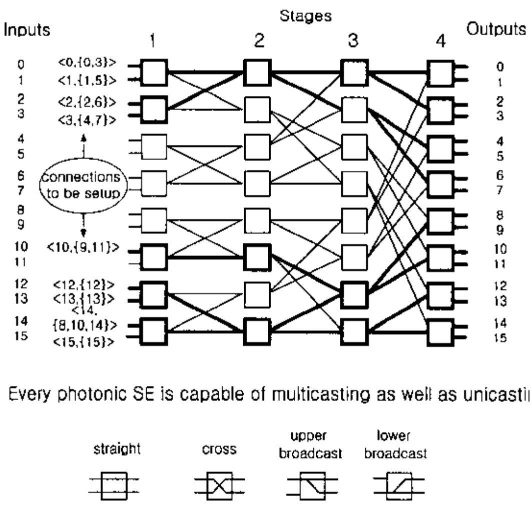


Fig. 1 A 16×16 reverse Baseline network

The *unique feature* of the Banyan-type networks is that the path between each input-output pair of the network is determined by the corresponding bit of the binary representation of the destination output[2]. If $i(1 \le i \le n)$-th bit is 0 then, the active input link of the SE at stage i is connected to the upper output link; otherwise the input is set up to the lower output link. Consider, for instance, the routing to destination output 5 in Fig. 1. Since the corresponding binary representation of the output is 0101, the sequence of decisions made by the SE at each stage is given as UP, DOWN, UP, and DOWN, respectively, from left to right. Thus, the path to the output from any source input can be easily and uniquely established by simple bit-hunting.

Denote by $\langle w,S \rangle$ a connection from an input w to a non-empty set S of destination outputs, $w \in \{0,1,...,N-1\}$, $S \subseteq \{0,1,...,N-1\}$. Then, $\langle w,S \rangle$ is one-to-one(i.e., *unicast*) if and only if $|S|=1$ and one-to-many(i.e., *multicast*) otherwise, where $|S|$ denotes the cardinality of S.

Photonic crosstalk occurs when two or more connections go through some SE in common. In Fig. 1, for instance, $\langle 0,\{0,3\} \rangle$ and $\langle 3,\{4,7\} \rangle$ use a common SE at stage 2. Hence, the output signals of the SE would be distorted. Moreover, $\langle 0,\{0,3\} \rangle$ and $\langle 1,\{1,5\} \rangle$ collide at links, respectively, between stage 1 and 2, 2 and 3, 3 and 4. The *link-blocking* also makes the crosstalk. If two connections intersect at a link between stage j and $j+1$, $1 \le j \le n-1$, evidently they use commonly the SE's at both stages. SE-disjoint routing will prevent the crosstalk problems.

We first consider a simple topological property of Banyan-type switching networks. The following holds due to the Buddy property[15] of the Banyan-type networks.

Observation 1: In the $N \times N(N=2^n)$ Banyan-type network, an SE at stage j, $1 \le j \le n$, is reachable to 2^j inputs and 2^{n-j+1} outputs, respectively. That is, there exists a back-to-back fully binary tree in which the root is an SE at stage j and the number of leaves, i.e., inputs(respectively, outputs) of the left(right) subtree is $2^j(2^{n-j+1})$ such that $2^j \cdot 2^{n-j+1} = 2N$(for instance, see Fig.2).

Let $C(i,j)$ denote the number of bits in common prefix between the n-bit binary representations of two integers i and j. For instance, C(1001,1011)=2, and C(1000,0100)=0. Given any $\langle w,R \rangle$ and $\langle x,S \rangle$, $w \ne x$, $R \cap S = \emptyset$, it follows that $0 \le C(w,x), C(r,s) \le n-1$

and $0 \le C(w,x)+C(r,s) \le 2(n-1)$ for any $r \in R$ and $s \in S$. With such notation a condition that connections will pass through a common SE is given as follows.

Lemma 1: In the $N \times N (N=2^n)$ photonic reverse baseline network, <w,R> and <x,S> use a common SE(i.e., make crosstalk) if and only if $C(w,x)+C(r,s) \ge n-1$ for some $r \in R$ and $s \in S$.

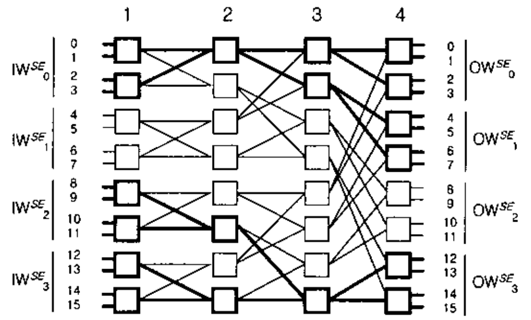
Proof: Necessity): Assume <w,R> and <x,S> pass through commonly some SE at stage $j(1 \le j \le n)$. Clearly, the common SE is reachable to inputs w and x, and also to some outputs $r \in R$ and $s \in S$. We note that inputs(outputs) of the reverse baseline network are consecutively numbered 0 thru $N-1$, respectively, from top to bottom. Considering Observation 1, the numerical difference between w and x is given as $1 \le |w-x| \le 2^j-1$[4]. Given n-bit binary representations of w and x, we have $C(w,x) \ge n-j$. Similarly, we have $1 \le |r-s| \le 2^{n-j+1}-1$. Therefore, $C(r,s) \ge n-(n-j+1) = j-1$ and finally, $C(w,x)+C(r,s) \ge n-1$. *Sufficiency)*: Let $C(w,x) \ge t$, $0 \le t \le n-1$, then, $1 \le |w-x| \le 2^{n-t}-1$. Since $C(r,s) \ge n-(t+1)$, we have $1 \le |r-s| \le 2^{t+1}-1$ for some $r \in R$ and $s \in S$. That is, there exists a back-to-back fully binary tree in which the root is the SE at stage $n-t$ and the number of inputs(outputs) of the left(right) subtree of $2^{n-t}(2^{t+1})$, as Observation 1. At least first $n-(t+1)$ bits of the n-bit binary representations of r and s are the same hence, the routing to the outputs r and s, respectively, from inputs w and x will result in intersection, at least, at stage $n-t$. □

Corollary 1: <w,R> and <x,S> intersect at stage $j(1 \le j \le n)$, for the first time, if and only if $C(w,x)=n-j$ and $C(r,s) \ge j-1$ for some $r \in R$ and $s \in S$.

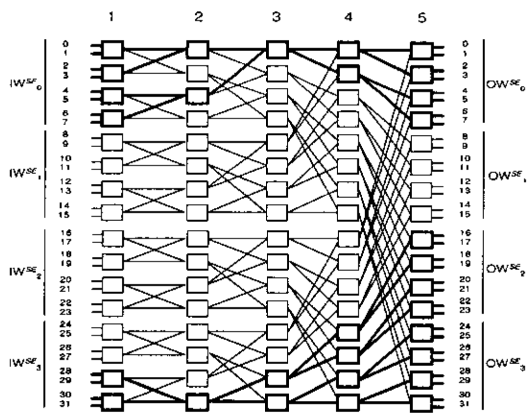
From Lemma 1, letting $C(w,x) \ge (n-1)/2$ and $C(r,s) \ge (n-1)/2$, it is seen that the maximum number of connections that go through an SE in common is $2^{\lfloor (n+1)/2 \rfloor}$ at the center stage $(n+1)/2$(respectively, $n/2$ or $(n/2)+1$) for odd(even) n where, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

4) For any set S, |S| denotes its cardinality. Notation |i-j| is used to denote the absolute numerical difference between two integers i and j. Thus, distinction between these two notations comes without confusion.

Definition 1: Given each input(output) set $\{0,1,\ldots,N-1\}$ of the reverse Baseline network, let IW^{SE}_f ($=OW^{SE}_f$) be $\{\delta_{SE} \cdot f, \delta_{SE} \cdot f+1, \ldots, \delta_{SE} \cdot f+(\delta_{SE}-1)\}$ where, $f \in \{0,1,\ldots,(N/\delta_{SE})-1\}$, $\delta_{SE}=2^{\lfloor (n+1)/2 \rfloor}$. Each IW^{SE}_f(OW^{SE}_f) is called *input(output) intersection window*. Hence, δ_{SE} is said to be *intersection window size*, i.e., $|IW^{SE}_f|=|OW^{SE}_f|=\delta_{SE}$.



a) n is even(n=4)



b) n is odd(n=5)

Fig. 2 Input intersection windows IW^{SE}_i and output intersection windows OW^{SE}_i in the $N \times N$ Banyan-type switching network

Fig. 2 shows input(output) intersection windows in the $N \times N$ Banyan-type switching networks in which N=16 and N=32, respectively. With definition of intersection window, the upper bound on the routing rounds in the SE-disjoint multicasting can be derived in terms of window size δ_{SE}. More importantly, splitting each multicast connection into several subconnections based on intersection windows helps us to easily analyze the worst case intersection in the multicast traffic. The next section will cover these in detail.

3. Graph Model for Crosstalk Connections

In this section we introduce a graph called SSG (SE-Sharing Graph) in order to represent the SE-sharing(i.e., crosstalk) relationship among the connections. We begin by defining subconnections.

Definition 2 : Given $\langle w,S\rangle$, let S be partitioned into a set of subsets $\{S^{SE}{}_f\}$ such that $S^{SE}{}_f=S\cap OW^{SE}{}_f$ and $S^{SE}{}_f \neq \emptyset$ for some $f\in\{0,1, ..., (N/\delta_{SE})-1\}$. Each $\langle w,S^{SE}{}_f\rangle$ is said to be *subconnection* of $\langle w,S\rangle$.

For the output set {8,10,14} of $\langle 14,\{8,10,14\}\rangle$ in Fig. 1, it is seen that $\{8,10\}\subset OW^{SE}{}_2$ and $\{14\}\subset OW^{SE}{}_3$. Hence, the connection has two unique subconnections $\langle 14,\{8,10\}\rangle$ and $\langle 14,\{14\}\rangle$. By definition a one-to-one connection is subconnection itself. Given $\langle w,S\rangle$ and $\{\langle w,S^{SE}{}_f\rangle\}$, it follows that $S^{SE}{}_f\subseteq OW^{SE}{}_f$, $1\leq|S^{SE}{}_f|\leq\delta_{SE}$ and that $S=\bigcup S^{SE}{}_f$, $\bigcap S^{SE}{}_f=\emptyset$, $1 \leq |\{\langle w,S^{SE}{}_f\rangle\}| \leq N/\delta_{SE}$.

Definition 3 : An SSG(SE-Sharing Graph), G_{SE} = (V_{SE},E_{SE}), is defined as follows. V_{SE} is a set of vertices corresponding to the subconnections, and E_{SE} is a set of edges such that an edge is drawn between two vertices corresponding to crosstalking subconnections.

Fig. 3 depicts G_{SE} from the connections shown in Fig. 1. In G_{SE}, there are no direct edges among the vertices that correspond to the subconnections with an identical input(for instance, see the vertices corresponding to $\langle 2,\{2\}\rangle$ and $\langle 2,\{6\}\rangle$). Because these subconnections are defined from a single one-to-many connection, they will never make the crosstalk even if they use some SE in common.

Definition 4: In G_{SE}, *physical degree* of a vertex v, denoted by $pd(v)$, is the number of vertices adjacent to v. *Logical degree* of v, $ld(v)$, is defined to be the number of vertices that are adjacent to v and correspond to subconnections with *different* inputs. Physical(respectively, logical) degree of G_{SE}, $pd(G_{SE})(ld(G_{SE}))$, is the maximal physical(logical) degree of a vertex in it.

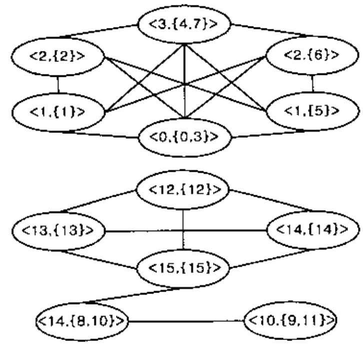


Fig. 3 G_{SE} from the connections shown in Fig. 1

In Fig. 3, $pd(\langle 3,\{4,7\}\rangle)=5$, $ld(\langle 3,\{4,7\}\rangle)=3$, $pd(G_{SE})=5$, and $ld(G_{SE})=3$, respectively. The upper bounds on $pd(G_{SE})$ and $ld(G_{SE})$ are respectively as follows.

Lemma 2: *It holds for any G_{SE} that $pd(G_{SE})\leq 0.5n\delta_{SE}$ for even n and $pd(G_{SE})\leq 0.25(n+1)\delta_{SE}$ for odd n, respectively, where $\delta_{SE}=2^{\lfloor n+1)/2\rfloor}$.*

Proof: We first consider odd n. Suppose an arbitrary subconnection $\langle w,S^{SE}{}_g\rangle$ such that $w\in IW^{SE}{}_f$, $S^{SE}{}_g\subseteq OW^{SE}{}_g$ for some $f,g\in\{0,1,...,(N/\delta_{SE})-1\}$. By bearing Lemma 1, we count all subconnections that will maximally intersect $\langle w,S^{SE}{}_g\rangle$ at stage 1, 2, ..., n, respectively(for the sake of convenience, readers may refer to Fig. 2 b)). Noting $|OW^{SE}{}_g|=\delta_{SE}$, it follows that $\langle w,S^{SE}{}_g\rangle$ and $\delta_{SE}-1$ other subconnections can intersect for any stage $j\geq(n+1)/2$, provided that $|S^{SE}{}_g|=1$. Next we consider the intersections at stage 1, 2, ..., $(n-1)/2$, respectively. An SE at stage 1 is reachable from 2 inputs and 2^n outputs(i.e., N/δ_{SE} output intersection windows) thus, $\langle w,S^{SE}{}_g\rangle$ may intersect $(N/\delta_{SE})-1$ other subconnections(note that constant "1" accounts for the subconnection destined for $OW^{SE}{}_g$ and this must be excluded from the counting since all such subconnections already considered). At stage 2 an SE is reachable from 2^2 inputs and 2^{n-1} outputs(i.e, $(N/2)(1/\delta_{SE})$ output intersection windows), respectively. Hence, excluding the subconnections considered at stage 1, it follows that $[(N/2^{2-1})(1/\delta_{SE})-1]2^{2-1}$ additional subconnections can intersect $<w,S^{SE}_g>$ at stage 2. Following the similar argument, it is seen that $[(N/2^{3-1})(1/\delta_{SE})-1]2^{3-1}$ additional subconnections at stage 3, ..., $[(N/2^{((n-1)/2)-1})(1/\delta_{SE})-1]2^{((n-1)/2)-1}$ additional subconnections at stage $(n-1)/2$. Therefore, the maximal number of subconnections that will intersect $<w,S^{SE}_g>$(i.e., the maximal physical degree of the vertex corresponding to $<w,S^{SE}_g>$) is given by $(\delta_{SE}-1)+\sum_{j=1}^{(n-1)/2}[(N/2^{j-1})(1/\delta_{SE})-1]\cdot 2^{j-1}=0.25(n+1)\delta_{SE}$. With the similar argument, it follows that $pd(G_{SE}) \le (\delta_{SE}-1)+\sum_{j=1}^{n/2}[(N/2^{j-1})(1/\delta_{SE})-1]\cdot 2^{j-1} = 0.5n\delta_{SE}$ for even n. □

Lemma 3: For any G_{SE}, it is given that $ld(G_{SE}) \le 2\delta_{SE}-2$ when n is even and $ld(G_{SE}) \le 1.5\delta_{SE}-2$ when n is odd, respectively.

Proof: By definition of logical degree of a vertex, the term $[(N/2^{j-1})(1/\delta_{SE})-1]$ of each formula given in the proof part of Lemma 2 becomes 1. This leads to that $ld(G_{SE}) \le (\delta_{SE}-1)+\sum_{j=1}^{(n-1)/2}\cdot 2^{j-1} = 1.5\delta_{SE} - 2$ for odd n, and $ld(G_{SE}) \le (\delta_{SE}-1)+\sum_{j=1}^{n/2}\cdot 2^{j-1} = 2\delta_{SE} - 2$ for even n, respectively. □

Note that $pd(G_{SE})=O(ld(G_{SE})\cdot \log_2 N)$ for sufficiently large N. Lemma 3 holds for any one-to-one connections in the Banyan-type switching networks, because every one-to-one connection is subconnection itself thus, $pd(G_{SE})=ld(G_{SE})$. Furthermore this is also true for any set of multicast connections $\{<w,S>\}$ in which the output set S of each $<w,S>$ is such that $S \subseteq OW^{SE}_f$ for some $f \in \{0,1,...,(N/\delta_{SE})-1\}$[8]. That is, the fanout capability of multicasting is restricted to the stage $\lceil (n+1)/2 \rceil$ thru n where, $\lceil x \rceil$ denotes the least integer greater than or equal to x. Any connection under such output(s) constraint will have only one subconnection regardless of whether it is one-to-one or one-to-many. The restriction can be considered as a routing algorithm and will be used for developing the wide-sense nonblocking networks in the successor paper[14].

Observations 2: It is true that $pd(G_{SE})(=ld(G_{SE})) \le 1.5\delta_{SE}-2$(respectively, $2\delta_{SE}-2$) when n is odd (even), provided that G_{SE} is obtained from any one-to-one connections or one-to-many connections $\{<w,S>\}$ in which S of each $<w,S>$ is such that $S \subseteq OW^{SE}_f$ for some $f \in \{0,1,...,(N/\delta_{SE})-1\}$.

4. Extension to Blocking Connections

So far we have considered the crosstalk relationship of the connections. A companion problem many researchers have been being interested in is the *link-disjoint*(i.e., nonblocking) routing for the Banyan-type switching networks[8]-[10].

Consider the $N \times N(N=2^n)$ Banyan-type multicasting network, as Fig.1. For the simplicity, we say that *a link is at stage j* if it is between stage j and $j+1$, $1 \le j \le n-1$. Cleary, link-blocking may happen at stage 1 thru $n-1$. The link at stage j is reachable to 2^j inputs and 2^{n-j} outputs, respectively. It is seen that at most, $2^{\lfloor n/2 \rfloor}$ connections can collide at the center stage $n/2$(respectively, $(n-1)/2$ or $(n+1)/2$) for even(odd) n. A condition similar to Lemma 1 is given as follows.

Corollary 2: In the $N \times N(N=2^n)$ reverse baseline network, connections $<w,R>$ and $<x,S>(w \ne x,\ R \cap S=\emptyset)$ collide at some link(i.e., block each other) if and only if $C(w,x)+C(r,s) \ge n$ for some $r \in R$ and $s \in S$.

By letting $\delta_L = 2^{\lfloor n/2 \rfloor}$ and replacing all occurrence of δ_{SE} in definitions given so far with it, we have another graph G_L that represents the blocking relationship among the connections, as in our previous work[8]. $G_L=(V_L,E_L)$ is defined as follows: V_L is a set of vertices corresponding to the subconnections defined based on the intersection window size δ_L, and E_L is a set of edges such that two vertices are interconnected with an edge if their corresponding subconnections intersect at some *link*. Note that $\delta_L=\delta_{SE}$ for even n and $\delta_L=\delta_{SE}/2$ for odd n, respectively. The upper bound on the physical degree of G_L, $pd(G_L)$, was given in [8]. The upper bound on the logical degree $ld(G_L)$ is as follows.

Corollary 3: For any G_L, it given that $ld(G_L) \le 1.5\delta_L-2$ for even n and $ld(G_L) \le 2\delta_L-2$ for odd n, respectively, where $\delta_L=2^{\lfloor n/2 \rfloor}$.

Proof: By Corollary 2 and analogy to Lemma 3,

$ld(G_L)$ is given as $ld(G_L) \leq (\delta_L - 1) + \sum_{j=1}^{(n/2)-1} \cdot 2^{j-1}$ for even n and $ld(G_L) \leq (\delta_L - 1) + \sum_{j=1}^{(n-1)/2} \cdot 2^{j-1}$ for odd n, respectively. Hence, the corollary follows. □

5. Conclusion

In this paper, we have presented the necessary and sufficient condition for connections to make crosstalk in the photonic Banyan-type multicasting networks. The worst case crosstalk has been characterized in terms of the degree of the graph representing the crosstalk among the connections. We note that the optimal scheduling problem is NP-complete[2]. In the successor paper[14] we will present an approximation algorithm that guarantees its scheduling length is less than double of the upper bound on the optimal length. Various nonblocking multicasting networks will be studied under the crosstalk-free constraint.

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