Robust H_∞ FIR Sampled-Data Filtering for Uncertain Time-Varying Systems with Unknown Nonlinearity

Hee-Seob Ryu, Byung-Moon Kwon, and Oh-Kyu Kwon

Abstract: The robust linear H_∞ FIR filter, which guarantees a prescribed H_∞ performance, is designed for continuous time-varying systems with unknown cone-bounded nonlinearity. The infinite horizon filtering for time-varying systems is investigated in terms of two Riccati equations by the finite moving horizon.

Keywords: Robust H_∞ FIR filter, cone-bounded nonlinearity, time-varying system, sampled-data systems

I. Introduction

Recently, the emergence of many new analysis tools that deal with the so-called H_∞ filtering has been witnessed [10]. The H_∞ filtering problem is concerned with designing estimators to minimize the H_∞ norm of the transfer function from the noise sources to the estimation error. However, the conventional H_∞ filters proposed are mainly limited to time-invariant systems. Therefore they can not be applied to general time-varying systems on the infinite horizon since one of two Riccati differential equations required to solve the problem can not be computed on the infinite horizon [6].

In practical systems, it is mainly continuous-time processes but output signal is measured by digital devices. The classical method of analyzing these systems is to develop a discrete-time method, based on the sampling frequency of the measurements. Digital filtering, smoothing and predicting devices built in this way tend to fail when the sampling frequency is too low and the system dynamics are relatively too fast because the inter-sampling behavior of the system may be overlooked. So, in the filtering problems for the continuous-time system, one is required to produce a continuous-time estimate of an analogue signal based on sampled-data measurements. In this situation, the filtering performance measure should be defined directly in terms of the continuous-time signals. We refer to this filtering approach as 'sampled-data filtering'.

This paper deals with the issue of the robust linear H_∞ filtering problem for uncertain nonlinear time-varying systems on the infinite horizon. The basic idea of the current paper is to formulate the robust linear H_∞ filtering problem on the moving horizon and to adopt the FIR (Finite Impulse Response) filter structure. The estimator of the current paper is rather a one step ahead predictor than a filter.

FIR filters are widely used in the signal processing area, and they were utilized in the estimation problem as the optimal FIR filters. Since the optimal FIR filters use the finite observations only over a finite preceding time interval, they can overcome the divergence problem and have the built-in BIBO (Bounded Input/Bounded Output) stability and the robustness to the numerical problems such as coefficient quantization errors and roundoff errors, which are well known properties of the FIR structure in signal processing area. Also note that IIR (Infinite Impulse Response) or recursive filter structure (e.g. Kalman filter) requires the initial conditions on each horizon, which is an impractical assumption, but that FIR filter structure does not requires the initial conditions. The optimal FIR filters are, however, presented so far not in the H_∞ setting but in the minimum variance formulation.

The linear H_∞ filter proposed is referred to as *robust linear H_∞ FIR sampled-data filtering* in the sense that it is a linear H_∞ filter with the FIR structure for uncertain systems.

II. Problem formulation and preliminaries

Consider the following class of nonlinear uncertain sampled-data time-varying systems:

$$\begin{aligned} \dot{x}(t) &= [A(t) + \triangle A(t)]x(t) + [G(t) + \triangle G(t)] \\ &\quad \cdot g[x(t)] + B(t)w(t),\ x(0) = x_0 \qquad (1) \\ z(t) &= L(t)x(t) \qquad (2) \\ z_d(i) &= L_d(i)x(i) \qquad (3) \\ y(i) &= [C(i) + \triangle C(i)]x(i) + [K(i) + \triangle K(i)] \\ &\quad \cdot k[x(i)] + D(i)v(i), \qquad (4) \end{aligned}$$

where $x(t) \in \Re^n$ is the state, x_0 is unknown initial state, $w(t) \in \Re^q$ is the process noise which belongs to $L_2[0,\infty)$, $y(i) \in \Re^m$ is the sampled measurement, $v(i) \in \Re^r$ is the measurement noise which belongs to $l_2(0,\infty)$, $z(t) \in \Re^p$ and $z(i) \in \Re^s$ are linear combinations of state variables to be estimated, i is an integer, $A(t)$, $B(t)$, $C(i)$, $D(i)$, $G(t)$, $K(i)$, $L(t)$ and $L_d(i)$ are known real time-varying bounded matrices of appropriate dimensions with $A(t)$, $B(t)$, $G(t)$ and $L(t)$ being piecewise continuous, and $\triangle A(t)$, $\triangle C(i)$, $\triangle G(t)$ and $\triangle K(i)$ represent real time-varying parameter uncertainties in A, C, G and K respectively and the mapping $g(\cdot) : \Re^n \to \Re^{n_g}$ and $k(\cdot) : \Re^m \to \Re^{m_k}$ are unknown nonlinearities. These admissible uncertainties are assumed to be of the form

$$\triangle A(t) = HF(t)E, \quad \triangle G(t) = H_G F_G(t) E_G \qquad (5)$$

$$\triangle C(i) = H_d F_d(i) E_d, \quad \triangle K(i) = H_K F_K(i) E_K, \qquad (6)$$

where $F(t) \in \Re^{i_a \times j_a}$, $F_d(i) \in \Re^{i_d \times i_d}$, $F_G(t) \in \Re^{i_G \times i_G}$ and $F_K(i) \in \Re^{i_K \times i_K}$ are unknown time-varying matrices satisfy-

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Manuscript received: Aug. 7, 2000, Accepted: Mar. 28, 2001.

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ing

$$F^T(t)F(t) \le I, \quad F_G^T(t)F_G(t) \le I, \; \forall\, t \qquad (7)$$

$$F_d^T(i)F_d(i) \le I, \quad F_K^T(i)F_K(i) \le I, \; \forall\, i \qquad (8)$$

with the elements of F and F_G being Lebesgue measureable, and E, E_d, E_G, E_K, H, H_d, H_G and H_K are known real constant bounded matrices of appropriate dimensions with E, E_G, H and H_G being piecewise continuous. The matrices $A(t)$, $B(t)$, $C(i)$, $D(i)$, $G(t)$, $K(i)$, $L(t)$ and $L_d(i)$ describe the nominal model of system (1)-(4). For the sake of notation simplification, in the sequel the dependence on t or i for all matrices will be omitted.

Note that nonlinear models of the form (1)-(4) can be used to represent many important physical systems. A typical example is a power system modelled in the form of a single machine-infinite bus [1]. The parameter uncertainty structure as in (5)-(8) has been widely used in the problems of robust control and robust filtering of uncertain systems [4],[13],[14] and many practical system possess parameter uncertainties which can be either exactly modelled, or overbounded by (5)-(6).

The admissible known nonlinearity functions $g(\cdot)$ and $k(\cdot)$ are assumed to satisfy the following assumptions.

Assumption 1: a) There exist known constant matrices W_g and W_k such that for all $x \in \Re^n$,

$$\|g(x))\| \le \|W_g x\|, \quad \|k(x)\| \le \|W_k x\|;$$

b) $[D(i) \quad H_d(i) \quad K(i) \quad H_k(i)]$ is of full the row rank for all $i \in (0, T)$.

Assumption 1b) means that the robust filtering problem is 'non-singular'. We observe that when there is no parameter uncertainty in the output matrix of system (1)-(4), Assumption 1b) reduces to $D(i)D^T(i) \ge 0$, which corresponds to a standard nonsingularity condition in the H_∞ filtering problem for the nominal system (1)-(4).

In the current paper, the FIR filter is defined by the form

$$\hat{x}(i+1 \mid i; T) = \sum_{i-T}^{i} M(i,k;T)y(k)$$
$$\hat{z}_d(i+1 \mid i;T) = L(i+1)\hat{x}(i+1 \mid i;T),$$

where $M(t, \cdot; T)$ is the finite impulse response with the finite duration T. The estimation error is defined by

$$e_d(i+1) = z_d(i+1) - \hat{z}_d(i+1 \mid i;T).$$

The H_∞ FIR filter is obtained by constructing its impulse response from that of the H_∞ filter on the finite moving horizon $[t-T, t]$. Then, the robust H_∞ FIR filtering problem we address is as follows:

Given a prescribed level of noise attenuation $\gamma > 0$ and an initial state weighting matrix $R = R^T > 0$, find a linear or linear causal filter $\mathcal{F}$ such that the estimation error dynamics, $z(t) - \hat{z}(t)$, is exponentially stable and satisfies H_∞ performance

$$\{\|z - \hat{z}\|^2 + \|z_d - \hat{z}_d\|^2\} < \gamma^2\{\|w\|^2_{[t-T,t]} + \|v\|^2_{(i-T,i)} + x_0^T R x_0\} \qquad (9)$$

holds for all admissible uncertainties and for any non-zero $(w, v, x_0) \in L_2[0,\infty) \oplus l_2(0,\infty) \oplus \Re^n$, where $x_0 = x(t-T)$ and $R = cov[x(t-T)]$.

Here, $\|\cdot\|_{[t-T,t]}$, $\|\cdot\|_{(t-T,t)}$ and $\|e\|^2$ will mean the L_2 norm over $[t-T,t]$, the l_2 norm over $(t-T,t)$ and e^Te, respectively. It is noted that the problem does not need the assumption of stabilizability or detectability of the system since it is formulated on the finite moving horizon. In the sequel, the bounded real lemma for linear time-varying systems with finite discrete jumps which will be used throughout the paper, is reviewed.

Consider the following linear time-varying system with finite discrete jumps:

$$(\Sigma_1): \dot{x}(t) = Ax(t) + Bw(t),\; t \ne i,\; x(0) = x_0 \qquad (10)$$
$$x(i) = A_d x(i^-) + B_d v(i),\; \forall\, i \in (0,T) \qquad (11)$$
$$z(t) = Cx(t) \qquad (12)$$
$$z_d(i) = C_d x(i^-), \qquad (13)$$

where $x \in \Re^n$, $w \in \Re^q$ and $v \in \Re^r$ belongs to $L_2[0,T]$ and $l_2(0,T)$, respectively, $z \in \Re^p$, $z_d \in \Re^s$, and A, A_d, B, B_d and C are known real time-varying bounded matrices with A, B and C being piecewise continuous. Next, introduce the following worst-case performance index for (Σ_1):

$$J(\Sigma_1) = \sup\left[\frac{\|z\|^2 + \|z_d\|^2}{\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0}\right]^{1/2}, \qquad (14)$$

where $R = R^T > 0$ is given weighting matrix for x_0 and the supremum is taken over all $(w, v, x_0) \in L_2[0,T] \oplus l_2(0,T) \oplus \Re^n$ such that $\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0 \ne 0$.

We now present a version of the bounded real lemma on finite horizon for interested filtering problem formulation of the system (Σ_1).

Lemma 1: [11]: Consider the system (Σ_1) and let $\gamma > 0$ be a given scalar. Then, the following statements are equivalent:

a) $J(\Sigma_1, R, T) < \gamma$;

b) There exists a bounded matrix function $P(t) = P^T(t) \ge 0$, $\forall\, t \in [0,T]$, such that

$$-\dot{P} = A^TP + PA + \gamma^{-2}PBB^TP + C^TC,\; t \ne i,$$
$$P(T) = 0 \qquad (15)$$
$$\gamma^2 I - B_d^T P(i^+) B_d > 0 \qquad (16)$$
$$P(i) = A_d^T P(i^+) A_d + A_d^T P(i^+) B_d [\gamma^2 I - B_d^T \cdot P(i^+) B_d]^{-1} B_d^T P(i^+) A_d + C_d^T C_d \qquad (17)$$
$$P(0^+) < \gamma^2 R; \qquad (18)$$

c)There exists a bounded matrix function $Q(t) = Q^T(t) > 0$, $\forall\, t \in [0,T]$, such that

$$-\dot{Q} > A^TQ + QA + \gamma^{-2}QBB^TQ + C^TC,\; t \ne i,$$
$$Q(T) > 0 \qquad (19)$$
$$\gamma^2 I - B_d^T Q(i^+) B_d > 0 \qquad (20)$$
$$Q(i) > A_d^T Q(i^+) A_d + A_d^T Q(i^+) B_d [\gamma^2 I - B_d^T \cdot Q(i^+) B_d]^{-1} B_d^T Q(i^+) A_d + C_d^T C_d \qquad (21)$$
$$Q(0^+) < \gamma^2 R. \qquad (22)$$

Lemma 2: [13]: Let A, E, F, H and M be real matrices of appropriate dimensions with M being symmetric. Then,

a) For any scalar $\epsilon > 0$ and for all matrices F satisfying $F^TF \le I$,

$$HFE + E^TF^TH^T \le \frac{1}{\epsilon}HH^T + \epsilon E^TE;$$

b) There exists a matrix $P = P^T > 0$ such that

$$[A + HFE]^T P[A + HFE] + M < 0$$

for all matrices F satisfying $F^T F \leq I$, if there exists some $\epsilon > 0$ such that the following conditions are satisfied

i) $\epsilon^{1/2} H^T P H < I$

ii) $A^T P A + A^T P H[\epsilon I - H^T P H]^{-1} H^T P A + \epsilon E^T E + M < 0$;

c) For any scalar $\epsilon > 0$ such that $\epsilon^2 E^T E \leq I$ and for all matrices F satisfying $F^T F \leq I$,

$$[A + HFE][A + HFE]^T \leq A(I - \epsilon E^T E)^{-1} A^T + \frac{1}{\epsilon} H H^T.$$

III. Robust H_∞ FIR sampled-data filters with unknown nonlinearity

In this section, the robust H_∞ FIR filtering problem for the system (1)-(4) is considered.

Theorem 1: Consider the system (1)-(4) satisfying Assumption 1. Given a scalar $\gamma > 0$ and an initial state weighting matrix $R = R^T > 0$, the robust H_∞ FIR sampled-data filtering problem over a moving horizon $[0, T]$ is solvable if there exist positive scalars ϵ_1, ϵ_2, ϵ_3 and ϵ_4 such that $\epsilon_2^2 E_G^T E_G < I$, $\epsilon_4^2 E_K^T E_K < I$ and the following conditions are satisfied:

a) There exists a bounded solution $P(t) = P^T(t) \geq 0$ over the moving horizon $[0, T]$ to the Riccati differential equation with jumps

$$\dot{P}(t) + A^T P(t) + P(t)A + \gamma^{-2} P(t)[\hat{B}\hat{B}^T + BB^T]P(t) + \epsilon_1^2 E^T E + W_g^T W_g = 0, \quad t \neq i \tag{23}$$

$$P(i) = P(i^+) + \epsilon_3^2 E_d^T E_d + W_k^T W_k \tag{24}$$

with terminal condition $P(T) = 0$ and such that $P(0^+) < \gamma^2 R$, where

$$\hat{B} = \left[\tfrac{\gamma}{\epsilon_1} H \quad \gamma G(I - \epsilon_2^2 E_G^T E_G)^{-1/2} \quad \tfrac{\gamma}{\epsilon_2} H_G \right]. \tag{25}$$

b) There exists a bounded solution $S(t)$ over the moving horizon $[0, T]$ to the Riccati differential equation with jumps

$$\dot{S}(t) = \hat{A}S(t) + S(t)\hat{A}^T + \gamma^{-2} S(t) L^T L S(t) + \hat{B}\hat{B}^T + BB^T, \tag{26}$$

$$S(i) = [S^{-1}(i^-) - \gamma^{-2} L_d^T L_d + C^T V^{-1} C]^{-1} \tag{27}$$

with initial condition $S(0) = [R - \gamma^{-2} P(0)]^{-1}$, where

$$\hat{A}(t) = A + (\gamma^{-2} BB^T + \epsilon_1^{-2} HH^T)P(t) \tag{28}$$

$$V = \hat{D}\hat{D}^T$$

$$\hat{D} = \left[D \quad \tfrac{\gamma}{\epsilon_3} H_d \quad \gamma K(I - \epsilon_4^2 E_K^T E_K)^{-1/2} \quad \tfrac{\gamma}{\epsilon_4} H_K \right]. \tag{29}$$

Moreover, if conditions a) and b) are satisfied, a suitable filter is given by

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) \tag{30}$$

$$\hat{x}(i) = \hat{x}(i^-) + S(i) C^T V^{-1}[y(i) - C\hat{x}(i^-)] \tag{31}$$

$$\hat{z}(t) = L\hat{x}(t) \tag{32}$$

$$\hat{z}_d = L_d \hat{x}(i). \tag{33}$$

Proof: First, associated with (1)-(4) and (30)-(33), we define $\tilde{x} \equiv x - \hat{x}$. Since $x(i) = x(i^-)$, from (1)-(4) and (30)-(33), we have that

$$\begin{aligned}\dot{\tilde{x}}(t) &= [A + \triangle A_e]\tilde{x}(t) + [\triangle A - \triangle A_e]x(t) \\ &\quad + Bw(t) + [G + \triangle G]g(x) - Gg(\hat{x}) \\ \tilde{x}(i) &= A_d \tilde{x}(i^-) + B_d \triangle Cx(i^-) + B_d K(k[x(i^-)] \\ &\quad - k[\hat{x}(i^-)]) + B_d \triangle Kk[x(i^-)] + B_d Dv(i),\end{aligned}$$

where

$$A_d = I - S(i) C^T V^{-1} C, B_d = -S(i) C^T V^{-1},$$
$$\triangle A_e(t) = (\gamma^{-2} BB^T + \epsilon_1^{-2} HH^T)P(t).$$

Hence, we have the following estimation error dynamics for the estimator error $z - \hat{z}$ and $z_d - \hat{z}_d$ is as follows

$$\dot{\eta}(t) = [A_e + H_e F(t) E_e]\eta(t) + B_e w(t) + [G_e + H_{ge} F_G E_{ge}] g_e[\eta(t)] \tag{34}$$

$$\eta(i) = [A_{de} + H_{de} F_d E_{de}]\eta(i^-) + B_{de} v(i) + [K_e + H_{ke} F_K E_{ke}] k_e[\eta(i^-)] \tag{35}$$

$$z(t) - \hat{z}(t) = L_e \eta(t) \tag{36}$$

$$z_d(i) - \hat{z}_d(i) = L_{de} \eta(i), \tag{37}$$

where $\eta = [x^T \quad \tilde{x}^T]^T$ and

$$A_e = \begin{bmatrix} A & 0 \\ -\triangle A_e & A + \triangle A_e \end{bmatrix}, A_{de} = \begin{bmatrix} I & 0 \\ 0 & A_d \end{bmatrix},$$

$$B_e = \begin{bmatrix} B \\ B \end{bmatrix}, B_{de} = \begin{bmatrix} 0 \\ B_d D \end{bmatrix}, H_{ke} = \begin{bmatrix} 0 \\ B_d H_K \end{bmatrix},$$

$$H_{de} = \begin{bmatrix} 0 \\ B_d H_d \end{bmatrix}, H_{ge} = \begin{bmatrix} H_G \\ H_G \end{bmatrix}, H_e = \begin{bmatrix} H \\ H \end{bmatrix},$$

$$E_e = [E \quad 0], \; E_{de} = [E_d \quad 0],$$

$$E_{ge} = E_G, E_{ke} = E_K,$$

$$L_e = [0 \quad L], \; L_{de} = [0 \quad L_d],$$

$$G_e = \begin{bmatrix} G \\ 0 \end{bmatrix}, K_e = \begin{bmatrix} 0 \\ B_d K \end{bmatrix},$$

$$g_e[\eta(t)] = g(x), k_e[\eta(i^-)] = k[x(i^-)].$$

Note that by Assumption 1,

$$\begin{aligned}\|g_e(\eta)\| &\leq \|W_g \eta\|, \forall \eta \in \Re^{2n} \\ \|k_e(\eta)\| &\leq \|\hat{W}_k \eta\|, \forall \eta \in \Re^{2n},\end{aligned} \tag{38}$$

where

$$\hat{W}_e = [\; W_g \quad 0 \;], \hat{W}_{de} = [\; W_k \quad 0 \;]. \tag{39}$$

From Theorem 3.1 in [5], condition b) is necessary and sufficient for the solvability of the moving horizon H_∞ FIR filtering problem for the linear system with sampled measurements

$$\dot{\xi}(t) = \hat{A}\xi(t) + [\hat{B} \quad B]\bar{w}(t) \tag{40}$$

$$\bar{y}(i) = C\xi(i) + \hat{D}\bar{v}(i) \tag{41}$$

$$\bar{z}(t) = L\xi(t) \tag{42}$$

$$\bar{z}_d(i) = L_d \xi(i), \tag{43}$$

where $\xi \in \Re^n$ is the state, ξ_0 is an unknown initial state, $\bar{w} \in \Re^{p+i+n_G}$ is the process noise, $\bar{y}(i) \in \Re^m$ is the sampled measurement, $\bar{v}(i) \in \Re^{q+\alpha+m_K}$ is the measurement noise,

$\bar{z} \in \Re^r$ and $\bar{z}_d \in \Re^s$ are linear combinations of the state variables to be estimated, and the filtering performance measure is given by

$$\sup\left\{\left[\frac{\|\bar{z}-\hat{\bar{z}}_e\|^2+\|\bar{z}_d-\hat{\bar{z}}_d\|^2}{\|\bar{w}\|^2_{[0,T]}+\|\bar{v}\|^2_{(0,T)}+\xi_0^T[R-\gamma^{-2}P(0)]\xi_0}\right]^{1/2}\right\} \quad (44)$$

where $\hat{\bar{z}}$ and $\hat{\bar{z}}_d$ are the estimates of $\bar{z}$ and $\bar{z}_d$, respectively. In the above, the supremum is taken over all $(\bar{w},\bar{v},\xi_0) \in L_2[0,\infty)\oplus l_2(0,\infty)\oplus\Re^n$ such that $\|\bar{w}\|^2_{[0,T]}+\|\bar{v}\|^2_{(0,T)}+\xi_0^T[R-\gamma^{-2}P(0)]\xi_0 \neq 0$. Also, observe that suitable estimates $\hat{\bar{z}}$ and $\hat{\bar{z}}_d$ are given by

$$\begin{aligned}(\Sigma_{e1}):\dot{\xi}_e(t) &= \hat{A}\xi_e(t), \quad t\neq i; \quad \xi_e(0)=0\\ \xi_e(i) &= \xi_e(i^-)+S(i)C^TV^{-1}[\bar{y}(i)-C\xi_e(i^-)]\\ \hat{\bar{z}}(t) &= L\xi_e(t)\\ \hat{\bar{z}}_d(i) &= L_d\bar{\xi}_e(i).\end{aligned}$$

Now, letting $\tilde{\xi}=\xi-\xi_e$, it follows from the system (Σ_{e1}) and (40)-(43) that

$$\begin{aligned}\dot{\tilde{\xi}}(t) &= \hat{A}\tilde{\xi}(t)+[\hat{B}\quad B]\bar{w}(t), \quad \tilde{\xi}(0)=\xi_0\\ \tilde{\xi}(i) &= A_d\tilde{\xi}(i^-)+B_d\hat{D}\bar{v}(i)\\ \bar{z}(t)-\hat{\bar{z}}(t) &= L\tilde{\xi}(t)\\ \bar{z}_d(i)-\hat{\bar{z}}_d(i) &= L_d\tilde{\xi}(i).\end{aligned}$$

Since the above system satisfies (44) by Lemma 1, this implies that there exists a bounded matrix $Z(t)=Z^T(t)\geq 0$, satisfying the following Riccati differential equation with jumps

$$\dot{Z}(t)+\hat{A}^TZ(t)+Z(t)\hat{A}+\gamma^{-2}Z(t)[\tilde{B}\tilde{B}^T+BB^T]\cdot Z(t)+L^TL=0, \quad t\neq i; \quad Z(T)=0 \quad (45)$$

$$\gamma^2I-\hat{D}^TB_d^TZ(i^+)B_d\hat{D}>0, \quad \forall i\in(0,T) \quad (46)$$

$$Z(i)=A_d^TZ(i^+)A_d+A_d^TZ(i^+)B_d\hat{D}[\gamma^2I-\hat{D}^T\cdot B_d^TZ(i^+)B_d\hat{D}]^{-1}\hat{D}^TB_d^TZ(i^+)A_d+L_d^TL_d \quad (47)$$

$$Z(0)<\gamma^2R-P(0). \quad (48)$$

Next, let

$$X(t)=\begin{bmatrix}P(t) & 0\\ 0 & Z(t)\end{bmatrix},$$

where $P(t)$ and $Z(t)$ are the non-negative definite solution of (23) and (24) and (45)-(48), respectively. Note that since $Z(0)<\gamma^2R-P(0)$, there exists a sufficiently small scalar $\delta>0$ such that

$$X(0)<X_0=\begin{bmatrix}P(0^+)+\delta I & 0\\ 0 & \gamma^2R-P(0^+)-\delta I\end{bmatrix}.$$

It is straightforward to verify that there exists a matrix $X(t)=X^T(t)\geq 0, \forall t\in[0,T]$ satisfying the following Riccati differential equation with jumps

$$\dot{X}(t)+A_e^TX(t)+X(t)A_e+X(t)\hat{B}_e\hat{B}_e^TX(t)+\hat{C}_e^TC_e=0, \quad t\neq i; \quad X(T)=0 \quad (49)$$

$$I-\hat{B}_{de}^TX(i^+)\hat{B}_{de}>0, \quad i\in(0,T) \quad (50)$$

$$X(i)=A_{de}^TX(i^+)A_{de}+A_{de}^TX(i^+)\hat{B}_{de}[I-\hat{B}_{de}^T\cdot X(i^+)\hat{B}_{de}]^{-1}\hat{B}_{de}^TX(i^+)A_{de}+\hat{C}_{de}^T\hat{C}_{de}, \quad (51)$$

$$X(0)<X_0, \quad (52)$$

where

$$\hat{B}_e=[\bar{B}_e\quad\gamma^{-1}B_e],\quad \hat{B}_{de}=[\bar{B}_{de}\quad\gamma^{-1}B_{de}],$$
$$\hat{C}_e=[\bar{C}_e\quad L_e]^T,\quad \hat{C}_{de}=[\bar{C}_{de}\quad L_{de}]^T,$$

and δ being a positive number with $\bar{B}_e$, $\bar{B}_{de}$, $\bar{C}_e$ and $\bar{C}_{de}$ such that

$$\bar{B}_e\bar{B}_e^T = G_e(I-\epsilon_2^2E_{ge}^TE_{ge})^{-1}G_e^T+\frac{1}{\epsilon_1^2}H_eH_e^T+\frac{1}{\epsilon_2^2}H_{ge}H_{ge}^T \quad (53)$$

$$\bar{B}_{de}\bar{B}_{de}^T = K_e(I-\epsilon_4^2E_{ke}^TE_{ke})^{-1}K_e^T+\frac{1}{\epsilon_3^2}H_{de}H_{de}^T+\frac{1}{\epsilon_4^2}H_{ke}H_{ke}^T \quad (54)$$

$$\bar{C}_e^T\bar{C}_e = \epsilon_1^2E_e^TE_e+\hat{W}_g^T\hat{W}_g \quad (55)$$

$$\bar{C}_{de}^T\bar{C}_{de} = \epsilon_3^2E_{de}^TE_{de}+\hat{W}_k^T\hat{W}_k, \quad (56)$$

where $\hat{W}_g$ and $\hat{W}_k$ are as in (39).

By Lemma 1, (49)-(52) implies that the system as below

$$\begin{aligned}(\Sigma_3):\dot{\xi}(t) &= A_e\xi(t)+\hat{B}_e\hat{w}(t) &(57)\\ \xi(i) &= A_{de}\xi(i^-)+\hat{B}_{de}\hat{v}(i) &(58)\\ z_e(t) &= \hat{C}_e\xi(t) &(59)\\ z_e(i) &= \hat{C}_{de}\xi(i) &(60)\end{aligned}$$

satisfies $J(\Sigma_3,\gamma^2,\hat{X}_0,T)<I$, where $\hat{X}_0=\gamma^{-2}X_0$. Finally, by considering the system (34)-(37) and (57)-(60), and the fact that the initial state of (34) satisfies $\eta^T(0)\hat{X}_0\eta(0)=x_0^TRx_0$, we conclude that the estimation error dynamics (34)-(37) satisfy

$$\{\|z-\hat{z}\|^2+\|z_d-\hat{z}_d\|^2\} < \gamma^2\{\|w\|^2_{[0,T]}+\|v\|^2_{(0,T)}+x_0^TRx_0\} \quad (61)$$

for all non-zero $(w,v,x_0)\in L_2[0,\infty)\oplus l_2(0,\infty)\oplus\Re^n$ and for all admissible uncertainties. ■

IV. Examples

To demonstrate the use of the above theory we consider the robust H_∞ FIR filter with unknown cone-bounded nonlinearity for a simple second-order problem. We show the advantage of the proposed technique by comparing its results with the corresponding results of the H_∞ nonlinear estimator of Shaked and Berman [10] and the extended Kalman filter(EKF), that has been widely used in the past in estimation of nonlinear systems.

Consider the time-invariant process with a saturating nonlinearity in the system dynamics

$$\begin{bmatrix}x_{1_{i+1}}\\ x_{2_{i+1}}\end{bmatrix}=\begin{bmatrix}\mu x_{1_i}\\ \arctan(\eta x_{1_i}+\lambda x_{2_i})\end{bmatrix}+H_1F(x_i)E(x_i)+B_kw_i, \quad (62)$$

where $\mu=0.91$, $\eta=-0.07$ and $\lambda=0.1$,

$$B_k=\begin{bmatrix}0.5\\ 1.5\end{bmatrix},\quad H_1=\begin{bmatrix}0.4\\ 0.4\end{bmatrix},$$

$E(x_i)=[0\ \ 1]\ x_i$ and $|\ F(x_i)\ |\leq I$, $\forall i\in[0,N]$ and $\forall x_i\in\Re^2$.

We assume here that the discretized measurement is described by

$$y_i=\cos(2x_{2_i})+3x_{2_i}+H_2F(x_i)E(x_i)+0.01w_i, \quad (63)$$

where $H_2 = 2$. We consider the time interval [0, N], where $N = 500$, and we are looking for an estimate of Lx_i, where $L = [1 \ \ 0]$. We also assumed that $\{w_i\}$ are uncorrelated standard gaussian white noise processes.

We have simulated the above three estimators for the worst values of the uncertainty F, namely for each estimator we describe the estimators. Figure 2, 3 and 4 show the estimation error, where $F = 1$ for the robust H_∞ FIR filter, the robust nonlinear H_∞ filter and the Extended Kalman filter and γ for the robust H_∞ FIR filter and robust nonlinear H_∞ filter are 4.326 and 1.419, respectively. Note that estimation error covariances of the proposed H_∞ FIR filter, the robust nonlinear H_∞ filter and the extended Kalman filter are 1.5478e-003, 1.7256e-003 and 3.261e-003. These result exemplify that the estimation performance of the robust H_∞ FIR filter is better than those obtained by the robust nonlinear H_∞ filter and by the Extended Kalman filter.

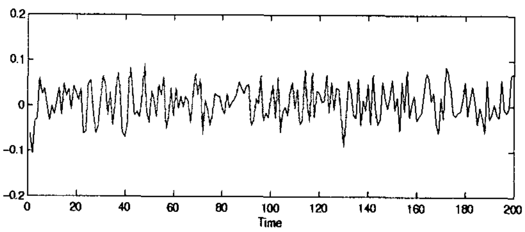

Fig. 1. Estimation error of the robust H_∞ FIR filter.

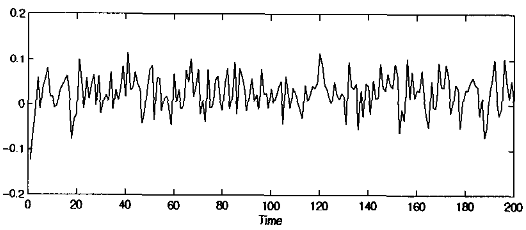

Fig. 2. Estimation error of the robust nonlinear H_∞ filter.

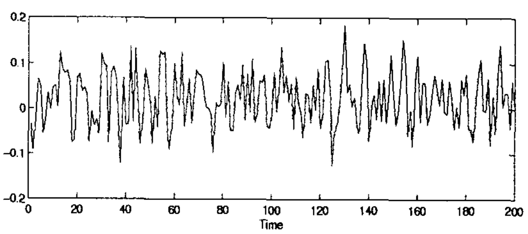

Fig. 3. Estimation error of the extended Kalman filter.

V. Conclusions

This paper has addressed the problem of robust H_∞ FIR filtering problem based on sampled measurements for a class of linear continuous-time systems subject to unknown initial state, real norm-bounded parameter uncertainty and unknown cone-bounded nonlinearity. Attention is focused on the simultaneous estimation of a continuous and discrete time-varying signal by using a performance measure which involves a mixed L_2/l_2 norm of the estimation error. Note that unlike the case where the nonlinearity $g(\cdot)$ and $k(\cdot)$ are unknown, the filter in the case where the nonlinearities are known Lipschitz nonlinearity is nonlinear filter [5]. We developed linear causal filter on the moving horizon and sampled measurements, which guarantees a prescribed H_∞ performance subject to unknown initial state, real norm-bounded parameter uncertainty and unknown cone-bounded nonlinearity.

References

[1] A. R. Bergen, "Power system analysis," *Prentice Hall, Englewood Cliffs*, NewJersey, U.S.A, 1986.

[2] C. E. de Souza, M. Fu, and L. Xie, "H_∞ analysis and synthesis of discrete-time systems with time-varying uncertainty," *IEEE Trans. Automatic Control*, vol. 38, pp. 459-462, 1993.

[3] C. E. de Souza, L. Xie, and Y. Wang, "H_∞ filtering for a class of uncertain nonlinear systems," *Systems and Control Letters*, vol. 20, pp. 419-429, 1993.

[4] C. E. de Souza and L. Xie, "Robust H_∞ filtering," *Control and Dynamic Systems*, vol. 65, pp. 323-377, 1994.

[5] H. S. Ryu, B. M. Kwon, and O. K. Kwon, "Robust nonlinear H_∞ FIR filtering for uncertain time-varying systems with Lipschitz nonlinearity," *Trans. Control, Automation and System Eng.*, vol. 2, pp. 255-261, 2000.

[6] O. K. Kwon, H. S. Ryu, and K. S. Yoo, "Robust H_∞ FIR filter for uncertain systems," *Proc. 13th IFAC World Congress*, vol. G, pp. 517-522, 1996.

[7] O. K. Kwon, C. E. de Souza, and H. S. Ryu, "Robust H_∞ FIR filter for discrete-time uncertain systems," *Proc. 35th CDC*, Kobe, Japan, 4819-4824, 1996.

[8] K. M. Nagpal and P. P. Kwargonekar, "Filtering and smoothing in an H_∞ setting," *IEEE Trans. Automatic Control*, vol. 36, pp. 152-166, 1991.

[9] U. Shaked and Y. Theodor, "H_∞-optimal estimation : a tutorial," *Proc. 31st IEEE CDC*, Arizona, U.S.A., 1992.

[10] U. Shaked and N. Bellman, "H_∞ nonlinear filtering of discrete-time processes," *IEEE Trans. Signal Processing*, vol. 43, pp. 2205-2209, 1995.

[11] P. Shi, L. Xie, and C. E. de Souza, "Output feedback control of sampled-data systems with parametric uncertainties," *Proc. 31st IEEE CDC*, Arizona, U.S.A., 1992.

[12] W. Sun, K. M. Nagpal, and P. P. Kwargonekar, "H_∞ control and filtering for sampled-data systems," *IEEE Trans. Automatic Control*, vol. 38, pp. 1162-1175, 1993.

[13] Y. Wang, L. Xie, and C. E. de Souza, "Robust control of class of uncertain systems," *System and Control Letters*, vol. 19, pp. 139-149, 1992.

[14] L. Xie, C. E. de Souza, and M. Fu, "H_∞ estimation for linear discrete-time uncertain systems," *Int. J. Robust and Nonlinear Control*, vol. 2, pp. 111-123, 1992.

[15] L. Xie, C. E. de Souza, and Y. Y. Wang, "Robust control of discrete time uncertain dynamical systems", *Automatica*, vol. 29, pp. 1133-1137, 1993.

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