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Jackknifed Cochran-Mantel-Haenszel Test for Conditional Independence in Sparse $2 \times 2 \times K$ Tables

Kwang Mo Jeong[1)]

Abstract

We are interested in the conditional independence in sparse $2 \times 2 \times K$ tables with very rare cell counts. The most popular test is Cochran-Mantel-Haenszel statistic when sample sizes are moderately large enough to guarantee the chi-square approximation. We will consider jackknifing the CMH test and also suggest an approximate normal distribution for the standardized jackknifed CMH statistic. The main focus of this paper is to improve the chi-squared approximation to the CMH test by using the asymptotic normality of the jackknifed CMH test when sample sizes are very sparse but K and $N \to \infty$. The performance of the proposed jackknifed test, in the sense of significance level control and power, will be compared with that of the CMH test through a Monte Carlo study.

Keywords : Conditional Independence, Sparse $2 \times 2 \times K$ Table, Jackknifing, Cochran-Mantel-Haenszel Statistic

1. Introduction

We are concerned with testing for conditional independence in sparse $2 \times 2 \times K$ tables with very rare cell counts. The most popular test is Cochran-Mantel-Haenszel (CMH) statistic when sample sizes are moderately large enough to guarantee the chi-square approximation. The CMH statistic is known to have an asymptotic chi-square distribution with 1 degree of freedom(df) under conditional independence of row and column variables given marginal totals.

Simonoff(1986) suggested jackknifing and bootstrapping goodness-of-fit statistics in sparse multinomials. Two well-known goodness-of-fit tests are the Pearson statistic X^2 and the likelihood ratio statistic (LRT) G^2 for testing null hypothesis of specific multinomial probabilities. Simonoff(1986) used nonparametric techniques of jackknifing and bootstrapping to

1) Professor, Research Institute of Information and Communication, Department of Statistics, Pusan National University, 609-735, Pusan, Korea,
E-mail : kmjung@hyowon.pusan.ac.kr

obtain the variance estimates of X^2 and G^2. The bootstrap, the parametric bootstrap, the jackknife and a "categorical jackknife" methods were compared via a Monte Carlo simulation to choose the best one. The jackknife estimate of variance was good in parametric models for multinomial data. Koehler(1986) also discussed the asymptotic normality of the goodness-of-fit statistics for loglinear models in sparse tables. Through a Monte Carlo study the traditional chi-square approximation was shown to be reasonably accurate for the X^2 for many sparse tables with some exceptional cases. The normal approximation appeared to be much more accurate than the chi-square approximation for the LRT G^2 but the bias of estimate was remained as a nuisance problem in sparse tables.

We are confronted with sparse tables when matched case-control studies are designed. For example one person in case group is matched with one control, in which sample sizes in each 2×2 table are very sparse. If K pairs of case-control are used in the experiment then the table structure is $2\times2\times K$. In this way a fixed small number of cases can be matched to another fixed number of controls and hence it forms $n:m$ matched case-control study. In this paper we will consider jackknifing the CMH test and also suggest an approximate normal distribution for the standardized jackknifed CMH statistic. That is, the critical points of the proposed test will be approximated using the standard normal distribution. The performance of the proposed jackknifed test, in the sense of significance level control and power, will be compared with that of the CMH test through a Monte Carlo study. The main focus of this paper is to improve the chi-squared approximation to the CMH test by using the asymptotic normality of the jackknifed CMH test when sample sizes are very sparse but K and $N\to\infty$.

2. Jackknifing CMH Statistic

Let ψ_i be an odds ratio of the ith stratum in $2\times2\times K$ table. When the odds ratios ψ_i, $i=1,2,\cdots,K$, are the same with a common value ψ over all strata the table denotes homogeneous association. If the common odds ratio ψ equals 1 we say that the row and column variables are conditionally independent given the marginal totals in each table. So we are interested in testing the hypothesis of conditional independence such as

$$H_0: \psi_1 = \psi_2 = \cdots = \psi_K = 1 \tag{2.1}$$

in sparse $2\times2\times K$ tables. The count n_{11k} has a hyper-geometric distribution given the marginal totals in kth partial table. Hence under H_0 the mean $E(n_{11k})$ is given by

$$E(n_{11k}) = n_{+1k}n_{1+k}/n_{++k} \tag{2.2}$$

and also the variance $Var(n_{11k})$ is given by

$$Var(n_{11k}) = n_{+1k}n_{+2k}n_{1+k}n_{2+k}/n_{++k}^{2}(n_{++k}-1), \tag{2.3}$$

where n_{1+k} means that $n_{1+k} = n_{11k} + n_{12k}$ and the others are defined in similar ways.

The CMH statistic is defined to be

$$M = \frac{[\sum_{k=1}^{K}\{n_{11k} - E(n_{11k})\}]^2}{\sum_{k=1}^{K} Var(n_{11k})}, \tag{2.4}$$

where $E(n_{11k})$ and $Var(n_{11k})$ are given in (2.2) and (2.3), respectively. We note that the statistic M is asymptotically chi-square distributed with 1 df under H_0.

We here suggest jackknifing the CMH statistic M by omitting every 2×2 partial table one by one in ordinary ways. Define $M_{(-i)}$ as the CMH statistic with the ith table omitted in $2\times2\times K$ table, that is,

$$M_{(-i)} = \frac{[\sum_{k\neq i}\{n_{11k} - E(n_{11k})\}]^2}{\sum_{k\neq i} Var(n_{11k})}. \tag{2.5}$$

The pseudo-values are defined as

$$M^{(i)} = K\,M - (K-1)M_{(-i)}, \quad i = 1, 2, \cdots, K.$$

Finally we define the jackknifed CMH statistic M_J to be

$$\begin{aligned} M_J &= \frac{1}{K}\sum_{i=1}^{K} M^{(i)} \\ &= KM - \frac{K-1}{K}\sum_{i=1}^{K} M_{(-i)} \end{aligned} \tag{2.6}$$

The estimate of variance of M_J in (2.6) is of the form

$$V_J = \frac{1}{K(K-1)}\sum_{i=1}^{K}(M_J - M^{(i)})^2. \tag{2.7}$$

We normalize the jackknifed statistic M_J using the variance estimate V_J of (2.7) as follows

$$Z_J = \frac{M_J - E(M_J)}{\sqrt{V_J}}$$

where $E(M_J)$ is the expectation of M_J under conditional independence of (2.1). We can compute $E(M_J)=1$ in a routine method under conditional independence. The test statistic M would be compared with the jackknifed statistic M_J in (2.6) in the respects of level control and powers via a Monte Carlo simulation for sparse $2\times 2\times K$ tables. The significance levels of M_J can be determined by comparing the standardized Z_J with quantiles of standard normal distribution. We first illustrate the proposed jackknifing method through the following two practical examples.

Example 2.1 Table 2.1, originally taken from Mantel(1963), refers to the effectiveness of immediately injected or $1\frac{1}{2}$-hour-delayed penicillin in protecting rabbits against lethal injection with β-hemolytic streptococci. The data was reanalyzed in Agresti(1990) to explain the CMH test versus model based goodness-of-fit tests such as the LRT and the Pearson chi-squared test.

Table 2.1 Effect of penicillin injection

penicillin level	response	delay	
		none	$1\frac{1}{2}$ h
1/8	cured	0	0
	died	6	5
1/4	cured	3	0
	died	3	6
1/2	cured	6	2
	died	0	4
1	cured	5	6
	died	1	0
4	cured	2	5
	died	0	0

The value of CMH statistic M is 5.657 with significance of 0.017. This means that the delay of injection time is a little strongly associated with the response of cured or died. On the other hand the normal approximation of jackknifed CMH statistic does not reject the conditional independence hypothesis with significance of 0.074 which is slightly greater than the level $\alpha=0.05$. As pointed out in Agresti(1990) the exact test of conditional independence has the significance of 0.040. Hence we note that the chi-square approximation of CMH test is liberal but a normal approximation to the jackknifed CMH statistic is a little more conservative compared to the exact test in the sense of Type I error rate.

Example 2.2 A data set consisting of 63 pairs of case-control groups is represented in the form of Table 2.2. A person of endometrial cancer was matched with 4 control subjects on the basis of residence, age, material status and date of entry into the community. The purpose of experiment was to find the effect of exogenous estrogens on the risk of endometrial cancer. The exposure denotes here the use of estrogens. Jewell (1984) also introduced this data to explain a common odds ratio estimator, which was originally reported by Mack, et al. (1976), can be represented as in Table 2.2.

Table 2.2. case-control data of 63 matched pairs

		number of controls exposed					total
		0	1	2	3	4	
cases	exposed	3	17	16	15	5	56
	non-exposed	0	4	1	1	1	7

Note: Mack, et al.(1976)

Each pair of case-control group can be expressed as a 2×2 table of $n=1$, $m=4$ with the number of exposures denoting cell counts in each group, and hence 63 pairs form a $2\times2\times63$ table. The value of CMH statistic is 31.16 with significance smaller than 0.0000, and the normalized jackknifed statistic has 7.84 with significnace smaller than 0.0000 but a little stronger evidence if we represent more digits. We conclude that both tests support the strong association of estrogens with endometrial cancer. The Mantel-Haenszel common odds ratio estimator of ψ is 8.46 and this also indicates very strong association between two variables. In this example not only $K=63$ is much larger than $K=5$ of Example 2.1 but also more sparse because $n=1$ over all strata.

3. Monte Carlo Simulation

3.1. Design of the Experiment

To form the data of $2\times2\times K$ tables we do a Monte Carlo study according to sample sizes of case-control groups, exposure probabilities of case group, odds ratios and the number of strata. Sample sizes (n, m) were taken to be (1,1),(2,1),(4,1),(8,1),(2,2), and (5,5). When $n=m$ the table is called balanced but for other cases it is unbalanced. We consider the odds ratios of 1.0, 1.5, 2.0, 2.5 and 3.0 so that the level controls and powers of tests are compared. The exposure probability π_1 of case group varies from 0.1(0.2) to 0.3(0.4) according to the number of strata. On the other hand the exposure probability π_0 of control group is computed by the relationship

$$\pi_0=\frac{\pi_1}{\pi_1+(1-\pi_1)\psi}$$

where ψ denotes a common odds ratio defined before. The cell count n_{11k} is generated from the binomial distribution with success probability π_1 among n trials. Similarly n_{12k} is generated from the binomial distribution with success probability π_0 among m trials. We note that the generated cell counts n_{11k} and n_{12k} are assumed to be independent binomial random variables. The significance level considered are α=0.10, 0.05 and 0.01. The common odds ratio of $\psi=1.0$ denotes the conditional independence in $2\times2\times K$ table. The other values of odds ratios not equal to 1.0 denote the alternatives to conditional independence.

3.2. Simulation Results

The powers of two tests, the CMH test M and the jackknifed CMH test M_J, are listed in Table 3.1 (a) through (c) and also in Table 3.2 (a) through (c) according to different values of π_1 and α. In each table we considered various values of sample sizes n and m, number of strata K, and also values of odds ratio ψ. In Table 3.1 (a) we can find that M_J is a little conservative in the sense of controlling level $\alpha=0.05$, on the other hand M sometimes fail in controlling it even for large K. We note that M_J is more powerful than M when sample sizes are unbalanced but for the balanced case the phenomenon is conversed. In Table 3.1 (b) of $\alpha=0.05$ the test M_J performs more nicely than M even if $n=m=1$ compared to the case of $\alpha=0.10$. Both of the two tests don't control level $\alpha=0.05$ when $n=2$ and $m=1$. The powers of two tests, as expected, increase as level α increases. As we see in Table 3.1 (c) of $\alpha=0.01$ both M and M_J are not good in level control, and this fact is worse for the test M_J in general.

Simulation results for $\pi_1=0.30$, in this case not so sparse as in $\pi_1=0.10$, are given in Table 3.2 (a) through (c) in similar ways. In Table 3.2 (a) the test M sometimes doesn't control $\alpha=0.05$ but its powers are greater than those of M for other cases. But when $\alpha=0.05$, which is listed in Table 3.2 (b), the jackknifed test M_J performs very well compared to M in the sense of level control and powers. Lastly we see many cases such that both M and M_J fail to control $\alpha=0.01$ but M_J appears to perform at least as good as M when the level is controlled. We also note that the powers for both tests are increased when $\pi_1=0.30$ compared to the case $\pi_1=0.10$ which is the value of rare exposure probability.

From the simulation results we can summarize that M_J performs well for unbalanced sample sizes with moderate values of $\alpha=0.05$ when $\pi_1=0.10$. Both of these tests are in general not good in level control when $\alpha=0.01$. We conclude that the jackknifed CMH test M_J can be a good alternative to the CMH test M for sparse $2\times2\times K$ tables.

Table 3.1 Powers of Two Test Statistics

(a) $\pi_1=0.10$, $\alpha=0.10$

n	m	K	test	odds ratio 1.0	1.5	2.0	2.5	3.0
1	1	30	M	0.099	0.136	0.196	0.258	0.316
			M_J	0.075	0.128	0.196	0.271	0.328
		40	M	0.087	0.149	0.235	0.319	0.398
			M_J	0.058	0.115	0.190	0.268	0.351
		50	M	0.090	0.180	0.293	0.398	0.467
			M_J	0.061	0.135	0.235	0.323	0.396
2	1	30	M	0.102	0.135	0.207	0.291	0.366
			M_J	0.098	0.152	0.241	0.341	0.425
		40	M	0.127	0.154	0.249	0.345	0.435
			M_J	0.114	0.167	0.267	0.366	0.435
		50	M	0.112	0.151	0.293	0.418	0.536
			M_J	0.082	0.149	0.294	0.429	0.548
4	1	30	M	0.082	0.123	0.203	0.274	0.342
			M_J	0.070	0.152	0.267	0.353	0.437
		40	M	0.101	0.140	0.274	0.389	0.485
			M_J	0.082	0.166	0.316	0.432	0.531
		50	M	0.111	0.183	0.330	0.471	0.584
			M_J	0.083	0.196	0.354	0.501	0.614
8	1	30	M	0.096	0.147	0.240	0.330	0.415
			M_J	0.083	0.177	0.310	0.409	0.512
		40	M	0.086	0.161	0.293	0.423	0.515
			M_J	0.078	0.203	0.358	0.491	0.583
		50	M	0.097	0.183	0.357	0.520	0.623
			M_J	0.077	0.207	0.392	0.558	0.665
2	2	30	M	0.074	0.173	0.326	0.439	0.498
			M_J	0.065	0.156	0.282	0.405	0.456
		40	M	0.091	0.193	0.362	0.521	0.600
			M_J	0.066	0.163	0.319	0.471	0.554
		50	M	0.103	0.256	0.424	0.597	0.683
			M_J	0.077	0.215	0.379	0.541	0.638
5	5	30	M	0.094	0.324	0.613	0.764	0.878
			M_J	0.071	0.282	0.540	0.716	0.847
		40	M	0.085	0.376	0.705	0.873	0.945
			M_J	0.062	0.324	0.647	0.832	0.920
		50	M	0.122	0.426	0.773	0.927	0.972
			M_J	0.091	0.382	0.719	0.904	0.960

Table 3.1 Powers of Test Statistics

(b) $\pi_1 = 0.10$, $\alpha = 0.05$

n	m	K	test	odds ratio 1.0	1.5	2.0	2.5	3.0
1	1	30	M	0.044	0.073	0.111	0.165	0.192
			M_J	0.067	0.121	0.186	0.259	0.317
		40	M	0.037	0.068	0.116	0.181	0.247
			M_J	0.044	0.087	0.156	0.228	0.307
		50	M	0.034	0.081	0.168	0.248	0.314
			M_J	0.037	0.084	0.180	0.271	0.340
2	1	30	M	0.054	0.064	0.096	0.150	0.195
			M_J	0.060	0.112	0.186	0.275	0.356
		40	M	0.065	0.087	0.141	0.217	0.274
			M_J	0.062	0.112	0.203	0.293	0.378
		50	M	0.049	0.081	0.177	0.274	0.355
			M_J	0.044	0.095	0.217	0.334	0.438
4	1	30	M	0.040	0.048	0.089	0.136	0.193
			M_J	0.042	9,115	0.201	0.285	0.359
		40	M	0.050	0.058	0.151	0.244	0.319
			M_J	0.046	0.106	0.248	0.361	0.466
		50	M	0.043	0.095	0.202	0.318	0.432
			M_J	0.044	0.136	0.276	0.418	0.540
8	1	30	M	0.042	0.057	0.109	0.179	0.230
			M_J	0.054	0.149	0.267	0.360	0.453
		40	M	0.042	0.080	0.170	0.261	0.338
			M_J	0.045	0.149	0.280	0.410	0.499
		50	M	0.044	0.095	0.218	0.339	0.465
			M_J	0.036	0.140	0.317	0.469	0.583
2	2	30	M	0.040	0.104	0.215	0.319	0.373
			M_J	0.036	0.101	0.210	0.314	0.373
		40	M	0.046	0.118	0.250	0.394	0.477
			M_J	0.052	0.108	0.237	0.386	0.472
		50	M	0.048	0.157	0.305	0.473	0.573
			M_J	0.042	0.131	0.287	0.446	0.551
5	5	30	M	0.052	0.220	0.466	0.659	0.801
			M_J	0.043	0.194	0.435	0.627	0.784
		40	M	0.038	0.269	0.590	0.795	0.886
			M_J	0.033	0.244	0.559	0.753	0.864
		50	M	0.065	0.317	0.662	0.869	0.945
			M_J	0.051	0.272	0.633	0.838	0.925

Table 3.1 Powers of Test Statistics

(c) $\pi_1=0.10$, $\alpha=0.01$

n	m	K	test	odds ratio 1.0	1.5	2.0	2.5	3.0
1	1	30	M	0.009	0.014	0.022	0.037	0.040
			M_J	0.036	0.070	0.107	0.166	0.208
		40	M	0.004	0.013	0.027	0.039	0.059
			M_J	0.014	0.040	0.086	0.132	0.198
		50	M	0.005	0.017	0.040	0.073	0.106
			M_J	0.016	0.039	0.098	0.165	0.225
2	1	30	M	0.010	0.011	0.017	0.019	0.026
			M_J	0.028	0.072	0.138	0.205	0.263
		40	M	0.008	0.016	0.033	0.058	0.072
			M_J	0.022	0.065	0.119	0.183	0.247
		50	M	0.007	0.021	0.046	0.078	0.108
			M_J	0.013	0.059	0.130	0.221	0.293
4	1	30	M	0.005	0.002	0.006	0.007	0.010
			M_J	0.018	0.068	0.122	0.178	0.242
		40	M	0.007	0.007	0.013	0.038	0.054
			M_J	0.015	0.047	0.143	0.236	0.318
		50	M	0.005	0.008	0.037	0.077	0.121
			M_J	0.012	0.068	0.176	0.287	0.400
8	1	30	M	0.004	0.002	0.003	0.006	0.008
			M_J	0.020	0.078	0.148	0.230	0.301
		40	M	0.012	0.010	0.029	0.051	0.067
			M_J	0.017	0.085	0.182	0.284	0.370
		50	M	0.012	0.015	0.048	0.081	0.125
			M_J	0.009	0.087	0.199	0.312	0.443
2	2	30	M	0.009	0.025	0.063	0.113	0.161
			M_J	0.014	0.044	0.106	0.183	0.240
		40	M	0.011	0.039	0.091	0.171	0.244
			M_J	0.014	0.059	0.134	0.217	0.310
		50	M	0.017	0.039	0.130	0.222	0.300
			M_J	0.015	0.047	0.154	0.253	0.356
5	5	30	M	0.012	0.068	0.220	0.405	0.560
			M_J	0.021	0.081	0.252	0.433	0.587
		40	M	0.006	0.102	0.331	0.577	0.705
			M_J	0.005	0.099	0.357	0.577	0.723
		50	M	0.007	0.118	0.418	0.683	0.829
			M_J	0.011	0.124	0.418	0.680	0.813

Table 3.2 Powers of Test Statistics

(a) $\pi_1 = 0.30$, $\alpha = 0.10$

n	m	K	test	odds ratio 1.0	1.5	2.0	2.5	3.0
1	1	30	M	0.090	0.180	0.306	0.434	0.528
			M_J	0.065	0.145	0.250	0.368	0.466
		40	M	0.086	0.190	0.343	0.508	0.637
			M_J	0.064	0.165	0.305	0.460	0.589
		50	M	0.092	0.220	0.444	0.610	0.735
			M_J	0.080	0.194	0.410	0.578	0.695
2	1	30	M	0.108	0.208	0.389	0.537	0.640
			M_J	0.085	0.190	0.365	0.517	0.624
		40	M	0.101	0.226	0.448	0.628	0.755
			M_J	0.078	0.185	0.409	0.595	0.721
		50	M	0.102	0.290	0.544	0.730	0.856
			M_J	0.075	0.256	0.507	0.691	0.818
4	1	30	M	0.098	0.218	0.412	0.573	0.702
			M_J	0.079	0.198	0.391	0.557	0.677
		40	M	0.111	0.257	0.487	0.677	0.812
			M_J	0.078	0.229	0.451	0.656	0.792
		50	M	0.108	0.279	0.587	0.779	0.901
			M_J	0.084	0.245	0.540	0.747	0.882
8	1	30	M	0.090	0.230	0.443	0.625	0.740
			M_J	0.070	0.212	0.423	0.613	0.731
		40	M	0.106	0.272	0.534	0.730	0.857
			M_J	0.074	0.260	0.507	0.702	0.846
		50	M	0.107	0.323	0.634	0.817	0.910
			M_J	0.077	0.286	0.595	0.793	0.893
2	2	30	M	0.118	0.270	0.494	0.683	0.812
			M_J	0.085	0.233	0.440	0.634	0.774
		40	M	0.100	0.326	0.607	0.803	0.897
			M_J	0.074	0.275	0.550	0.752	0.868
		50	M	0.102	0.368	0.697	0.870	0.947
			M_J	0.082	0.310	0.645	0.842	0.921
5	5	30	M	0.098	0.492	0.844	0.966	0.993
			M_J	0.076	0.437	0.791	0.956	0.989
		40	M	0.094	0.568	0.929	0.991	1.000
			M_J	0.063	0.507	0.901	0.986	0.999
		50	M	0.100	0.668	0.960	0.995	1.000
			M_J	0.071	0.610	0.941	0.993	0.999

Table 3.2 Powers of Test Statistics

(b) $\pi_1=0.30$, $\alpha=0.05$

n	m	K	test	odds ratio 1.0	1.5	2.0	2.5	3.0
1	1	30	M	0.038	0.099	0.194	0.270	0.376
			M_J	0.039	0.102	0.198	0.273	0.383
		40	M	0.050	0.117	0.237	0.383	0.501
			M_J	0.043	0.108	0.232	0.376	0.494
		50	M	0.055	0.151	0.340	0.506	0.629
			M_J	0.048	0.130	0.310	0.480	0.605
2	1	30	M	0.057	0.123	0.264	0.404	0.518
			M_J	0.053	0.133	0.285	0.422	0.537
		40	M	0.051	0.142	0.310	0.491	0.625
			M_J	0.046	0.136	0.308	0.490	0.637
		50	M	0.048	0.206	0.423	0.606	0.737
			M_J	0.035	0.193	0.405	0.596	0.735
4	1	30	M	0.058	0.134	0.273	0.425	0.543
			M_J	0.045	0.140	0.294	0.448	0.583
		40	M	0.061	0.161	0.342	0.551	0.709
			M_J	0.051	0.158	0.357	0.573	0.730
		50	M	0.051	0.180	0.428	0.654	0.812
			M_J	0.039	0.181	0.423	0.654	0.817
8	1	30	M	0.044	0.145	0.300	0.483	0.612
			M_J	0.039	0.154	0.328	0.525	0.650
		40	M	0.044	0.179	0.410	0.595	0.749
			M_J	0.029	0.194	0.428	0.612	0.769
		50	M	0.055	0.207	0.485	0.724	0.851
			M_J	0.044	0.212	0.494	0.722	0.849
2	2	30	M	0.057	0.175	0.377	0.569	0.723
			M_J	0.044	0.154	0.349	0.526	0.689
		40	M	0.047	0.224	0.489	0.700	0.813
			M_J	0.033	0.193	0.443	0.667	0.787
		50	M	0.061	0.265	0.584	0.803	0.895
			M_J	0.050	0.229	0.525	0.759	0.875
5	5	30	M	0.047	0.367	0.748	0.929	0.984
			M_J	0.043	0.327	0.705	0.911	0.972
		40	M	0.040	0.442	0.873	0.978	0.999
			M_J	0.029	0.400	0.838	0.969	0.993
		50	M	0.051	0.560	0.929	0.988	0.999
			M_J	0.038	0.513	0.908	0.983	0.999

Table 3.2 Powers of Test Statistics

(c) $\pi_1=0.30$, $\alpha=0.01$

n	m	K	test	odds ratio 1.0	1.5	2.0	2.5	3.0
1	1	30	M	0.009	0.018	0.054	0.106	0.153
			M_J	0.016	0.036	0.097	0.159	0.235
		40	M	0.012	0.035	0.082	0.150	0.238
			M_J	0.013	0.046	0.109	0.199	0.306
		50	M	0.009	0.036	0.123	0.228	0.350
			M_J	0.010	0.049	0.159	0.278	0.407
2	1	30	M	0.008	0.036	0.094	0.151	0.239
			M_J	0.016	0.069	0.156	0.257	0.365
		40	M	0.009	0.049	0.128	0.219	0.328
			M_J	0.010	0.068	0.169	0.288	0.433
		50	M	0.011	0.058	0.186	0.336	0.480
			M_J	0.011	0.076	0.231	0.404	0.558
4	1	30	M	0.010	0.025	0.091	0.180	0.281
			M_J	0.009	0.069	0.177	0.300	0.419
		40	M	0.015	0.052	0.145	0.259	0.409
			M_J	0.021	0.078	0.213	0.367	0.541
		50	M	0.014	0.064	0.225	0.381	0.546
			M_J	0.014	0.085	0.277	0.463	0.639
8	1	30	M	0.010	0.040	0.109	0.210	0.314
			M_J	0.014	0.085	0.186	0.345	0.484
		40	M	0.008	0.052	0.170	0.326	0.467
			M_J	0.008	0.087	0.277	0.446	0.596
		50	M	0.104	0.069	0.228	0.453	0.624
			M_J	0.017	0.099	0.317	0.554	0.720
2	2	30	M	0.011	0.057	0.174	0.323	0.455
			M_J	0.012	0.070	0.187	0.343	0.494
		40	M	0.006	0.087	0.236	0.460	0.623
			M_J	0.006	0.089	0.254	0.460	0.628
		50	M	0.011	0.099	0.333	0.568	0.729
			M_J	0.013	0.098	0.336	0.561	0.725
5	5	30	M	0.010	0.171	0.527	0.804	0.933
			M_J	0.014	0.178	0.506	0.800	0.903
		40	M	0.005	0.251	0.685	0.923	0.975
			M_J	0.007	0.227	0.658	0.905	0.971
		50	M	0.011	0.314	0.802	0.958	0.995
			M_J	0.009	0.296	0.775	0.955	0.994

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